# LINEAR STOCHASTIC DYNAMIC MODEL OF ARTIC AND ANTARCTIC SEA ICE VARIABILITY

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#### ABSTRACT

A dynamical model of sea ice variability is of particular interest in global climate modelling. In order to achieve a good description the dynamics of the complete system ocean, sea ice and atmosphere has to be included in the model.

In a first step a dynamical model of the atmosphere - sea ice feedback branch is developed.

Since the internal time-scales of sea ice and atmosphere are very different (sea ice: several months, atmosphere: several days) and the atmospheric system is highly variable a stochastic description of the interaction seems to be appropriate.

The available data of sea ice extent in both polar regions show that sea ice variability can be explained as the integral response to random atmospheric forcing. This model is essentially a Vector Markov-process in which the diagonal elements of the feedback matrix describe the local feedback of the ice with itself and the non-diagonal elements describe the interaction of the ice in different geographical regions. These coefficients are mainly due to internal ice - ice interactions and ice - ocean - ice or ice - atmosphere - ice feedback.

## 1. DATA SET

Our Artic data (Lemke, Trinkl and Hasselmann, 1980) is based on the monthly northern hemisphere sea ice maps published by the United Kingdom Meteorological Office, Bracknell.

The data was digitized in 10-degree longitudinal sectors by counting the number of one-degree rectangles of sea ice within each one-degree band of latitude within a given 10-degree longitudinal sector. The ocean was regarded as ice-covered if the MO map indicated at least 70% ice cover. Open pack ice, also indicated on the maps, was not included.

This data was then used to determine the total sea ice area within a 10-degree longitudinal sector. The 36 time series cover an 11 year period from January 1966 to December 1976.

The Antarctic data (Jan. 4th, 1973 - Feb. 22nd, 1979) is based on the weekly southern hemisphere sea ice charts published by the U.S. Fleet Weather Facility (FLEWEAFAC), Suitland, Maryland.

# 2. DATA ANALYSIS

The results of our data analysis of these time series of sea ice cover may be summarized as follows:

The annual sea ice variation shows the well known strong regional variability. The anomaly variance of the sea ice cover exhibits approximately the same longitudinal dependence as the annual cycle, but is only 1/3 as large (in terms of the r.m.s. area). The expansion of the time series into empirical orthogonal functions (EOF's) shows, that for both poles lower-order EOF's are associated generally with larger spatial scales. From the auto-

correlation functions of the EOF amplitudes it is seen, that lower-order EOF amplitudes have larger relaxation times (of the order of several months). So large spatial variations are associated with large relaxation times.

The auto-spectra of the sea ice cover in the 10-degree longitudinal sectors as well as the auto-spectra of the lower-order EOF amplitudes show a red behavior, i.e. an enhancement towards smaller frequencies.

The main purpose of this work is to test whether the observed spectra might support the hypothesis, that sea ice variability may be explained as the response of a slow system to continuous random atmospheric forcing.

# 3. THE MODEL

Since the internal time scales of the sea ice (several months) and the atmosphere (several days) are very different and the atmospheric system is highly variable a stochastic model of the interaction between sea ice and atmosphere is appropriate.

The response of a slow system to white noise forcing is described by a first order Markov process:

$$y(t) + \lambda y(t) = x(t)$$
 (1)

where y denotes the slow system (sea ice), x the white noise forcing and  $\lambda$  is a stabilizing feedback parameter.

The spectrum of the first order Markov process is given by

$$G(\omega) = \frac{F(0)}{\omega^2 + \lambda^2}$$
 (2)

where F(o) is the spectrum of the white noise forcing at zero frequency.

The white noise forcing may be identified as in previous applications of the stochastic forcing climate model (Hasselmann, 1976; Frankignoul and Hasselmann, 1977; Lemke, 1977; Reynolds, 1978) with the atmospheric forcing associated with short-time weather disturbances. The stabilizing feedback parameter  $\lambda$  may be attributed to the heat transfer from the ice to either the atmosphere or the ocean; existing parameterizations for these transfer rates all yield a linear stabilizing feedback term for small deviations of the ice-sheet area.

The level of the atmospheric forcing F(o) and the feedback parameter  $\lambda$  are determined by fitting the model spectra (2) to the observed spectra.

The simple first order Markov process (1) with local forcing and local feedback yields a satisfactory description if the auto-spectra both for the EOF representation and for the original representation of the ice anomalies in individual sectors.

Although these models provide some useful numbers on the general magnitude of the forcing and feedback rates, they need to be generalized in order to explain the structure of the nondiagonal elements of the cross - spectral matrix.

A generalization may be achieved by introducing a correlation between the ice cover in different sectors. Eq. (1) then has to be replaced by a vector Markov process

$$\dot{y}_{i}(t) + \lambda_{ij}y_{j}(t) = x_{i}(t)$$
(3)

$$G_{ij}(\omega) = H_{ik}H_{jl}^*F_{kl}(0)$$
 (4)

where  $H = (i\omega I - \lambda)^{-1}$  (I = unit matrix) and  $F_{kl}$  (o) is the cross-spectral forcing at zero frequency.

In view of the narrow width of the cross correlation function as a function of sector lag, only the diagonal elements and the two closest off-diagonal elements were included in the feedback matrix  $\lambda$ . The diagonal elements of  $\lambda$  mainly describe the local feedback of the ice with itself (internal or via ocean or atmosphere), and the two adjacent diagonals describe the interaction with the next neighbour longitude.

Comparing (3) with the linear discrete form of a differential equation that includes local feedback as well as lateral diffusion and advection of ice anomalies, it is evident that the symmetric part of the next neighbour interaction determines the diffusion coefficient and the antisymmetric part describes the advection, modified by a term due to the longitudinal gradient of the diffusion coefficient.

This model now includes three feedback parameters for each longitudinal gridpoint: the local feedback  $\lambda_{\mbox{ii}}$ , the diffusion coefficient D and the advection  $\mbox{u}_{\mbox{i}}$ .

Since the average diameter of a high or a low in the atmosphere is of the order of 500 km the cross-spectral matrix  $F_{ij}$  of the atmospheric forcing is taken to consist of three diagonals only, as well as the feedback matrix  $\lambda$ .

In order to find the simplest model that can describe the data at a certain significance level, a hierarchy of models is tested. The simplest model includes local feedback  $\lambda_{\mbox{\sc ii}}$  only. The next higher order model includes additional diffusion or advection, and the highest order model includes both diffusion and advection.

## 4. RESULTS

The data show that rotational symmetric models, i.e.local feedback, advection and diffusion are assumed to be independent of longitude, have to be rejected at a 95 % significance level.

Accordingly, at each longitudinal gridpoint a local feed-back, diffusion and advection parameter are determined together with two parameters describing the cross-spectral forcing of the white noise. The optimal parameters are given by fitting the cross-spectral matrix of sea ice anomalies.

The local response times are generally larger in the Artic Ocean (2 months) than in the Southern Ocean (5 weeks) where upwelling and strong ocean currents provide a more intense thermal contact between sea ice and ocean. This increased ice-ocean interaction around Antartica is expressed also in the diffusion and advection parameters that account for the lateral interaction between sea ice anomalies in different regions. The mean diffusion parameter, describing the symmetric part of the interaction of sea ice with the adjacent eastern and western neighbour longitudinal gridpoint, is of the order of 0.8·108cm²/sec for Arctic and 3·108 cm²/ sec for Antarctic sea ice. The mean absolute value of the lateral advection amounts to 3.6 cm/sec in the Arctic Ocean compared to 11 cm/sec in the Southern Ocean.

The advection pattern roughly agrees with the oceanic surface circulation at the ice edge at both poles.

The errorfunction accounting for the deviation of the models in the hierarchy from the measurements, clearly

indicates, that lateral diffusion and advection of sea ice anomalies play an important role in sea ice dynamics. The errorfunction in the Arctic is in general smaller than in the Southern Ocean, denoting that some characteristics of sea ice dynamics, which is not present in the Arctic Ocean, is not covered by the model (3).

For Arctic sea ice the model (3) including local feedback, diffusion and advection provides a valid description at the 80 % significance level.

Except at four longitudes in the Bellinghausen Sea (90 W, 100 W) and the western Weddell Sea (50 W, 40 W), which obviously demand additional dynamical parameterisations, the Markov-model (3) can be accepted at the 95 % significance level for explaining Antarctic sea ice anomalies, too.

### 5. CONCLUSIONS

It is shown, that sea ice variability can be explained as the response of a slow system to random atmospheric forcing. The response is described by local feedback and by lateral advection and diffusion, accounting for the lateral interaction of sea ice anomalies at different longitudes.

This coupling is mainly due to internal ice-ice interaction and ice-ocean-ice or ice-atmosphere-ice feedback.

## REFERENCES

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