TECHNICAL MEMORANDUM

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A proposal for modifying the Kessler rain-scheme

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1. Introduction

The problems encountered so far at ECMWF with the parameterization of moist processes indicate that we will perhaps need in the future to incorporate a rain-scheme in our parameterization package. Most simplified rain-schemes used in numerical weather prediction research are of the so-called "Kessler type" ("On the distribution and continuity of water substance in atmospheric circulation"; Kessler, Meteorological Monographs; Vol.10 1969). Kessler's model uses very simple equations and a useful separation of the processes inside and outside clouds, but unfortunately the formulation is not very suitable for efficient programming.
The aim of this paper is to propose a scheme following Kessler's ideas, using the same original data but allowing efficient coding.

2. The Kessler scheme

a) Raindrop's size distribution

Kessler uses the Marshall-Palmer distribution

$$N(D) = N_0 \, e^{-\lambda D} \tag{1}$$

D is the diameter of a raindrop
N(D) . dD would be the number of drops per unit volume to have a diameter between D and D + dD
N_0 is a universal constant
λ is the parameter characterising the rainfall type.

b) Raindrop's fall speed

Kessler takes

$$V = K_1 \sqrt{\frac{D\rho_0}{\rho}} \tag{2}$$

as fitted from measurements

V is the fall velocity (positive downwards) of a rain drop

ρ is the air density, ρ_0 a standard value of ρ related to the constant coefficient K_1

Combining (1) and (2) we can find a relation between the rainflux R (positive downwards) and λ

$$R = \int_0^\infty K_1 \sqrt{\frac{\rho_0}{\rho}} D^{\frac{1}{2}} N_0 e^{-\lambda D} \frac{\Pi D^3}{6v} dD = \frac{\Pi K_1 N_0}{6v} \sqrt{\frac{\rho_0}{\rho}} \frac{\Gamma(9/2)}{\lambda^{9/2}} \quad (3)$$

v being the massic volume of liquid water.

c) Raindrop's evaporation rate

Kessler uses

$$\frac{dM}{dt} = - K_2 \; D^{8/5}(\rho_s - \rho_a) \quad (4)$$

as fitted from numerical values for the more exact formula.

$$\frac{dM}{dt} = - 2\Pi D \,(1 + K_3 D)\, \mathcal{D} \,(\rho_b - \rho_a) \quad (5)$$

M is the mass of a drop of diameter D

ρ_s is the saturation water vapour density at the temperature of the air,

ρ_a is the water vapour density in the air

$\mathcal{D}$ is the water vapour diffusion coefficient in the air

ρ_b is the density of saturated water vapour at the surface of the drop ($\simeq$ wet bulb temperature's saturation density)

K_2 and K_3 are constant coefficients

Combining (1) (3) and (5) one obtains for the total rate of evaporation

$$\frac{dR}{dz} = \frac{d\rho_R}{dt} = - \int_0^\infty N_0 \, e^{-\lambda D} \, K_2 D^{8/5} (\rho_s - \rho_a) \, dD$$
$$= - N_0 K_2 (\rho_s - \rho_a) \, \frac{\Gamma(13/5)}{\lambda^{13/5}} = -\alpha(\rho, \rho_0)(\rho_s - \rho_a) R^{26/45} \qquad (6)$$

ρ_R is the rain water density in the air, z the vertical coordinate increasing downwards.

The final constant α depends on ρ/ρ_0 through (2)

d) Raindrop's collection of cloud water

In the collection of cloud water by falling raindrops Kessler assumes the collection efficiency E to be 1 and obtains therefore

$$\frac{dR}{dz} = \frac{d\rho_R}{dt} = \int_0^\infty N_0 e^{-\lambda D} \, K_1 \sqrt{\frac{\rho_0}{\rho}} \, D^{\frac{1}{2}} \, \frac{\Pi D^2}{4} \, E \rho_\ell \, dD$$
$$= \frac{\Pi N_0 K_1}{4} \sqrt{\frac{\rho_0}{\rho}} \, \rho_\ell \, \frac{\Gamma(7/2)}{\lambda^{7/2}} = \beta \, (\rho, \rho_0) \rho_\ell \, R^{7/9} \qquad (7)$$

ρ_ℓ is the liquid water density in clouds

β depends on ρ/ρ_0 through (2)

e) Autoconversion of cloud water into rainwater

In the absence of any simplified theory equivalent to what was shown previously Kessler arbitrarily expresses

$$\frac{dR}{dz} = - \frac{d\rho_\ell}{dt} = K_4 \, (\rho_\ell - \rho_{crit}) \qquad (8)$$

ρ_{crit} is a critical value for ρ_ℓ
K_4 is a constant coefficient (inverse of a relaxation time)

3. Proposed modified scheme

a) Remarks

- One can first see that if we express (ρ_s - ρ_a) and ρ_ℓ as product of the air density ρ by specific humidities (q_s - q) and q_ℓ the dR/dz can be replaced by a more suitable dR/dp.

- The power of R in the right-hand side of (6) and (7) can easily be incorporated into the derivative of the left-hand side, leaving a right-hand side independent of the rainfall rate and therefore of all what happened in the atmosphere above.
  This will simplify the computational process.

- The exponents 26/45 and 7/9 will require an expensive double "y^x" for each computation. On the other hand they are quite close to 0.5 and 1 respectively. A modification of (2) (4) and of E = 1 can make the exponents take these new values.

- There is no simple analytical combination of (7) and (8) in the clouds which has the interesting property described above of eliminating R of the right-hand side.A redefinition of the autoconvection as the collection by a fictitious flux to be added to R in the left-hand side of the equation will be very similar to a relaxation process.

b) New fittings of data

We first want to replace (2) by an expression of the type

$$V = k\left(\frac{p}{p_0}, \frac{T}{T_0}\right) D.$$

Fitting to the same sources as Kessler (Smithsonian Meteorological Table 114) gives us

$$V = 3510 \frac{p_0}{p} \frac{T}{T_0} D \quad (\text{MKS}, p_0 = 100000, T_0 = 293.15) \tag{9}$$

To replace (4) we seek a fitting of the type

$$\frac{dM}{dt} = -k\left(\frac{p}{p_0}, \frac{T}{T_0}\right) D^{3/2} \quad (q_s - q)$$

Contrary to Kessler we do not try to fit the whole expression (5) given in Smithsonian Meteorological Tables No. 117A-B but, since the expressions given in 117B for $\mathcal{D}(\rho_b - \rho_a)$ look suspect (linear in 1-relative humidity as they should not be), we fit our expression to the data of 117 A for $2\pi D(1+K_3 D)$ and of Table 113 for $\mathcal{D}$ assuming $\rho_b \simeq \rho_s$. We obtain

$$\frac{dM}{dt} = -0.0230 \frac{p_0}{p} \left(\frac{T}{T_0}\right)^{1.23} (q_s - q)$$

$$(\text{MKS}, p_0=100000, T_0=293.15) \tag{10}$$

Our fittings are surely not as good as those by Kessler where the choice of the exponent was part of the fitting process, but nevertheless the errors are not likely to affect dramatically a model with such crude simplifications.

Finally, we fit data out of "The physic of clouds", Mason, Oxford University Press 1957, for the collection efficiency to have

$$E = 1 - e^{-\gamma D}$$

We obtain $\gamma = 1030\left(\frac{p_0}{p}\right)^2\left(\frac{T}{T_0}\right)^{0.42}$

$$(\text{ MKS, } p_0 = 100000,\ T_0 = 293.15) \qquad (11)$$

c) New expressions

Using (9) and (10) instead of (9) and (4), following the two first remarks in 3a) and incorporating the p and T dependencies in a more general sigma dependency, we get instead of (6)

$$\frac{d\sqrt{R}}{dp} = -\ 6.584\ 10^{-4} \quad \sigma^{-0.36}\ (q_s - q) \qquad (12)$$

To compute an equivalent to (7) we incorporate the new value of E parallely to (9) and we make the (false) assumption that $\gamma \ll \lambda$ (equivalent to $E = \gamma D$) and we get

$$\frac{d\ell nR}{dp} = 1.613\ 10^{-1} \quad \sigma^{-1.92} \quad q_\ell \qquad (13)$$

For the autoconversion of cloud water we arbitrarily choose to put the fictitious flux at a value of $6.665\ 10^{-5}$ kg/m^{2}s independent of σ . This independency means that the full effect of the $\sigma^{-1.92}$ term on the right-hand side will be felt not only in the collection process but also in the autoconversion. We hope this will help to simulate the more efficient rain mechanism at lower temperatures (Bergeron's process). The equivalent relaxation time for $\sigma = 1$ is 2 hours 40 minutes. This parameter should be the one used to tune the scheme.

Our final set of equations is therefore

$$\left.\begin{aligned} \frac{d\sqrt{R}}{dp} &= -\ 6.584\ 10^{-4}\ \sigma^{-0.36}\ (q_s - q) \\ \frac{d\ell n\ (R + 6.665\ 10^{-5})}{dp} &= 1.613\ 10^{-1}\ \sigma^{-1.92}\ q_\ell \end{aligned}\right\} \quad (14)$$

d) Practical implementation of the scheme

For every cloudy layer, knowing its thickness Δp and its cloud water specific humidity q_ℓ one can deduce the flux R_b at the bottom of the layer from the one R_t at the top of the layer

$$R_b = \exp(\ell n(R_t + 6.665\ 10^{-5}) + 1.613\ 10^{-1}\ \sigma^{-1.92}\ \Delta p\ q_\ell) - 6.665\ 10^{-5} \quad (15)$$

For every cloud-free layer there is the supplementary difficulty to ensure that $\sqrt{R}$ has not become negative across the layer

$$R_b = \left[\max\ (0,\ \sqrt{R_t} - 6.584\ 10^{-4}\ \sigma^{-0.36}\ \Delta p(q_s - q)\right]^2 \quad (16)$$

If we want to introduce partial cloudiness into the scheme we simply need to have for every layer two parallel computations with (15) and (16) q_ℓ being replaced by q_ℓ/C and $q_s - q$ by $(q_s - q)/(1-C)$ where C is the cloud cover. At the interface of two layers we assume maximum overlapping of cloudy parts and two linear combinations of the two fluxes at the bottom of the upper layer (for cloudy and non cloudy parts) give us the two new fluxes at the top of the lower layer (for cloudy and non cloudy parts again).

4. Conclusion

The scheme proposed here should not be too far away in its results from the well tested Kessler scheme (they both use the same basic data), but it allows very efficient computing. Introduction of partial cloud cover is straightforward . Preliminary tests have started with the "confusion scheme" (J.-F. Louis,ECMWF Seminar 1977, p. 366-367) implemented in ECMWF's global grid point model.