European Centre
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A Comprehensive Radiation Scheme Designed for Fast Computation

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A COMPREHENSIVE RADIATION SCHEME DESIGNED FOR FAST COMPUTATION

by

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N O T E :

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1. Introduction

From the point of view of radiation the needs of a General Circulation Model (GCM) and of a Medium-Range Forecasting Model (MFM) are not the same. Let us take an example: there are two important equilibria that a radiation scheme applied to the real state of the atmosphere should represent. First, the energy balance at the top of the atmosphere (for the whole earth and a sufficiently long time: averaged net flux equal zero at $p = 0$). Secondly, the radiative equilibrium in the parts of the atmosphere, especially the stratosphere, where the radiation is the only important diabatic phenomenon (for a sufficiently long time: averaged net flux constant with height).

For a GCM only the second condition is important: if we have an equilibrium with a net flux constant but not equal to zero, the ocean provides the infinite source or sink of energy which enables the atmosphere to remain in the right state of temperature.

For a MFM the situation is the reverse: at a given moment of its history the atmosphere is not in equilibrium but tends to come back to it, and the speed of this change, which we shall represent in a forecast, is governed by the internal energy, controlled by the flux balance at the top of the atmosphere.

Therefore, and also to allow an immediate effect for a maximum of feed back processes, the accent is put, in the scheme described here, on the fluxes and on overall effects without overrating of the input information rather than on careful computation of some local phenomena with neglect of their interaction.

However, the basic formulation of the problem is the same as in GCM's and we have a similar task of balancing accuracy and time consumption. Thus we have to apply many of the methods already used in GCM radiation codes to simplify the treatment of the basic equations (see for example MANABE and STRICKLER (1964), KATAYAMA (1974), TIEDTKE and GELEYN (1976) or the comparison of these three schemes in the Research Paper 1/16/E/RD4/019/1976).

After describing the radiative transfer equation we shall explain in this paper the few simple hypotheses whose application to this equation allows a fast computation of the radiative fluxes without any arbitrary suppression of phenomena. Then we shall show some results which justify our choices and some comparisons with another radiation code.

2. The monochromatic equation of radiative transfer

At a given frequency ν we can write

$$\mu . \frac{\partial I_\nu(t_\nu,\mu,\phi)}{\partial t_\nu} = I_\nu(t_\nu,\mu,\phi) - \frac{1-k_\nu(t_\nu)}{4\pi} . \Big[S_{o\nu} . e^{-t_\nu/\mu_o} . \tag{1}$$

$$P_\nu(t_\nu,(\mu,\phi,-\mu_o,\phi_o)) + \int_o^{2\pi}\int_{-1}^{+1} P_\nu(t_\nu,(\mu,\phi,\mu',\phi')).$$

$$I_\nu(t_\nu,\mu',\phi').d\mu'.d\phi' \Big] - k_\nu(t_\nu).B_\nu(T(t_\nu))$$

with I_ν : intensity of the diffuse radiation

$S_{o\nu}$: intensity of the solar parallel radiation at the top of the atmosphere. The multiplication by e^{-t_ν/μ_o} gives us the local intensity as solution of the equation for parallel radiation: $-\mu_o . \frac{dS_\nu(t_\nu)}{dt_\nu} = S_\nu(t_\nu)$

t_ν : optical thickness of the atmosphere used as vertical coordinate : 0 at the top and increasing downward

μ : cosine of the angle between the direction of radiation and the upward directed vertical

ϕ : azimuth angle of the direction of radiation

$-\mu_o, \phi_o$: μ and ϕ for the solar parallel radiation

k_ν : ratio of absorption to extinction (absorption plus scattering)

P_ν : scattering phase function normalised to the mean value 1 and with axial symmetry:

$$P(\mu,\phi,\mu'\phi') = P(\mu.\mu' + \sqrt{(1-\mu^2).(1-\mu'^2)}.\cos(\mu-\mu')) = P(\cos\theta)$$

B_ν : Planck function

T : Temperature

The four terms on the right hand side of (1) represent respectively

- The loss of energy by absorption and scattering
- The gain of energy coming from the scattered solar parallel radiation
- The gain of energy coming from the scattering of radiation from other directions
- The gain of energy through thermal emission

Supposing we know the vertical profiles of t_ν, T, k_ν and $P_\nu(\cos\theta)$ and the boundary condition for I_ν at the bottom of the atmosphere (at the top $S_{o\nu}\mu_o\phi_o$ are only dependent on geography and time) we can compute I_ν everywhere and therefore the spectral upward net flux is

$$F_\nu(t_\nu) = \int_o^{2\pi}\int_{-1}^{+1} I_\nu(t_\nu,\mu,\phi).\mu.d\mu.d\phi - \mu_o.S_{\nu o}.e^{-t_\nu/\mu_o} \quad (2)$$

A spectral integration allows us to compute our final goal: the net upward radiation flux

$$F(p) = \int_o^{\infty} F_\nu(t_\nu(p))d\nu \quad (3)$$

with p pressure as vertical coordinate

The signification of all the previous equations and definitions is more or less intuitive. For more complete information see CHANDRASEKHAR (1950).

The exact solution of the radiative transfer equation involves three types of integration: over angles, over the vertical coordinate and over the wave length spectrum. We shall describe here successively the way of solving the three problems arising from these integrations.

3. The spectral integration problem

We first suppose that we can separate the whole spectrum into two intervals: the long waves where we put $S_{o\nu} = 0$ and the short waves with $B_\nu(T) \equiv 0$. Because of the difference between the radiative temperatures of the sun (5750°K) and of the earth (254°K) this simplification is very reliable.

Let us then suppose that we are able to do the necessary computations to solve the monochromatic problem in both spectral ranges. In order to avoid a great number of such computations at different frequencies we have to find how to determine and use t, k and P(cosθ) representative for wide parts of the spectrum.

The solution of the monochromatic equation being of negative exponential type (as we shall see later on), the main problem comes from the highly non-linear nature of the exponential operator: (as $e^{-\frac{a+b}{2}} \neq \frac{e^{-a}+e^{-b}}{2}$ a strong and a weak extinction cannot be combined in an intermediate one): the use of spectrally averaged coefficients of absorption and scattering is only valid when the real coefficients have the same order of magnitude throughout the considered spectral interval.

We suppose that this is the case for cloud-aerosols absorption and scattering (less than one order of magnitude variation for the extinction coefficients) and Rayleigh scattering (only present in short waves) in a small number of domains (3 for long waves, 2 for short waves) and we have grey effects except for gaseous absorption in these intervals.

The experimental data were taken from ZDUNKOWSKI, KORB and NIELSEN (1967).

The aerosols are included in the scheme more to give a possibility of fitting the results of the model and to smooth the transition between cloudy and non-cloudy conditions than to represent the poorly known effect of natural aerosols (the dynamical model does not give their geographical distribution and their optical properties are uncertain).

There is first a dry effect proportional to the quantity of aerosols (given climatologically and idealised) with a constant absorption coefficient throughout the whole spectrum and a scattering coefficient increasing the Rayleigh effect and modifying its phase function in short waves. Furthermore, outside of the clouds, we assume empirically an adsorption of water proportional to the quantity of aerosols and to $U/(1-U)$ (U being the relative humidity). The optical properties of this smog are the same as those of clouds. We hope that the use of averaged coefficients for cloud and aerosols will not create a bigger error than the error caused by the poor knowledge that we have from their optical properties themselves.

The Rayleigh effect, although highly non-linear (coefficients proportional to ν^4) is sufficiently small so that we can choose empirical coefficients for which the effect of the first scattering is well parameterised (by taking into account the zenith angle of the sun) without having important errors for the subsequent scatterings.

An extra difficulty arises in the case of gaseous absorption, the coefficients depending strongly on temperature and pressure. (There is also a temperature dependence for the other effects in the long wave domains, due to the change of shape of the Planck function with temperature. However, it can be well parameterized by a linear dependence of the spectral coefficients on the inverse of temperature). For the gases, their line-type absorption spectrum obviously makes the averaging of coefficients hopeless, since strong absorption and no absorption at all are present together in the same parts of the spectrum.

We must therefore use empirical transmission functions for the gases. The theory of gaseous absorption (see for example GOODY (1964)) shows that these transmission functions can be expressed over some spectral intervals as the product of the individual transmission functions for the different gases; but we can group the effect of all the gases which have a constant mixing ratio throughout the atmosphere as if it would be the effect of the most important of them: CO_2. So we have only three gases to consider: water vapour, ozone and carbon dioxide.

Furthermore, we shall use for each of them the two parameters scaling approximation (Curtis Godson approximation): the transmission function is expressed in terms of the unreduced amount of absorber $u = \int r dp$ (r being the mixing ratio of the gas) and of the reduced amount $u_r = \int rp/\sqrt{T}\, dp$. (The dimensions of u and u_r does not matter; the product with absorption coefficients has only to be dimensionless).

For a narrow spectral range one can compute the transmission τ from

$$- \ln \tau = \frac{au}{\sqrt{1+bu^2/u_r}} + cu_r \qquad (4)$$

The term cu_r represents the absorption of the continuum; for weak absorption the first term on the right hand side of (4) becomes au and for strong absorption $a\sqrt{u_r/b}$. These two formulations are the ones given by the theory of band absorption. The coefficients a, b and c depend on temperature.

By analogy to this form we choose an empirical transmission for the five spectral domains or for sub-intervals of them as

$$\tau = \prod_{H_2O,\ CO_2,\ O_3} \left(\frac{1}{1+\frac{au}{\sqrt{1+bu^2/u_r}} + cu_r} \right) = \tau_T(u_{H_2O}, u_{rH_2O}, u_{CO_2}, u_{rCO_2}, u_{O_3}, u_{rO_3})$$

a, b, c depending linearly on $1/T$ are fitted to experimental data.

But to use these transmission functions we need to know the encountered unreduced and reduced amounts of H_2O, CO_2 and O_3 along the different radiation paths. We can reduce our search to the evaluation of the mean value for each of these 6 amounts and introduce them in the transmission functions. Since these are still non linear we make there an error but a smaller one than by averaging the coefficients a priori.

Let us see in detail this evaluation in the case of short wave radiation. We first make a monochromatic computation without any gaseous absorption, the resulting flux at the reference level being F_0; the way this result is obtained will be shown in part 4. F_0 can represent either the solar parallel flux or the upward or the downward diffuse flux. Now we add each gas (H_2O, CO_2 and O_3) in both reduced and unreduced amounts (6 cases) with an arbitrary but very small absorption coefficient k_i; the result is F_i. We can say that the mean encountered amount of this absorber type u_i is given by $u_i = \frac{1}{K_i} \cdot \frac{Fo-Fi}{F_i}$ (5)

(From $F_i = F_o e^{-k_i u_i} \simeq F_o/(1 + k_i u_i)$). Finally we compute the real flux with $F = F_o \cdot \tau(u_i, i = 1,6)$.

In the long wave part of the spectrum the problem is more complicated. There is not a single external source but every absorption is accompanied by an equivalent emission depending on temperature through the Planck function. Hence to evaluate the amounts of absorbers we have to compare runs with and without gases in an isothermal case (only the optical thickness matters, not the Planck function which is B* throughout the atmosphere). We get the fluxes (see part 4 again) $F_o{}^*$ and $F_i{}^*$ (either upward or downward diffuse fluxes)

u^*_i is given by $u^*_i = \frac{1}{K_i} \cdot \frac{F_o{}^* - F_i{}^*}{F_i{}^* - \pi B^*}$ (6)

$$\left(\text{From } F_i{}^* - \pi B^* = (F_o{}^* - \pi B^*) \cdot e^{-k_i u_i} \simeq \frac{F_o{}^* - \pi B^*}{1 + k_i u_i}\right)$$

and we get $\tau^* (u^*_i, i = 1,6)$

The ratio $F^*_o/\pi B^*$ is the emissivity ε^* in the isothermal case without absorber.

We need then to compute the flux F_o in the case without absorber but with the actual temperature state of the atmosphere. There B_r is the Planck function in the reference level.

Finally we compute the real flux F by making an analogy to the short waves. We had there F as the result of the transmission without scattering of F_o through a layer of transmissivity τ. Here F is the result of the transmission through a layer of transmissivity $\tau = \tau^*$ with, at the origin, a flux F_o provided by an emission with emissivity $\varepsilon = \varepsilon^*$. For the computation we suppose, as we shall do in each case, that the Planck function varies linearly with the optical thickness taken as vertical coordinate: $t = -\ln\tau^*$. We obtain

$$F = \pi B_r + (F_o - F_o/\varepsilon^*) \cdot \tau^* + (F_o/\varepsilon^* - \pi B_r) \cdot (\tau^* - 1)/\ln\tau^* \quad (7)$$

when we integrate the simplified version of (1).

$$\frac{dF}{dt} = F - \pi B = F - \pi(B_o + B't) \quad (8)$$

in which the scattering effects have been suppressed and the intensities replaced by the fluxes (explanation in Appendix A).

4. The vertical integration problem

Following the principle expressed at the beginning we do not try to extract more information from the input parameters than they can give us. Therefore, we suppose that each layer is a vertically homogeneous absorbing and scattering medium and as an interpolation assumption that in each spectral interval of the long wave domain the Planck functions vary linearly with the optical thicknesses through every layer.

We make now the so-called Two-stream Eddington approximation. At each level both upward and downward diffuse radiation fields are hemispherically isotropic (I depends only on the sign of μ).

We can now compute a matrix solution of the radiative transfer equation for each layer: the outgoing fluxes depend linearly on the incoming ones and (in the long wave domain) on the black body fluxes at the boundaries.

$$\begin{bmatrix} S_b \\ F_{1t} \\ F_{2b} \end{bmatrix} = \begin{bmatrix} a_1 & o & o \\ a_2 & a_4 & a_6 \\ a_3 & a_5 & a_7 \end{bmatrix} \cdot \begin{bmatrix} S_t \\ F_{1b} \\ F_{2t} \end{bmatrix} \qquad (9)$$

t for top b for bottom

S solar parallel flux

F_1 solar upward diffuse flux

F_2 " downward " "

and

$$\begin{bmatrix} F_{1t} \\ F_{2b} \end{bmatrix} = \begin{bmatrix} b_1 & b_3 \\ b_2 & b_4 \end{bmatrix} \cdot \begin{bmatrix} F_{1b} \\ F_{2t} \end{bmatrix} + \begin{bmatrix} b_5 & b_7 \\ b_6 & b_8 \end{bmatrix} \cdot \begin{bmatrix} \pi B_b \\ \pi B_t \end{bmatrix} \qquad (10)$$

F_1 thermal upward flux

F_2 " downward "

See appendix A for the computation of the a and b coefficients:

The b coefficients are functions of
- the optical thickness of the layer Δt
- the ratio absorption/extinction k
- an integral factor of the phase function:

$$A_1 = \int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_o^{1} \frac{\underline{P}(\underline{\mu},\underline{\phi},\underline{\mu'},\underline{\phi'})}{8\pi^2} d\mu d\phi d\mu' d\phi'$$

The a coefficients are functions of the same parameters and of

- the cosine of the solar zenith angle μ_o
- a second integral factor of the phase function depending on μ_o:

$$A_3(\mu_o) = \int_o^{2\pi}\int_o^1 \frac{P(\mu,\phi,-\mu_o,\phi_o)}{4\pi}\, d\mu d\phi$$

To compute the mean encountered amounts of gaseous absorbers we should recompute 6 times the a and b coefficients. Since the changes in optical thicknesses $k_i . u_i$ are arbitrarily small we can avoid this amount of new computations if we take analytically the derivative a' and b' of the coefficients a and b with respect to the absorption optical thickness $(k\Delta t)$ under the condition $(1-k)\Delta t$ constant.

The new a and b are given by $a + a'k_i\ u_i$ and $b + b'\ k_i\ u_i$

See appendix B for the computations of the a' and b' coefficients.

Each layer is thus characterised in each spectral interval by 30 coefficients.

A little supplementary treatment is needed when we have a cloudy layer with partial coverage. We compute the a and b coefficients for both cloudy and clear parts. Then we distinguish two cases. If the layer is alone between two clear sky layers, we simply do a linear combination of the coefficients with the amount of cloudiness and its complement to 1 as weights.

If there are several adjacent cloudy layers, building a so-called "cloud" we compute the coefficients which, if the layers were homogeneous would give the same results as those obtained in the following way: we suppose that the overlapping of the adjacent cloudy parts is maximal, and so we have n + 1 vertical distributions of cloudy and non-cloudy parts (n number of layer in the cloud); we compute the results inside the "cloud" for each combination for arbitrary incoming fluxes and finally combine linearly the results with the weights given by the geometry of the "cloud" and eliminate the arbitrary incoming fluxes from the equations. (See Appendix C)

When in each spectral interval, for each case (with and without gaseous absorption, isotherm or not) we have the a and b coefficients of each layer, we can compute all the fluxes through the atmosphere as resulting from a linear system. (An example can be seen in Appendix C).

For this we only need the boundary conditions which are for short wave fluxes:

. S_∞ given by astronomical considerations

. $F_{2\infty} = 0$

. $F_{1z=o} = A\ell \; F_{2z=o} + A'\ell(\mu o)S_{z=o}$

$A\ell$ and $A'\ell$ ground albedos for diffuse and parallel radiation

and for long wave fluxes:

. $F_{2\infty} = 0$

. $F_{1z=o} = \varepsilon\pi B_{z=o} + (1-\varepsilon)F_{2z=o}$

ε emissivity of the ground.

(We suppose for simplicity that $A\ell$, $A'\ell$ and ε are the same in the different spectral intervals).

5. The angle integration problem

As seen before, we make the hypothesis of hemispheric isotropy. Therefore we need a magnification factor for diffuse fluxes which multiplies the quantities of absorbing and scattering media computed for a vertical beam. For all effects except gaseous absorption we take this factor equal to 2. This value is the one for small effects as seen in Appendix A. We choose it because the involved effects are either small (outside of clouds) or strong (in the clouds) and then the fluxes do not depend on the quantities of acting media any longer. For gaseous absorption we have two different factors: 2 for the unreduced amount of gases and 25/16 for the reduced amounts. This later value is the one we obtain as a limit for transmission zero in the form $\tau = 1/(1+a\sqrt{u_r/b})$ (see part 3).

Thus our magnification factor diminishes with increasing absorption as it is the case in nature. The usually accepted value of 5/3 falls between our two values.

As all our computation of a and b coefficients are done with the magnification factor 2 we need to correct the quantities $k_i.u_i$ for reduced amounts of absorbers. We multiply them by a factor 25/32 for the linear computation of the coefficients $a_{4...7}$ and $b_{1...8}$. For the coefficient a_1 we have no modification to do since it concerns a parallel beam. For a_2

and a_3 we assume (only for this purpose) that there is only a single scattering taking place in the middle of the layer. On the way in we have a path length proportional to $1/\mu_o$ and a multiplying factor 1. On the way out the path length is proportional to 2 and the factor is 25/32. So our final factor is

$$\left(\frac{1}{\mu_o} \cdot (1) + 2 \cdot \left(\frac{25}{32}\right)\right) / \left(\frac{1}{\mu_o} + 2\right)$$

6. First results of the model

It should first be noticed that the empirical transmission functions for gases used here are not yet definitive (there is no division in sub-intervals and we have still to introduce the self broadening effects)but they already give a good idea of the possibilities of the scheme. For the basic data which help us to determine these functions we use McCLATCHEY et al (1973) and VIGROUX (1953).

There are two determinant assumptions in the model - the direct use of a multiple scattering method instead of an emissivity type (with mathematical separation of scattering and absorption as, for example, in the GFDL radiation scheme).

- the simplification in the long waves $\varepsilon = \varepsilon^*$ $\tau = \tau^*$ which is in a certain sense the equivalent in our formalism of the so-called "cooling to boundaries" approximation. (In the latter approximation, for the computation of the fluxes at a given reference level one assumes that the atmosphere is isothermal with the temperature of the reference level).

We will try here to justify these two choices by showing the influence they have on the results of the model for a great amount of possible atmospheric configurations. We apply our model to a set of 142 atmospheres (with 15 layers) whose characteristics of temperature, humidity and cloud coverage are randomly distributed around reasonable profiles (see Appendix D) with the help of the random number generator of the computer. The distribution of the solar zenith angle is also random between -1 and +1. The values for ozone and carbon dioxide do not vary and are taken from observations (McCLATCHEY et al (1972)). The number of 142 is the one for which the averaged results have the best flux balance at the top of the atmosphere (net flux as small as possible) for a computing time less than 1 minute.

It is interesting to note that this is accompanied by a good cooling-heating balance at p = o too. This can be seen on Fig. 1. In this figure we have computed the results of cooling-heating rates of the model (full lines) and of a modified version in which there is no more scattering of the diffuse radiation (A1 ≡ 1 in Appendix A), for long wave and short wave separately and for their sum. At the top and the bottom the net fluxes (in W/m^2) are indicated.

One can see that neglecting the multiple scattering leads to errors of the order of 25% for the divergences and of 50% for the fluxes (relative to the values of long wave or short wave fluxes before they cancel by summation). Although the errors are larger for short waves than for long waves, these latter are still important, particularly in the middle of the atmosphere where both differences are additive, whereas in the boundary layer (with more long wave cooling of the last layers without reflection of "warm" radiation from the upper levels) and in the stratosphere (with the short wave absorption by ozone of multiple-scattered radiation on its way back to space) they tend to cancel each other.

Considering only the results of the model for the net radiation we can see an important cooling in the boundary layer, an almost constant cooling rate throughout the troposphere, another increase of cooling (it will create the tropopause which does not exist in our data) at the bottom of the stratosphere, and finally, as already pointed out, an equilibrium at the top.

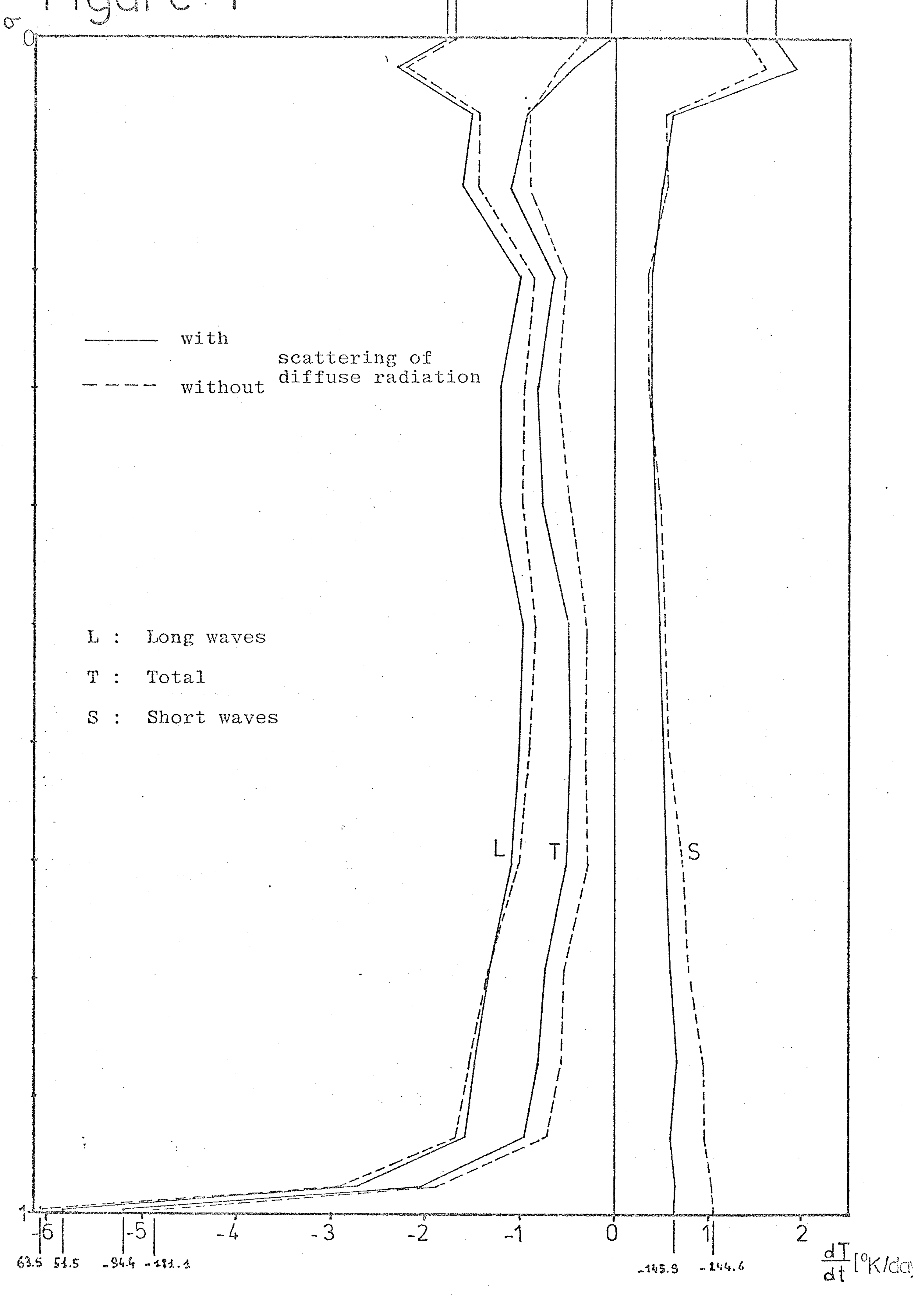
However, at $p = 0$ both long wave and short wave fluxes are too low (right value ≈ 237). This is probably due to the absence of a positive lapse rate of temperature in the stratosphere and to a too high liquid water content of the clouds.

For the second point mentioned above, there is no possibility in the framework of the code to see what would be the results with temperature dependent emissivity and transmissivity. However, it is possible to compute the changes in the fluxes for small k_i coefficients in the real temperature state of the atmosphere as we have done for the isothermal case (with the use of a' and b' coefficients). Therefore, we can compare these tendencies with the one predicted by the model when

$$\tau^* = \frac{1}{1 + k_i \, u_i} \quad \text{in equation (7).}$$

For the same set of 142 random atmospheres the correlation coefficient between the two sets of results is 0.838. This result might be improved by using real data where the temperature profiles are probably more regular than the one we get from the random generation. However, it seems already worth doing this hypothesis considering the amount of integral computation which is saved (instead of varying with the square of the number of levels, the amount of computation for long wave fluxes varies with this number itself).

Figure 1



Fluxes
216.4
216.5
-103.8
0.7
-324.7
-245.7
(W/m²)
σ
0
1
with
without
scattering of diffuse radiation
L : Long waves
T : Total
S : Short waves
L
T
S
-6
-5
-4
-3
-2
-1
0
1
2
63.5
51.5
-94.4
-181.1
-145.9
-144.6
dT/dt [°K/day]

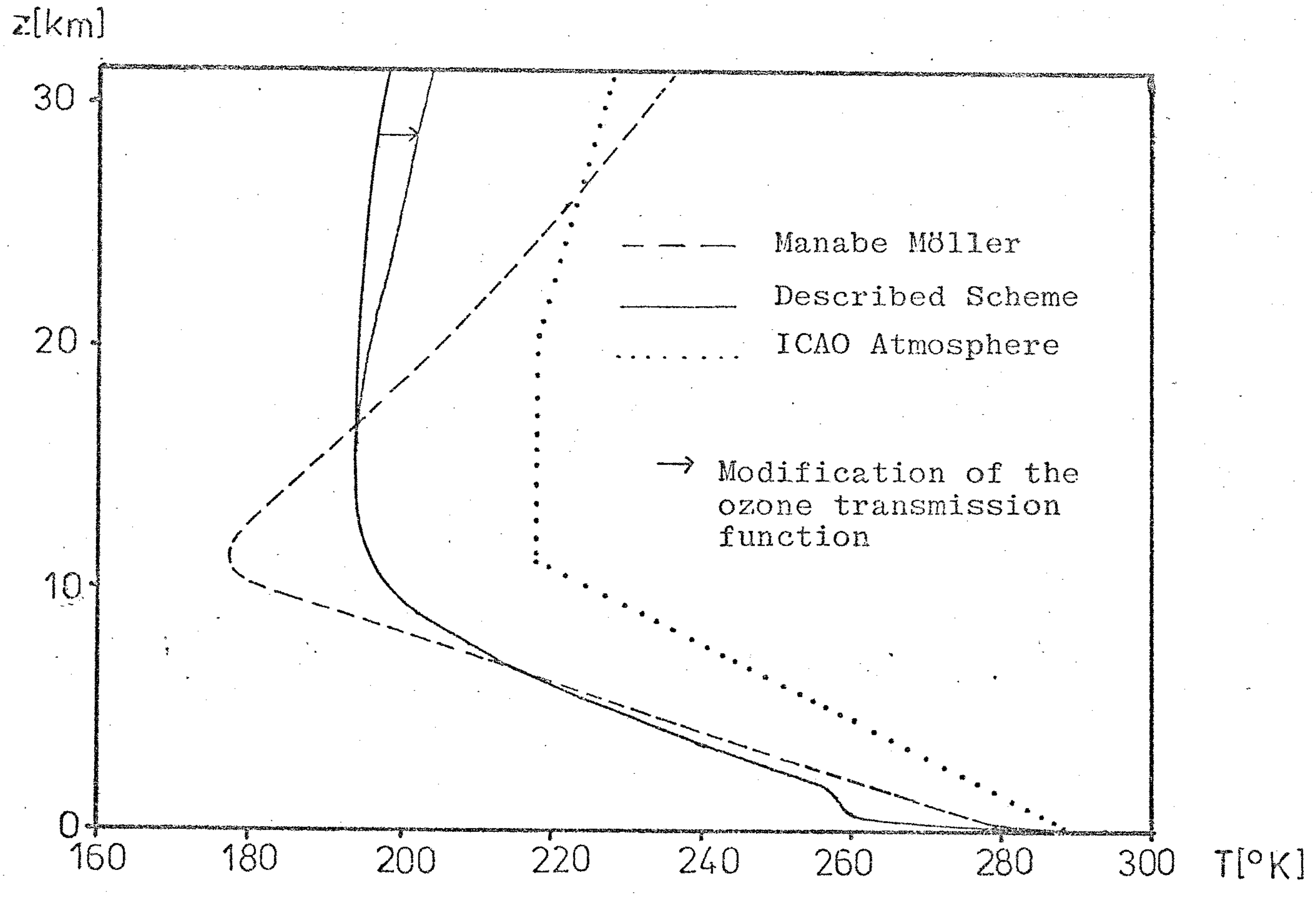
7. Comparison with the Manabe Möller experiment

In order to compare our scheme with that used in the GFDL model, we recalculated the MANABE-MÖLLER (1961) experiment on radiative equilibrium. The results are shown on figures 2, 3 and 4, corresponding to figures 12, 14 and 15 of the original paper. There are two input elements in our scheme to which the results are quite sensitive and which are unknown to the M.M. model: the emissivity of the soil and the saturation humidity (for the aerosols). We took arbitrarily the second from the standard atmospheric temperatures and the first equal to 0.99.

The scheme used here is not exactly the one described in the paper, since we had to suppress the dependence of the coefficients on temperature: for very low temperatures, some of them become negative and even if we set them to zero, this creates a computational instability.

On Figure 2 we can see that both schemes agree well in the troposphere, but that the stratospheric results are totally different. Furthermore, our scheme shows a very strong boundary effect (which in some extreme cases can lead to an inversion). However, this is a consequence of the strong boundary cooling already noticed in Figure 1 and which is an observable feature (see for example GAMP and

Figure 2



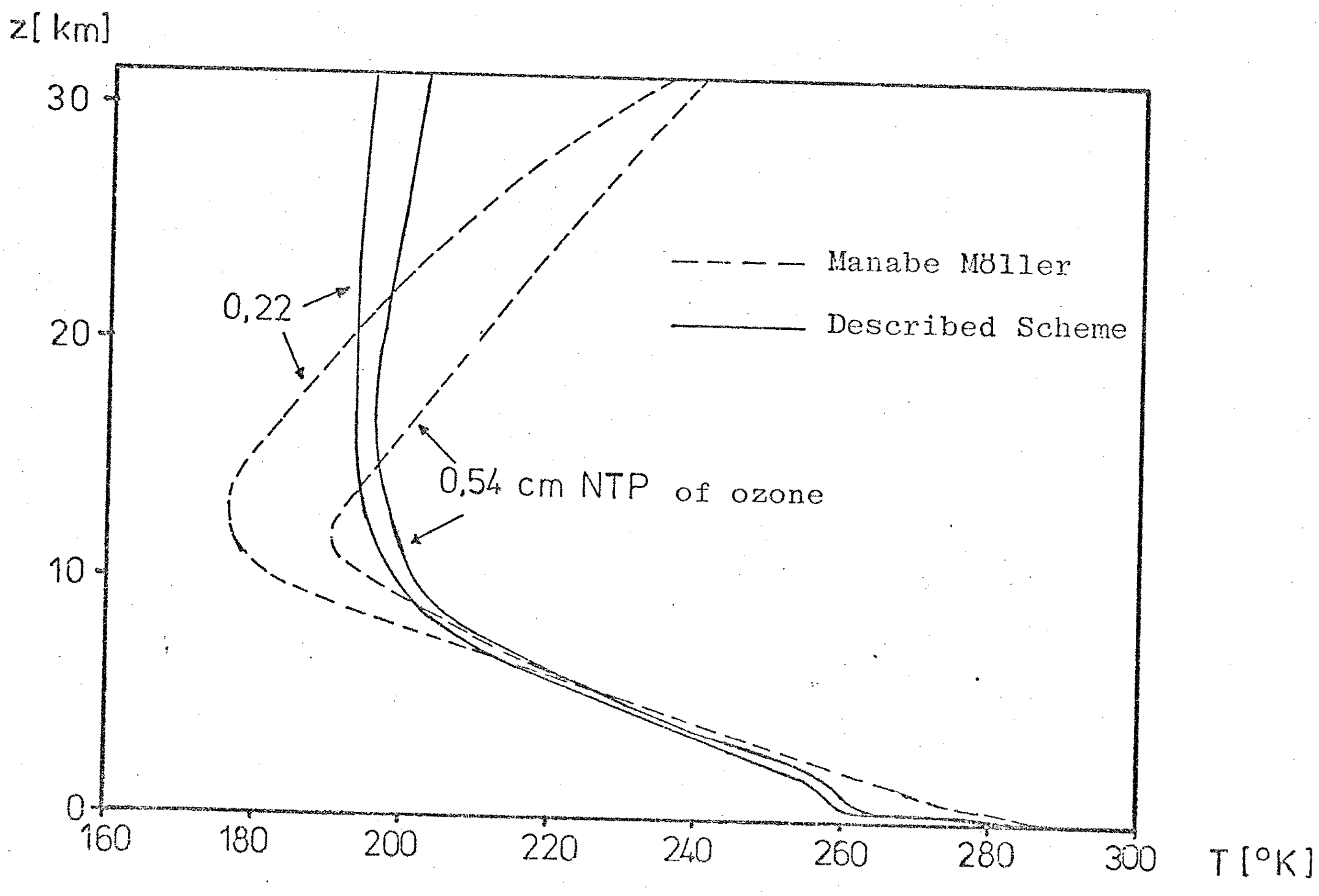
HEINRICH (1976). Thus we can already say that, if our scheme is to be used in a dynamical-physical model, this latter must have a treatment of the boundary layer, including the effect of stability in order to avoid the creation of too strongly unstable temperature gradients.

In Figures 3 and 4 we investigate the effect of a change only in the absorber quantities (3: ozone - 4: water vapour without change of the relative humidity) on the equilibrium conditions. The same remarks as for Figure 1 apply, but our scheme is more sensitive to water vapour and less sensitive to ozone than the M.M. one. The most important thing to notice is that the effect of the changes is more local in the M.M. case and spread throughout the atmosphere in our case. This will lead to a stronger computational stability in our scheme which is already proven by the fact that our critical time step for the computation of radiative equilibrium is about 8 to 16 times larger than the 12 hour time step given by M.M.

This difference of behaviour of the two schemes probably lies in their basic conceptions: - M.M.'s computes exactly what happens for a unique and a priori idealised photon path

- our scheme takes into account all photon paths but only

Figure 3



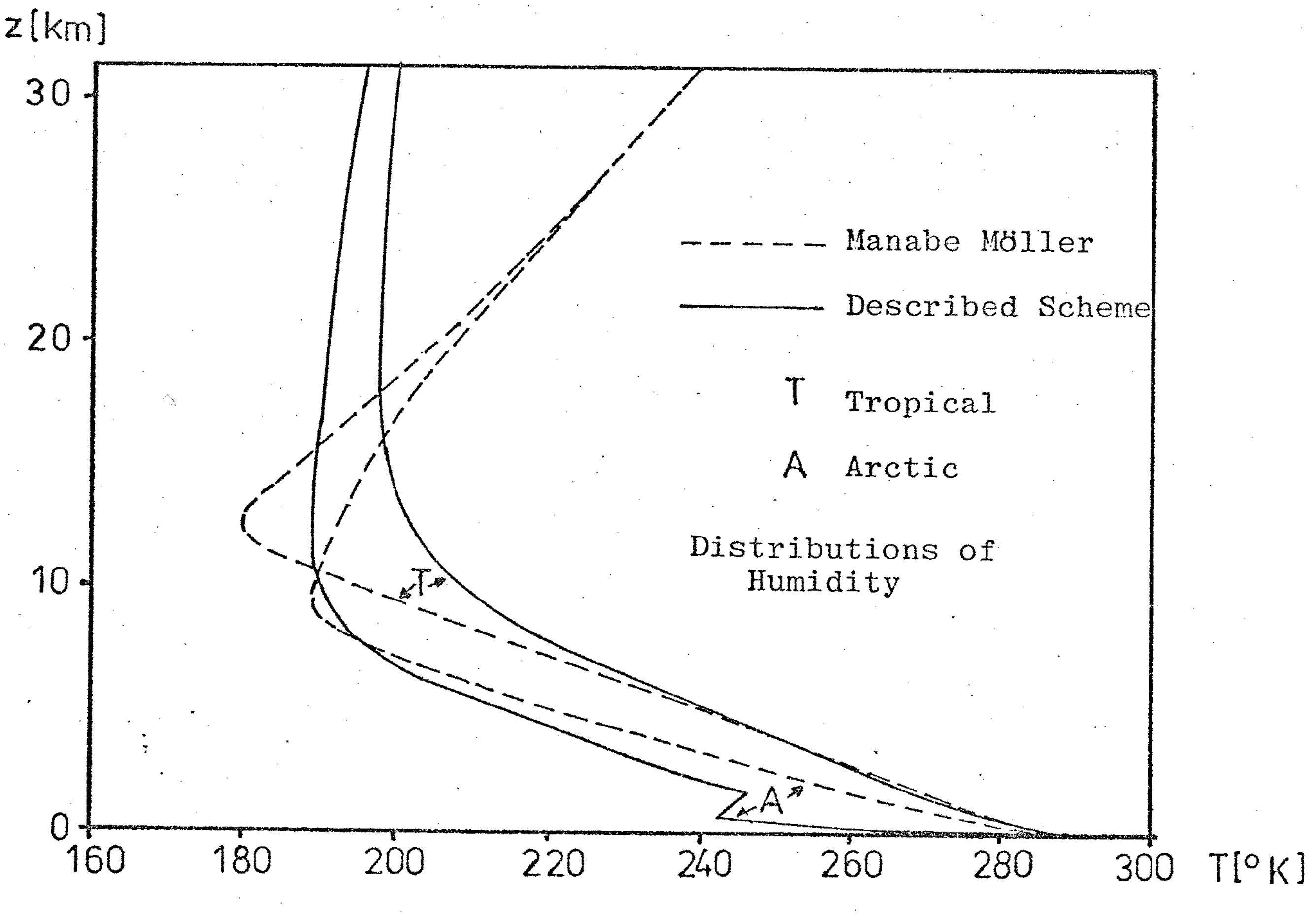
makes computation on averaged properties. Therefore every input parameter has an influence on every flux but strong local effects are somewhat smoothed by the averaging process.

The M.M. results have stratospheric values of temperature closer to the observed ones (represented in Figure 2 by the ICAO Standard Atmosphere), but our model has better lapse rates. Which of the two solutions is the more realistic is difficult to say, since the other physical effects will change the conditions of the equilibrium.

We can explain the discrepancies in the stratosphere with three reasons :

- our model does not separate the effects of the ultra violet and visible absorption bands of O_3 (it will later) and therefore our heating rates are too low above 25km. and too strong below. A test with a transmission function of a different type but taking into account both bands gives us an evaluation of the error (Figure 2).

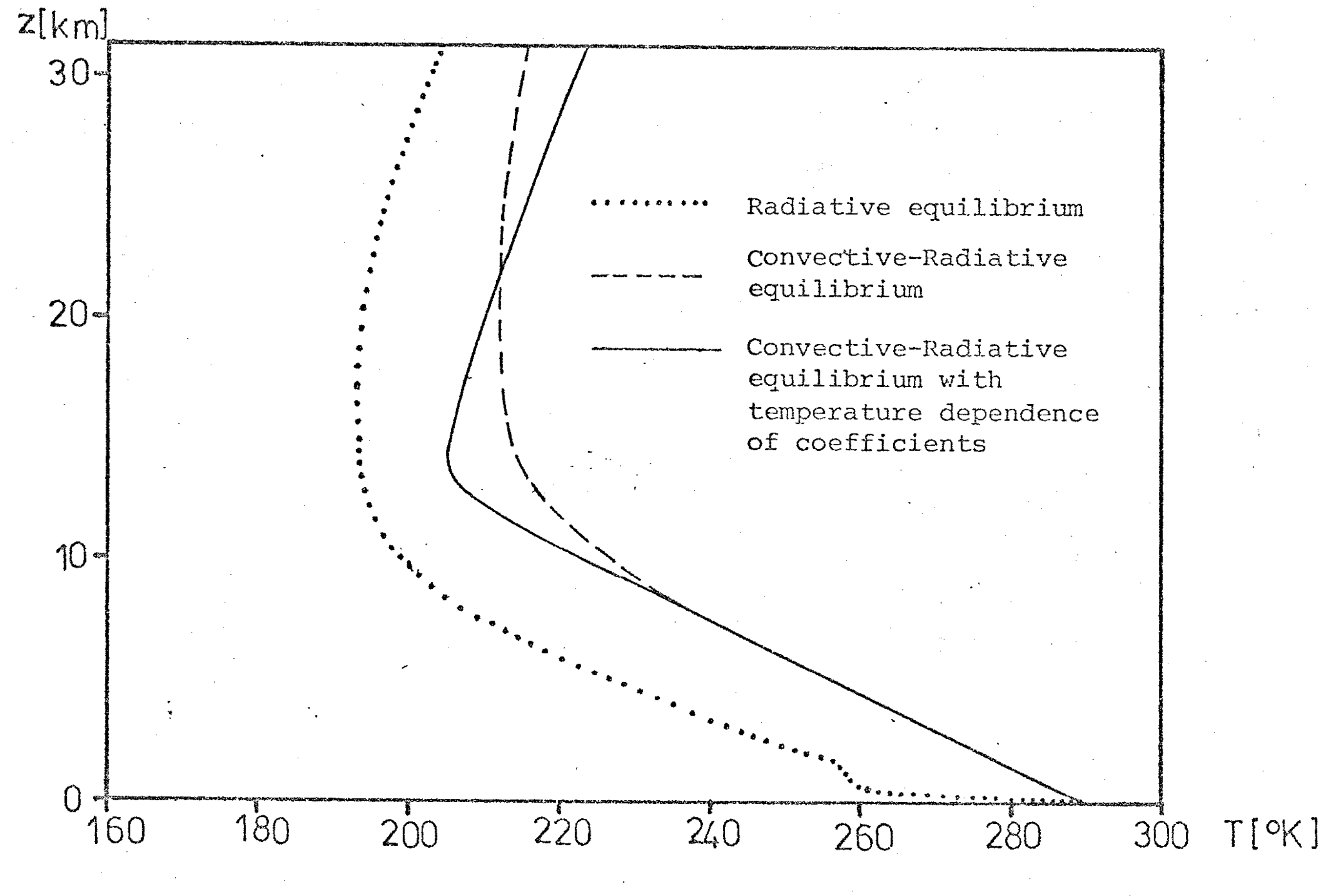
Figure 4



- The GFDL scheme has an upper boundary condition dF/dp = 0, which we cannot introduce in our computation since our temperatures are not at the same levels as theirs. (This feature also creates for us numerical problems for the computation of a convective-radiative equilibrium with a free soil temperature and this explains why we have to limit our present comparison to the first of the GFDL papers on equilibrium temperatures). In any case, in MANABE and STRICKLER (1964) with new transmission functions the stratospheric temperature and lapse rate are reduced in a slightly different experiment (free soil temperature as only change of conditions: Figure 1 of the M.S. paper).

- As already seen, the temperature effects are less local in our computations than in M.M.'s. This can also be seen if we compute a convective-radiative equilibrium with fixed soil temperature. The effect will not be a convective lapse rate extended until it reaches an unmodified radiative equilibrium, but rather a displacement of the whole stratospheric profile (See Figure 5).

Figure 5



With these higher temperatures, however, we can return to our original model by reintroducing the dependence of coefficients on temperature. We then get a better stratospheric lapse rate and, for the first time, a well defined tropopause (See Figure 5 again in which all three curves are computed with the modified ozone transmission function).

8. Conclusion

The scheme described here produces reasonable results (not only as shown for an average of different situations, but also for each of them) and allows the treatment of all sorts of feed-back mechanism between radiation and other meteorological processes without too high computational cost (9 runs per second for 15 levels on a CDC 6600 compared to 25 runs per second for 9 levels for the GFDL scheme; it is impossible to give too much comparative signification to these figures since our scheme is less sensitive to the number of levels than the other (See 6)). The use of the multiple scattering method and the suppression of vertical integrations seems to provide good numerical stability. However, the great number of degrees of freedom could perhaps lead to physical instability if some feed-back between radiation and dynamic are not completely included in the model as mentioned for example in 7 for the boundary level.

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APPENDIX A

Calculation of the a and b coefficients introduced in 4

We consider one layer in which the optical thickness t varies from 0 at the top to T at the bottom, k and P (cosθ) are constant and B is linear in t.

We write the equation (1) in the form

$$\frac{\partial I(t,\mu,\phi)}{\partial t} = \frac{1}{\mu}\left[I(t,\mu,\phi) - \frac{1-k}{4\pi}\left\{I_o.P(\mu,\phi,-\mu_o,\phi_o)\right.\right.$$

$$\left.\left. + \int_o^{2\pi}\int_{-1}^{+1} P(\mu,\phi,\mu',\phi').I(t,\mu',\phi').d\mu'.d\phi'\right\} - k.B(t)\right]$$

We suppose the radiation field hemispherically isotropic with I_1 (upward) and I_2 (downward) and we apply the operator $\int_o^{2\pi}\int_o^{+1} \mu.d\mu.d\phi$ to our equation:

$$\frac{dI_1(t)}{dt}.\int_o^{2\pi}\int_o^{1} \mu.d\mu.d\phi = I_1(t).\int_o^{2\pi}\int_o^{1} d\mu.d\phi - \frac{1-k}{4\pi}.I_o.$$

$$\int_o^{2\pi}\int_o^{1} P(\mu,\phi,-\mu_o,\phi_o).d\mu.d\phi - \frac{1-k}{4\pi}.I_1(t).\int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_o^{1} P(\mu,\phi,\mu',\phi').$$

$$d\mu'.d\phi'.d\mu.d\phi - \frac{1-k}{4\pi}.I_2(t).\int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_{-1}^{o} P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi$$

$$- k.B(t).\int_o^{2\pi}\int_o^{1} d\mu d\phi$$

But we have $F_1 = \int_o^{2\pi}\int_o^{1} I_1.\mu.d\mu.d\phi = \pi I_1$ $\quad F_2 = \pi I_2$

Therefore

$$\frac{dF_1(t)}{dt} = F_1(t).\left[2 - \frac{1-k}{4\pi^2}.\int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_o^{1} P(\mu,\phi,\mu',\phi').\right.$$

$$d\mu'.d\phi'.d\mu.d\phi\Big] - F_2(t).\left[\frac{1-k}{4\pi^2}.\int_0^{2\pi}\int_0^1\int_0^{2\pi}\int_{-1}^{0} P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi\right]$$

$$- I_o(t).\left[\frac{1-k}{4\pi}.\int_0^{2\pi}\int_0^1 P(\mu,\phi,-\mu_o,\phi_o)d\mu.d\phi\right] - 2k.\pi B(t)$$

Similarly with the operator $\int_0^{2\pi}\int_{-1}^{0} \mu.d\mu.d\phi$

$$\frac{dF_2(t)}{dt} = F_1(t).\left[\frac{1-k}{4\pi^2}.\int_0^{2\pi}\int_{-1}^{0}\int_0^{2\pi}\int_0^1 P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi\right]$$

$$-F_2(t).\left[2 - \frac{1-k}{4\pi^2}.\int_0^{2\pi}\int_{-1}^{0}\int_0^{2\pi}\int_{-1}^{0} P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi\right]$$

$$+ I_o(t)\left[\frac{1-k}{4\pi}\int_0^{2\pi}\int_{-1}^{0} P(\mu,\phi,-\mu_o,\phi_o).d\mu.d\phi\right] + 2k\pi B(t)$$

But since $P(\mu,\phi,\mu',\phi') = P(\mu\mu'+\sqrt{(1-\mu^2)(1-\mu'^2)}.\cos(\phi-\phi'))$

$= P(-\mu,\phi,-\mu',\phi')$ we have

$$\int_0^{2\pi}\int_0^1\int_0^{2\pi}\int_0^1 P(\mu,\phi,\mu',\phi')d\mu'.d\phi'.d\mu.d\phi = \int_0^{2\pi}\int_{-1}^{0}\int_0^{2\pi}\int_{-1}^{0} P(\mu,\phi,\mu',\phi').$$

$d\mu'.d\phi'.d\mu.d\phi$

On the other hand

$$\int_0^{2\pi}\int_0^1\int_0^{2\pi}\int_{-1}^{0} P(\mu,\phi,\mu',\phi')\ d\mu'.d\phi'.d\mu.d\phi = \int_0^{2\pi}\int_{-1}^{0}\int_0^{2\pi}\int_0^1 P(\mu,\phi,\mu',\phi')$$

$d\mu'.d\phi'.d\mu.d\phi$

So we can write

$$\frac{dF_1(t)}{dt} = \alpha_1 F_1(t) - \alpha_2 F_2(t) - \alpha_3 I_o(t) - 2k\pi B(t)$$

$$\frac{dF_2(t)}{dt} = \alpha_2 F_1(t) - \alpha_1 F_2(t) + \alpha_4 I_o(t) + 2k\pi B(t)$$

Since $\int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_o^{1} P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi + \int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_{-1}^{o}$

$$P(\mu,\phi,\mu',\phi').d\mu'.d\phi'.d\mu.d\phi = \int_o^{2\pi}\int_o^{1}\int_o^{2\pi}\int_{-1}^{+1} P(\mu,\phi,\mu',\phi').$$

$$d\mu'.d\phi'.d\mu.d\phi = \int_o^{2\pi}\int_o^{1} 4\pi d\mu d\phi = 8\pi^2$$

and

$$\int_o^{2\pi}\int_o^{1} P(\mu,\phi,-\mu_o,\phi_o).d\mu.d\phi + \int_o^{2\pi}\int_{-1}^{o} P(\mu,\phi,-\mu_o,\phi_o).d\mu.d\phi =$$

$$\int_o^{2\pi}\int_{-1}^{1} P(\mu,\phi,-\mu_o,\phi_o).d\mu.d\phi = 4\pi$$

We get for the equations and the definition of their coefficients

$$\alpha_1 = 2(1 - (1 - k)A_1) \quad \alpha_2 = 2(1 - k)A_2 \text{ with } A_1 + A_2 = 1$$

$$\alpha_3 = (1 - k)A_3(\mu_o) \qquad \alpha_4 = (1 - k)A_4(\mu_o) \text{ with } A_3(\mu_o) + A_4(\mu_o) \equiv 1$$

$$\frac{dF_1(t)}{dt} = \alpha_1(F_1(t) - \pi B(t)) - \alpha_2(F_2 - \pi B(t)) - \alpha_3 I_o(t)$$

$$\frac{dF_2(t)}{dt} = \alpha_2(F_1(t) - \pi B(t)) - \alpha_1(F_2 - \pi B(t)) + \alpha_4 I_o(t)$$

$$I_o(t) = I_o(0)e^{-t/\mu_o}$$

A. Long wave case

$$I_o \equiv 0 \qquad B(t) = B_o + B't$$

let us take $F_1^* = F_1 - \pi B \qquad F_2^* = F_2 - \pi B$

we have $\dfrac{dF_1^*(t)}{dt} = \alpha_1 F_1^*(t) - \alpha_2 F_2^*(t) - \pi B'$

$$\frac{dF_2^*(t)}{dt} = \alpha_2 F_1^*(t) - \alpha_1 F_1^*(t) - \pi B'$$

let us now take $F_1^{**} = F_1^* - \pi B'/(\alpha_1+\alpha_2)$; $\quad F_2^{**} = F_2^* + \pi B'/(\alpha_1+\alpha_2)$

we get $\dfrac{dF_1^{**}(t)}{dt} = \alpha_1 F_1^{**}(t) - \alpha_2 F_2^{**}(t)$

$$\frac{dF_2^{**}(t)}{dt} = \alpha_2 F_1^{**}(t) - \alpha_1 F_2^{**}(t)$$

a) General case $\quad \alpha_1 \neq \alpha_2$

We combine both equations in

$$\frac{d}{dt}(F_1^{**} - \beta F_2^{**}) = F_1^{**}(\alpha_1 - \beta\alpha_2) - F_2^{**}(\alpha_2 - \beta\alpha_1)$$

The homogeneous solutions are obtained for

$$\beta = \frac{\alpha_2 - \beta\alpha_1}{\alpha_1 - \beta\alpha_2} \qquad \beta^2\alpha_2 - 2\beta\alpha_1 + \alpha_2 = 0 \qquad \beta = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 - \alpha_2^2}}{\alpha_2}$$

let us take $\varepsilon = \sqrt{\alpha_1^2 - \alpha_2^2} \qquad \beta_1 = \dfrac{\alpha_1 - \varepsilon}{\alpha_2} \qquad \beta_2 = \dfrac{\alpha_1 + \varepsilon}{\alpha_2} = 1/\beta_1$

We obtain $\frac{d}{dt}(F_1^{**} - \beta_1 F_2^{**}) = \varepsilon(F_1^{**} - \beta_1 F_2^{**}) \Rightarrow$

$$F_1^{**} - \beta_1 F_2^{**} = C_1 e^{\varepsilon t}$$

$$\frac{d}{dt}(F_1^{**} - \beta_2 F_2^{**}) = -\varepsilon\,(F_1^{**} - \beta_2 F_2^{**}) \Rightarrow$$

$$F_1^{**} - \beta_2 F_2^{**} = C_2 e^{-\varepsilon t}$$

$$F_1^{**} = (C_1\beta_2 e^{\varepsilon t} - C_2\beta_1 e^{-\varepsilon t}) / (\beta_2 - \beta_1)$$

$$F_2^{**} = (C_1 e^{\varepsilon t} - C_2 e^{-\varepsilon t}) / (\beta_2 - \beta_1)$$

With the boundary conditions

$$C_1\beta_2 e^{\varepsilon T} - C_2\beta_1 e^{-\varepsilon T} = (\beta_2 - \beta_1)\, F_1^{**}(T)$$

$$C_1 - C_2 = (\beta_2 - \beta_1)\, F_2^{**}(0)$$

Hence our equations are

$$F_1^{**}(0) = ((\beta_2 - \beta_1)F_1^{**}(T) + (e^{\varepsilon T} - e^{-\varepsilon T})F_2^{**}(0))/(\beta_2 e^{\varepsilon T} - \beta_1 e^{-\varepsilon T})$$

$$F_2^{**}(T) = ((e^{\varepsilon T} - e^{-\varepsilon T})F_1^{**}(T) + (\beta_2 - \beta_1)F_2^{**}(0))/(\beta_2 e^{\varepsilon T} - \beta_1 e^{-\varepsilon T})$$

Therefore

$$b_1 = b_4 = \tau_1 \cdot \frac{1 - \beta_1^2}{1 - (\beta_1\tau_1)^2} \qquad \varepsilon = \sqrt{\alpha_1^2 - \alpha_2^2}$$

$$b_2 = b_3 = \beta_1 \cdot \frac{1 - \tau_1^2}{1 - (\beta_1\tau_1)^2} \qquad \beta_1 = (\alpha_1 - \varepsilon)/\alpha_2$$

$$b_5 = b_8 = \frac{1 - b_1 + b_2}{(\alpha_1 + \alpha_2)T} - b_1 \qquad \tau_1 = e^{-\varepsilon T}$$

$$b_6 = b_7 = 1 - b_2 - \frac{1 - b_1 + b_2}{(\alpha_1 + \alpha_2)T}$$

b) Case without absorption

$$\alpha_1 = \alpha_2 = \alpha \iff k = o$$

$$\frac{dF_1^{**}}{dt} = \alpha(F_1^{**} - F_2^{**}) = \frac{dF_2^{**}}{dt} \Rightarrow F_1^{**} = F_2^{**} + C$$

With the boundary condition $C = F_1^{**}(T) - F_2^{**}(0) - C\alpha T$

$$F_1^{**}(0) = (F_1^{**}(T) + \alpha T.F_2^{**}(0))/(1+ \alpha T)$$

$$F_2^{**}(T) = (\alpha T.F_1^{**}(T) + F_2^{**}(0))/(1 + \alpha T)$$

therefore

$$b_1 = b_4 = 1/(1 + \alpha T)$$

$$b_2 = b_3 = \alpha T/(1 + \alpha T)$$

$$b_5 = b_6 = b_7 = b_8 = 0$$

B. Short wave case

$$B(t) \equiv 0$$

The parallel solar flux S is given by $S(t) = \mu_o.I_o(t)$

From the equation for parallel radiation $a_1 = e^{-T/\mu_o}$

and a_4, a_5, a_6, a_7 have the same expressions as b_1, b_2, b_3, b_4 since a diffuse radiation cannot become parallel again.

If S = 0 we have the same equation for F_1 and F_2 as in the longwave case for F_1^{**} and F_2^{**}

Let us try to derive the same expression valid for any S

$$F_1^{\odot} = F_1 - \gamma_1 S$$

$$F_2^{\odot} = F_2 - \gamma_2 S$$

$$\frac{dF_1^{\odot}}{dt} = \alpha_1 F_1^{\odot} - \alpha_2 F_2^{\odot} + I_o(\gamma_1 + \alpha_1\gamma_1\mu_o - \alpha_2\gamma_2\mu_o - \alpha_3)$$

$$\frac{dF_2^{\odot}}{dt} = \alpha_2 F_1^{\odot} - \alpha_1 F_2^{\odot} + I_o(\gamma_2 + \alpha_2\gamma_1\mu_o - \alpha_1\gamma_2\mu_o + \alpha_4)$$

We seek γ_1 and γ_2 so that

$$\gamma_1(1 + \alpha_1\mu_o) - \gamma_2\alpha_2\mu_o = \alpha_3$$

$$\gamma_1\alpha_2\mu_o + \gamma_2(1 - \alpha_1\mu_o) = -\alpha_4$$

The discriminant of the system is $1 - \alpha_1^2\mu_o^2 + \alpha_2^2\mu_o^2 = 1-\varepsilon^2\mu_o^2$

a) <u>General case</u> $\varepsilon\mu_o \neq 1$

$$\gamma_1 = \frac{\alpha_3 - \mu_o(\alpha_1\alpha_3 + \alpha_2\alpha_4)}{1 - \varepsilon^2\mu_o^2} \qquad \gamma_2 = \frac{-\alpha_4 - \mu_o(\alpha_1\alpha_4 + \alpha_2\alpha_3)}{1 - \varepsilon^2\mu_o^2}$$

and

$$a_2 = -a_5\gamma_2 - a_4\gamma_1 a_1 + \gamma_1$$

$$a_3 = -a_4\gamma_2 - a_5\gamma_1 a_1 + \gamma_2 a_1$$

b) <u>Resonance case</u> $\varepsilon\mu_o = 1$

We no longer have a solution with γ_1 and γ_2 constants.

We seek now solutions with $\gamma_1 = \gamma_1^o + \gamma'_1(t/\mu_o)$

and $\gamma_2 = \gamma_2^o + \gamma'_2(t/\mu_o)$

The equations are now

$$(\gamma_1^o + \gamma'_1\frac{t}{\mu_o}).(1 + \alpha_1\mu_o) - (\gamma_2^o + \gamma'_2\frac{t}{\mu_o})\,\alpha_2\mu_o = \alpha_3 + \gamma'_1$$

$$(\gamma_1^o + \gamma'_1\frac{t}{\mu_o}).\alpha_2\mu_o + (\gamma_2^o + \gamma'_2\frac{t}{\mu_o})\,(1 - \alpha_1\mu_o) = -\alpha_4 + \gamma'_2$$

$$\frac{\gamma'_1}{\gamma'_2} = \frac{\alpha_2\mu_o}{1+\alpha_1\mu_o} = \frac{1 - \alpha_1\mu_o}{-\alpha_2\mu_o} = \frac{-\alpha_4 + \gamma'_2}{\alpha_3 + \gamma'_1}$$

We obtain

$$\gamma'_1 = \frac{-\alpha_3 + \mu_o(\alpha_1\alpha_3 + \alpha_2\alpha_4)}{2}$$

$$\gamma'_2 = \frac{\alpha_4 + \mu_o(\alpha_1\alpha_4 + \alpha_2\alpha_3)}{2}$$

The choice of one of the two γ^o is then arbitrary. Among the infinity of solutions the most symmetrical one is

$$\gamma_1^o = \mu_o \frac{\alpha_1\alpha_3 + \alpha_2\alpha_4}{2} \qquad \gamma_2^o = \mu_o \frac{\alpha_1\alpha_4 + \alpha_2\alpha_3}{2}$$

We get the results

$$a_2 = - a_5\gamma_2^o - a_4 \left(\gamma_1^o + \gamma'_1 \frac{T}{\mu_o}\right)a_1 + \gamma_1^o$$

$$a_3 = - a_4\gamma_2^o - a_5 \left(\gamma_1^o + \gamma'_1 \frac{T}{\mu_o}\right)a_1 + \left(\gamma_2^o + \gamma'_2 \frac{T}{\mu_o}\right)a_1$$

APPENDIX B

Calculation of the a' and b' coefficients introduced in 4

A. Long wave, general case

Our three input parameters are

T		optical thickness
$w = 1-k$		single scattering albedo
and $A1$		integral factor of the scattering phase function

Let us symbolise by D the differential operator d( )/ d(kT) with (1-k)T constant. (kT is the absorption optical thickness which will be increased and (1-k)T the scattering optical thickness which will not be affected in the process).

We have $DT = 1$ $Dw = -w/T$ and $DA_1 = 0$

The expressions for the computation of b_1 and b_2 are:

$$\alpha_1 = 2.(1 - w.A_1) \qquad \alpha_2 = 2.w.(1 - A_1) \qquad \varepsilon = \sqrt{\alpha_1^2 - \alpha_2^2}$$

$$\beta_1 = (\alpha_1 - \varepsilon)/\alpha_2 \qquad \tau_1 = e^{-\varepsilon T}$$

$$b_1 = \tau_1.(1 - \beta_1^2)/(1 - \beta_1^2.\tau_1^2) \qquad b_2 = \beta_1.(1 - \tau_1^2)/(1 - \beta_1^2.\tau_1^2)$$

A differentiation step by step leads us to

$$b'_1 = Db_1 = (4\beta_1\tau_1 \frac{b_2}{T} - 2(1 + \beta_1^2\tau_1^2)\alpha_1 b_1)/(\varepsilon(1 - \beta_1^2\tau_1^2))$$

$$b'_2 = Db_2 = (4\beta_1\tau_1\alpha_1 b_1 - 2(1 + \beta_1^2\tau_1^2) \frac{b_2}{T})/(\varepsilon(1 - \beta_1^2\tau_1^2))$$

For the computation of b_5 and b_6 we have

$$\lambda = 1/((\alpha_1 + \alpha_2).T)$$

$$b_5 = (1 - b_1 + b_2).\lambda - b_1 \qquad b_6 = 1 - b_2 - (1 - b_1 + b_2)\lambda$$

The result of the differentiation is here

$$b'_5 = Db_5 = (b'_2 - b'_1).\lambda - 2.\lambda^2.(1 - b_1 + b_2) - b'_1$$

$$b'_6 = Db_6 = (b'_1 - b'_2).\lambda + 2.\lambda^2.(1 - b_1 + b_2) - b'_2$$

B. Long wave, case without absorption

Although we know the formal expressions of b_1 and b_2 we can no longer take their derivative as in the previous case, since the introduction of a kT is in contradiction with the condition k = o. We must therefore compute a limited expansion of b_1 and b_2 in the neighbourhood of the values obtained for k = o.

We have, for the input parameters, with kT = x

$$T = To + x \qquad w = 1 - x/To + x^2/To^2 \ldots \qquad A1 \text{ constant}$$

We get with $u = \alpha T$

$$b_1 = \frac{1}{1 + u}\left(1 - x\,\frac{8 + 8u + (8/3)u^2}{4 + 4u}\right)$$

$$b_2 = \frac{u}{1 + u}\left(1 - x\,\frac{8 + (16/3)u}{4 + 4u}\right)$$

Thus:

$$b'_1 = -(1 + b_1^2 - b_2^2/3) \qquad b'_2 = -(1 - b_1^2 + b_2^2/3)$$

The results for b'_5 and b'_6 are the same as in part A.
with $\lambda = 1/(2u)$

Thus:

$$b'_5 = b_1 + 2b_2/3 \qquad b'_6 = b_1 + 4b_2/3$$

C. Short wave, general case

We have two supplementary input parameters. μo cosine of the solar zenith angle and its dependent integral factor of the phase function $A3(\mu o)$.

Both of them are unchanged in the differentiation process : $D\mu_o = 0$ $DA_3 = 0$

The results for a'_4 and a'_5 are the same as those for b'_1 and b'_2

As $a_1 = e^{-T/\mu_o}$

$$a'_1 = -a_1/\mu_o$$

The expressions for the computation of a_2 and a_3 are

$$\alpha_3 = wA_3 \qquad \alpha_4 = w(1 - A_3)$$

$$\gamma_1 = -\frac{(\alpha_1\alpha_3 + \alpha_2\alpha_4)\mu_o - \alpha_3}{1 - \varepsilon^2\mu_o^2} \qquad \gamma_2 = -\frac{(\alpha_1\alpha_4 + \alpha_2\alpha_3)\mu_o + \alpha_4}{1 - \varepsilon^2\mu_o^2}$$

$$a_2 = -a_5\gamma_2 - a_4\gamma_1 a_1 - \gamma_1 \qquad a_3 = -a_4\gamma_2 - a_5\gamma_1 a_1 - \gamma_2 a_1$$

Thus to compute a'_2 and a'_3 we only need to know $D\gamma_1$ and $D\gamma_2$.

The differentiation step by step leads to

$$D\gamma_1 = \gamma_1(2\mu_o^2(2\alpha_1-\varepsilon^2)/(1-\varepsilon^2\mu_o^2) - 2)/T - \alpha_3(2\mu_o-1)/(T(1-\varepsilon^2\mu_o^2))$$

$$D\gamma_2 = \gamma_2(2\mu_o^2(2\alpha_1-\varepsilon^2)/(1-\varepsilon^2\mu_o^2) - 2)/T - \alpha_4(2\mu_o+1)/(T(1-\varepsilon^2\mu_o^2))$$

and finally

$$a'_2 = -a'_5\gamma_2 - a_5 D\gamma_2 - a'_4\gamma_1 a_1 - a_4(D\gamma_1 a_1 + \gamma_1 a'_1) + D\gamma_1$$

$$a'_3 = -a'_4\gamma_2 - a_4 D\gamma_2 - a'_5\gamma_1 a_1 - a_5(D\gamma_1 a_1 + \gamma_1 a'_1) + D\gamma_2 a_1 + \gamma_2 a'_1$$

D. Short wave, resonance case

The results obtained in C. for a'_1 a'_4 and a'_5 remains valid.

But for a'_2 and a'_3, as in B., we can no longer derive the final expressions of a_2 and a_3. But in this case the calculation of the derivative is simplified since we can compute a limited expansion by varying an independent parameter, namely μ_o.

We take $\mu_o = (1 + y)/\varepsilon$ and the results for a'_2 and a'_3 are of the type $(0 + my^2)/(0 + ny^2) = m/n$

The result is quite complicated :

With the $\gamma_1^o \gamma_2^o \gamma'_1 \gamma'_2$ from Appendix A

and

$$\delta_1^o = -\frac{1}{T}.((4\gamma_1^o + 2\gamma'_1).\alpha_1\mu_o^2 - \alpha_3.(6\mu_o - 1)/4)$$

$$\delta_2^o = -\frac{1}{T}.((4\gamma_2^o + 2\gamma'_2).\alpha_1\mu_o^2 - \alpha_4.(6\mu_o + 1)/4)$$

$$\delta'_1 = -\frac{1}{T}.((2\gamma_1^o + 4\gamma'_1).\alpha_1\mu_o^2 - \gamma_1^o - \alpha_3.(2\mu_o - 1)/2)$$

$$\delta'_2 = -\frac{1}{T}.((2\gamma_2^o + 4\gamma'_2).\alpha_1\mu_o^2 - \gamma_2^o - \alpha_4.(2\mu_o + 1)/2)$$

$$\delta''_1 = -\frac{1}{T}.\gamma'_1.(2\alpha_1\mu_o^2 - 1)$$

$$\delta''_2 = -\frac{1}{T}.\gamma'_2\ (2\alpha_1\mu_o^2 - 1)$$

we have

$$a'_2 = -a_5\delta_2^o - a'_5(\gamma'_2 + 2\gamma_2^o) - a_4\Big[a_1(\delta_1^o + (\delta'_1 - \delta''_1)\frac{T}{\mu_o} + \frac{\delta''_1}{2}\frac{T^2}{\mu_o^2})$$
$$+ a'_1(2\gamma_1^o - \gamma'_1 + 2\gamma'_1\frac{T}{\mu_o})\Big] - a'_4 a_1(\gamma'_1 + 2\gamma_1^o + 2\gamma'_1\frac{T}{\mu_o}) + \delta_1^o$$

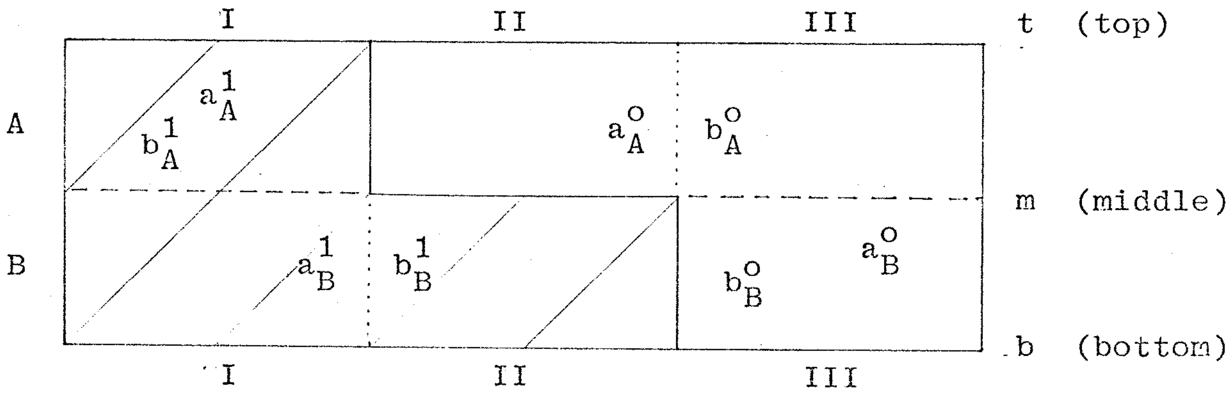
$$a'_3 = -a_4\delta_2^o - a'_4(\gamma'_2 + 2\gamma_2^o) - a_5\Big[a_1(\delta_1^o + (\delta'_1 - \delta''_1)\frac{T}{\mu_o} + \frac{\delta''_1}{2}\frac{T^2}{\mu_o^2})$$
$$+ a'_1(2\gamma_1^o - \gamma'_1 + 2\gamma'_1\frac{T}{\mu_o})\Big] - a'_5 a_1(\gamma'_1 + 2\gamma_1^o + 2\gamma'_1\frac{T}{\mu_o})$$
$$+ a_1(\delta_2^o + (\delta'_2 - \delta''_2)\frac{T}{\mu_o} + \frac{\delta''_2}{2}\frac{T^2}{\mu_o^2}) + a'_1(2\gamma_2^o - \gamma'_2 + 2\gamma'_2\frac{T}{\mu_o})$$

APPENDIX C

Combination of partially covered cloud layers

We are treating here the simplest case of two adjacent cloudy layers A(above) and B(below) with partial cloud coverages C_A and C_B (here $C_A<C_B$) and we are looking for the a and b coefficients of the layer A knowing the ones for cloudy (1) and clear (0) conditions in both layers. All of the following can easily be extended to any layer in any cloud configuration.

The geometry of our "cloud" is (with the coefficients in the corresponding spaces)



A) Short wave fluxes With a* and b* representing a^1 and b^1 or a^o and b^o depending on case k = I, II or III (different vertical repartitions of cloudy and non cloudy conditions) we can write our linear equation system for the fluxes (see equation (9))

$$
\begin{bmatrix}
1 & & & & & & & & \\
 & 1 & & & & & & & \\
-a_A^{*6} & -a_A^{*2} & 1 & & & -a_A^{*4} & & & \\
-a_A^{*7} & -a_A^{*3} & & 1 & & -a_A^{*5} & & & \\
 & -a_A^{*1} & & & 1 & & & & \\
 & & & -a_B^{*6} & -a_B^{*2} & 1 & & & -a_B^{*4} \\
 & & & -a_B^{*7} & -a_B^{*3} & & 1 & & -a_B^{*5} \\
 & & & & -a_B^{*1} & & & 1 & \\
 & & & & & & & & 1
\end{bmatrix}
\begin{bmatrix}
F_{2t} \\ S_t \\ F_{1t} \\ F_{2m} \\ S_m \\ F_{1m} \\ F_{2b} \\ S_b \\ F_{1b}
\end{bmatrix}^k
=
\begin{bmatrix}
F_{2t} \\ S_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{1b}
\end{bmatrix}
$$

We can solve it in each case to get for the layer A matrix equations of the type

$$\begin{bmatrix} S_t \\ F_{1m} \\ F_{2t} \end{bmatrix}^k = \begin{bmatrix} c \end{bmatrix}^k \cdot \begin{bmatrix} S_t \\ F_{1b} \\ F_{2t} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} S_m \\ F_{1t} \\ F_{2m} \end{bmatrix}^k = \begin{bmatrix} d \end{bmatrix}^k \cdot \begin{bmatrix} S_t \\ F_{1b} \\ F_{2t} \end{bmatrix}$$

for the incoming and outgoing fluxes.

Thus with linear combination and elimination of the fluxes S_t F_{1b} and F_{2t} we obtain the matrix solution from equation (9)

$$\begin{bmatrix} a_A \end{bmatrix} = \begin{bmatrix} C_A\begin{bmatrix} d \end{bmatrix}^{I} + (C_B - C_A)\begin{bmatrix} d \end{bmatrix}^{II} + (1 - C_B)\begin{bmatrix} d \end{bmatrix}^{III} \end{bmatrix} \cdot \begin{bmatrix} C_A\begin{bmatrix} c \end{bmatrix}^{I} + (C_B - C_A)\begin{bmatrix} c \end{bmatrix}^{II} + (1 - C_B)\begin{bmatrix} c \end{bmatrix}^{III} \end{bmatrix}^{-1}$$

<u>B) Long wave fluxes</u> The computation of coefficients $b_{1...4}$ is the same as the one described above for the $a_{1.....7}$

For the other ones we change the formalism of equation (10) and replace the product $\begin{bmatrix} b_5 & b_7 \\ b_6 & b_8 \end{bmatrix} \cdot \begin{bmatrix} \pi B_b \\ \pi B_t \end{bmatrix}$ by the resulting vector $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

Then we first make a computation similar to the one already done for the determination of the $b_{1..4}$ coefficients, without incoming fluxes but with local emission (see equation (10)).

$$\begin{bmatrix} 1 & & & & & \\ -b^*_A3 & 1 & & -b^*_A1 & & \\ -b^*_A4 & & 1 & -b^*_A2 & & \\ & & -b^*_B3 & 1 & & -b^*_B1 \\ & & -b^*_B4 & & 1 & -b^*_B2 \\ & & & & & 1 \end{bmatrix} \cdot \begin{bmatrix} F_{2t} \\ F_{1t} \\ F_{2m} \\ F_{1m} \\ F_{2b} \\ F_{1b} \end{bmatrix}^k = \begin{bmatrix} 0 \\ e^*_A1 \\ e^*_A2 \\ e^*_B1 \\ e^*_B2 \\ 0 \end{bmatrix}$$

Then we again make a linear combination and eliminate the unwanted fluxes from equation (10).

$$\begin{bmatrix} e_A 1 \\ \\ e_A 2 \end{bmatrix} = \left[C_A \begin{bmatrix} F_{1t} \\ \\ F_{2m} \end{bmatrix}^{I} + (C_B - C_A) \begin{bmatrix} F_{1t} \\ \\ F_{2m} \end{bmatrix}^{II} + (1 - C_B) \begin{bmatrix} F_{1t} \\ \\ F_{2m} \end{bmatrix}^{III} \right]$$

$$- \begin{bmatrix} b_A 1 & b_A 3 \\ \\ b_A 2 & b_A 4 \end{bmatrix} \cdot \left[C_A \begin{bmatrix} F_{1m} \\ \\ F_{2t} \end{bmatrix}^{I} + (C_B - C_A) \begin{bmatrix} F_{1m} \\ \\ F_{2t} \end{bmatrix}^{II} + (1 - C_B) \begin{bmatrix} F_{1m} \\ \\ F_{2t} \end{bmatrix}^{III} \right]$$

APPENDIX D

Generation of random atmospheres

There are 15 levels equally spaced between 0 and 1 in the coordinate system q with $\sigma = p/p_s = \sin^2(q\pi/2)$

We start from the ground with $p_s = 1013.25$ mb. and $T_s = 288.15^\circ K$ and going upwards for each layer we generate randomly the temperature lapse rate dT/dz and two relative humidities U_1 and U_2 under the following conditions:

$\frac{dT}{dz} = \frac{g}{c_p}(f_1 - 1)$ f_1 having a log-normal distribution with mean value $1-\sigma$ and variance σ/α

$U_{1,2} = f_2$ f_2 having a log-normal distribution with mean value σ and variance $(1-\sigma).\alpha$

U_1 and U_2 represent a maximum and a minimum relative humidity and we have assumed a rectangular distribution between these values for the relative humidities in the layer.

This gives us the cloud cover and the mixing ratios of water vapour and liquid water (with the temperature linearly interpolated with respect to pressure in the middle, for the coordinate q, of the layer).

The arbitrary parameter α is adjusted so that the mean cloud cover is equal to 0.5. For the computation of the parameter we suppose that adjacent cloudy layers have a maximum overlapping of their cloudy parts and that distinct "clouds" are randomly distributed with respect to each other. Thus we have to take $\alpha = 0.76$.

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