EFFECT OF DIABATIC PROCESSES ON TRANSIENT MID-LATITUDE WAVES

Brian J. Hoskins
Department of Meteorology
University of Reading
Reading, United Kingdom

1. INTRODUCTION

There is no doubt that diabatic processes are important in the transient behaviour of mid-latitude disturbances. However, the theory available to make this statement quantitative is mostly lacking. Consequently, this lecture is largely devoted to providing a simple foundation for estimating the importance of the various heat sources and sinks.

We take as our starting point the quasi-geostrophic, Boussinesq approximations of the equations for the vertical component of vorticity ξ and potential temperature θ.

$$D_g\xi_g + \beta v_g = f\,\frac{dw}{dz}, \tag{1}$$

$$D_g\theta + w\,\frac{d\theta}{dz} = Q, \tag{2}$$

where $D_g = \frac{\partial}{dt} + \underset{\sim}{v}_g \cdot \nabla$; Q represents the diabatic processes. The hydrostatic relation linking ξ_g and θ is

$$f\,\frac{d\xi g}{dz} = \frac{g}{\theta_o}\,\nabla_h{}^2\theta \tag{3}$$

Eliminating the time derivative between (1) and (2) using (3) gives the "omega"-equation

$$f^2\,\frac{\partial^2 w}{\partial z^2} + N^2\,\nabla_h^2\,w = S + \frac{g}{\theta_o}\,\nabla_h^2\,Q \tag{4}$$

where $N^2 = \frac{g}{\theta_o}\,\frac{d\bar{\theta}}{dz}$ and for the large scale ascent in baroclinic waves we may use the Sutcliffe approximation (Hoskins et al 1978) for the dynamical forcing of vertical motion:

$$S \simeq 2f\,\frac{\partial \underset{\sim}{v}_g}{\partial z} \cdot \nabla\xi_g \tag{5}$$

A direct measure of the relative importance of adiabatic and diabatic processes in forcing vertical motion is provided by the two terms on the right hand side of (4). As boundary conditions for (4) we may consider w to be small at some level representing the tropopause and at $z=0$ w is zero or, if a surface Ekman layer of depth δ is included, $w = \frac{1}{2}\,\delta\xi_g$.

The omega-equation is purely diagnostic. To obtain a prognostic equation w may be eliminated between (1) and (2), giving the quasi-geostrophic potential vorticity (q) equation

$$D_g \left[\beta y + \xi_g + f\left(\frac{\theta}{\bar{\theta}_z}\right)_z \right] = f\frac{g}{\theta_o} \frac{\partial Q}{\partial z} \qquad (6)$$

The crucial quantity q is changed only by the vertical derivative of the diabatic heating, stressing the importance of knowing its vertical distribution.

From the omega-equation and the potential vorticity equation there is a natural ratio between the height scale H and horizontal length scale L:

$$H/L \sim f/N \sim 10^{-2}$$

Finally, in this section, we note that many physical processes have an on-off nature. For example the simplest parameterisation of latent heat release might be

$$\begin{aligned} Q &= \alpha w \quad (w > 0) \\ &= 0 \quad (w < 0) \end{aligned}$$

Taking w to be of the form $w_o \cos kx$, Fourier analysis gives

$$Q = \alpha w_o \left(\frac{1}{\pi} + \tfrac{1}{2} \cos kx + \text{higher harmonics}\right)$$

For inclusion in linear analysis we thus set $Q = \frac{\alpha}{2} w$ for all x, neglecting the zonal warming and the higher harmonics.

2. EFFECT OF DECREASED STATIC STABILITY ON BAROCLINIC INSTABILITY

Frequently when considering the possible influence of diabatic processes one needs to be able to gauge the effects of changed static stability on baroclinic instability. The only firm indications of this are given by linear quasi-geostrophic theory for y-independent basic flows. Sketched in Fig. 1a are the growth rate curves given by Eady theory ($\beta = 0$) for typical parameters and for the same parameters except that N^2 is reduced by a factor of 2. The minimum e-folding time decreases by N^2 from about 1.3 days to .92 days and the wavelength at which this occurs is also reduced by N^2 from about 4000 km to 2900 km.

Inclusion of the β-effect gives the Green problem for which growth rate curves for $10^4 N^2 = 2, 1, \cdot 5\ s^{-2}$ are shown in Fig. 1b. It should be noted that for longer wavelengths the north-south scale of the normal modes are chosen to maximise the growth rates - a fixed meridional scale is not considered. The maximum

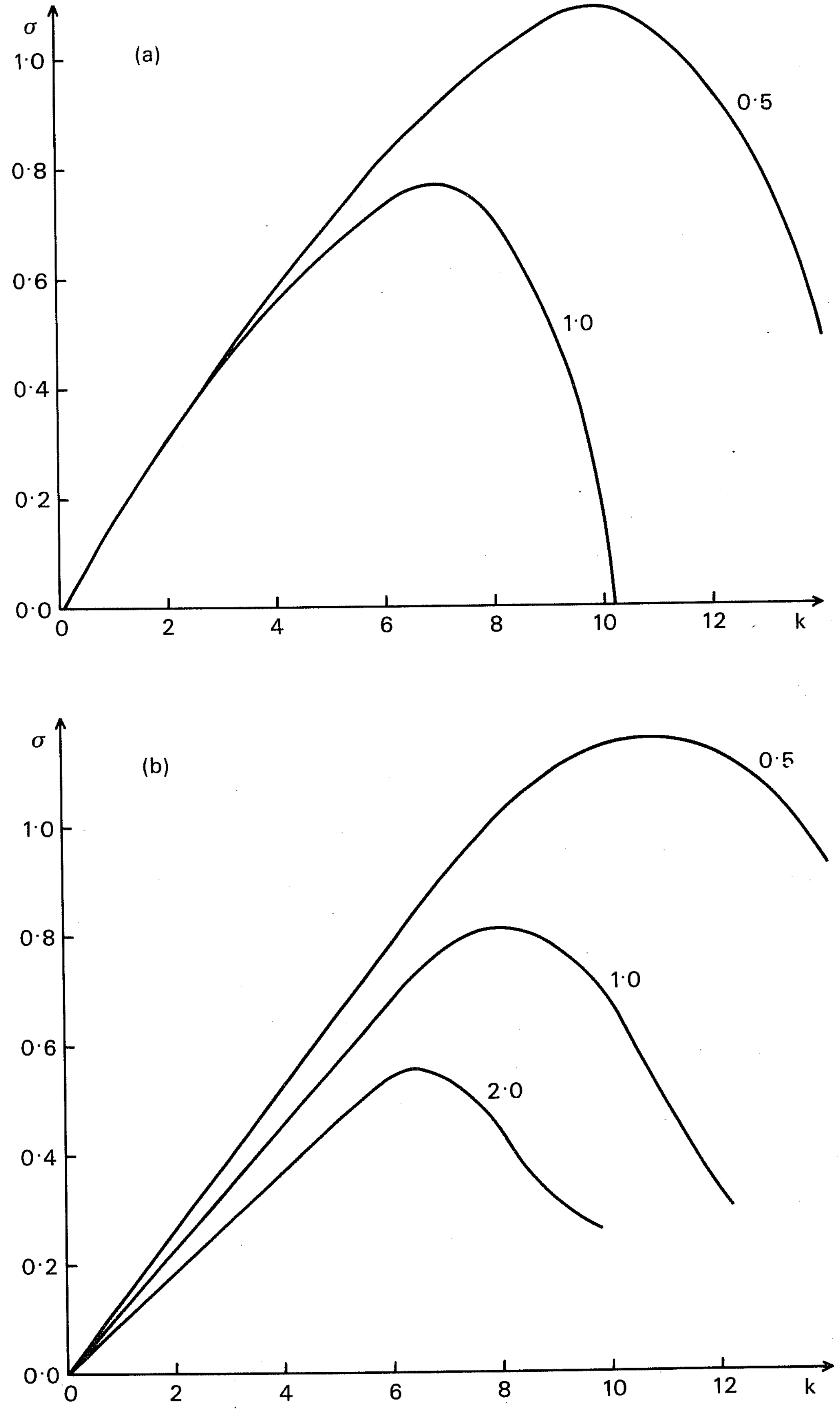


Fig. 1 Curves showing growth rates (days^{-1}) versus zonal wavenumber for typical parameters in (a) Eady problem and (b) Green problem.

The different curves are for N = 2.0, 1.0, and 0.5 x $10^{-4}s^{-2}$.

growth rate increases by factors slightly more than N^2 and the wavelengths at which this maximum occurs is reduced by less than N^2. Short wavelength disturbances can be much influenced by reduced static stability. For $N^2 = 1.10^{-4}\ s^{-2}$ the wavenumber 12 mode is confined near the surface and has an e-folding time of 3 days. For $N^2 = .5.10^{-4}\ s^{-2}$ it extends up to the tropopause and grows by a factor of e in 22½ hours. Of interest also is the fact that wavelengths 5 and longer also increase their growth rates by factors 1.23 and 1.15. Thus even a large-wavelength disturbance whose structure is already established may be expected to grow significantly quicker if the effective static stability is reduced.

3. POSSIBLE EFFECTS OF DIABATIC PROCESSES

In order to estimate the effects of diabatic processes on transient mid-latitude disturbances it is probably necessary to split the problem into three: initiation, growth and decay. In this section an attempt is made to evaluate and deal with diabatic processes in these three stages. The account is incomplete and at best only meant to be suggestive.

3.1 Initiation

Probably the only diabatic effect that could play a role in initiating depressions is convection. Tracton (1973) has found good evidence that this was the case in certain cases in which a forecast model had predicted development but its rapid onset was not predicted. Fig. 2 shows a possible sequence of events when there is convective activity producing mid-level heating in the hatched region, a few hundred kilometers across. The ageostrophic circulation implied by Eqn. 4 is sketched in Fig. 2a. In circles are shown the implied vorticity sources and in square boxes the temperature sources. If there is a positive value of $\frac{d\bar{u}}{dz}$, the vorticity and thermal patterns that may be expected to develop are sketched in Fig. 2b. The vertical motion implied by the source term $S = 2f \frac{d\bar{u}}{dz} \frac{\partial \xi}{\partial x}$ is also shown. The whole structure is now very much that of a system able to grow through baroclinic instability.

3.2 Growth

It is useful when considering the various diabatic processes to consider the typical linear baroclinic wave structure shown in Fig. 3.

(1) Large scale latent heat release. The large scale cloud and rain tend to be concentrated as shown in Fig. 3. Qualitatively one may argue that the response to the heating will be as in Fig. 2a and will thus enhance the upward vertical motion, low level trough and upper ridge. Attempting to be more quantitative we may use the simple parameterisation described in Section 1. If the effective static stability in the moist ascending air is negligible, then the overall static

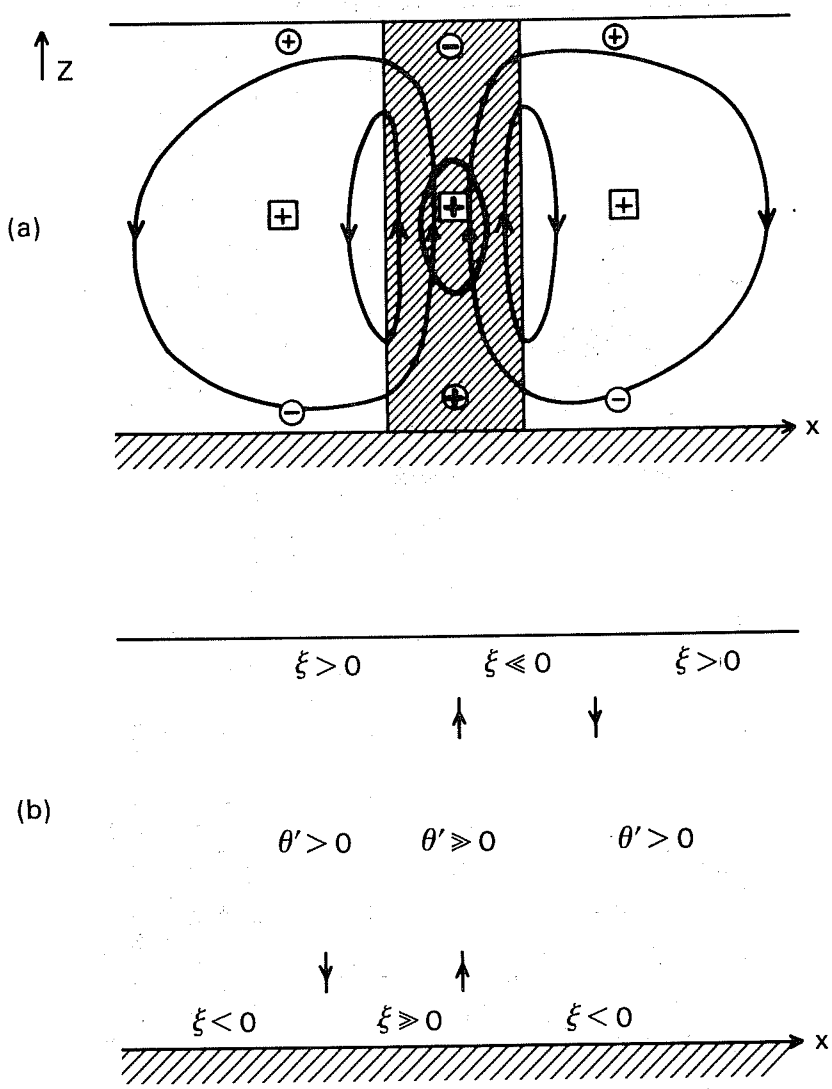


Fig. 2 (a) The response of the troposphere to heating in the hatched area.
(b) The induced fields showing a baroclinic wave structure. More details are given in the text.

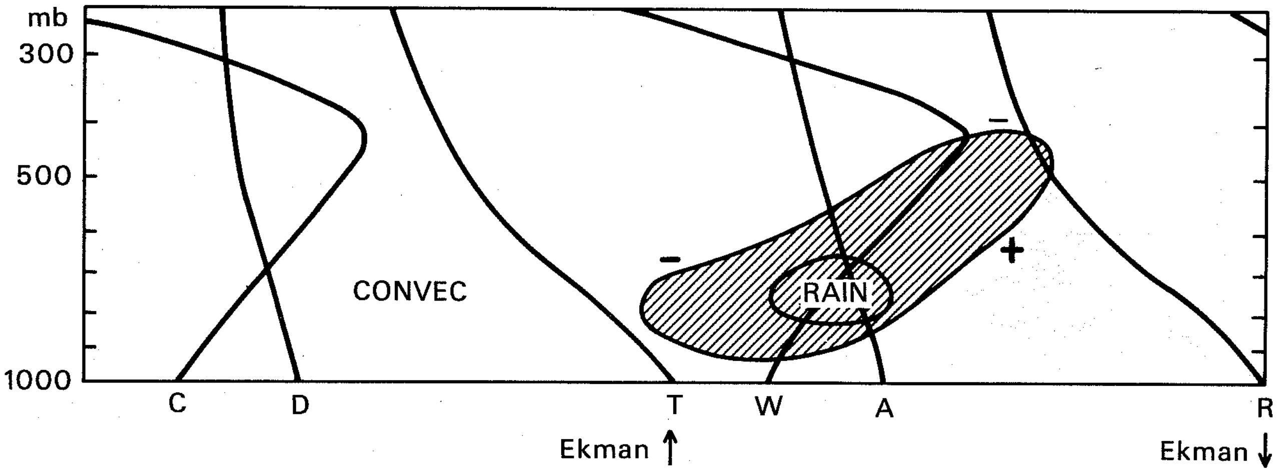


Fig. 3 A typical linear baroclinic wave structure plus the regions of occurrence of some physical processes. T and R denote the position of the trough and ridge at each level. Similarly A and D, and W and C denote the positions of the maximum ascent and descent, and warmth and cold. The hatched area is rain and cloud and the positive and negative signs give radiative heating and cooling.

stability to use in a linear calculation should be halved. The effect of halving the static stability has been described in Section 2, in particular in Fig. 1b. Thus large scale latent heat release may lead to greatly increased growth rates at wavelengths 2-3000 km and even a 20% increase at long wavelengths. This is in agreement with the findings of Gall (1976).

(2) Convection. A proportion of the precipitation near and ahead of the surface trough may be of a convective nature and so, as described in (1), is helpful to the development. However the convective heating in the cold air behind the surface trough will generally damp the baroclinic wave structure. As described by Tokioka (1973) for 1000 km length scale disturbances on a low Richardson number (2-5) flow convection can be a major ingredient in growing waves. However these waves have a different structure - in particular the pressure field tilts eastwards with height.

(3) Boundary layer. Off the east coasts of the major continents, boundary processes are clearly crucial in maintaining the baroclinity on which eddies may grow. However in doing so they also tend to damp the low-level temperature wave in any growing system. The boundary layer also plays a crucial role in the convective heating in the cold air discussed above. The role of boundary layer friction in damping baroclinic instability through Ekman pumping has been documented (e.g. Barcilon 1964). At the surface trough there is upward motion at the top of the boundary layer leading to divergence in the free atmosphere. It seems possible that a stable boundary layer could act as a buffer between the free atmosphere and the rough surface, thereby drastically reducing the Ekman pumping. The extra vertical motion associated with the boundary layer pumping will imply larger condensation and thus increase the effects described under (1) and (4).

(4) Long-wave radiation. In the absence of cloud, radiative effects will clearly provide a slight damping on the temperature wave. The more interesting and, perhaps, important effects arise when cloud has been produced. Referring to Fig. 3, we may expect the upper cloud ahead of the surface trough to give more warming below than cooling above. The lower cloud near the surface trough will have dominant cooling. The total effect on the development of the system may be guessed at using ideas described above, but the net result could depend crucially on the actual distribution of cloud and might vary greatly from one system to another.

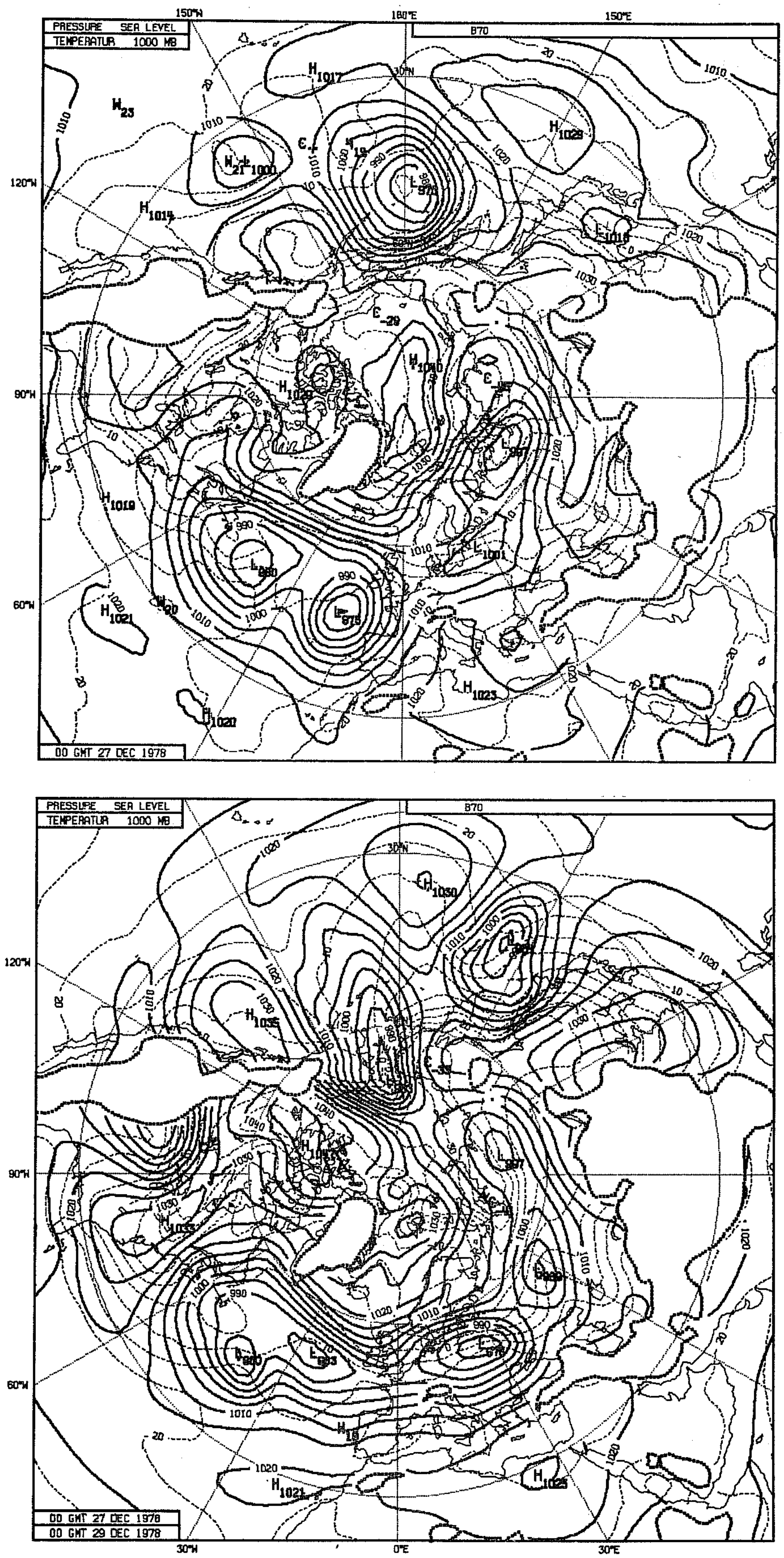
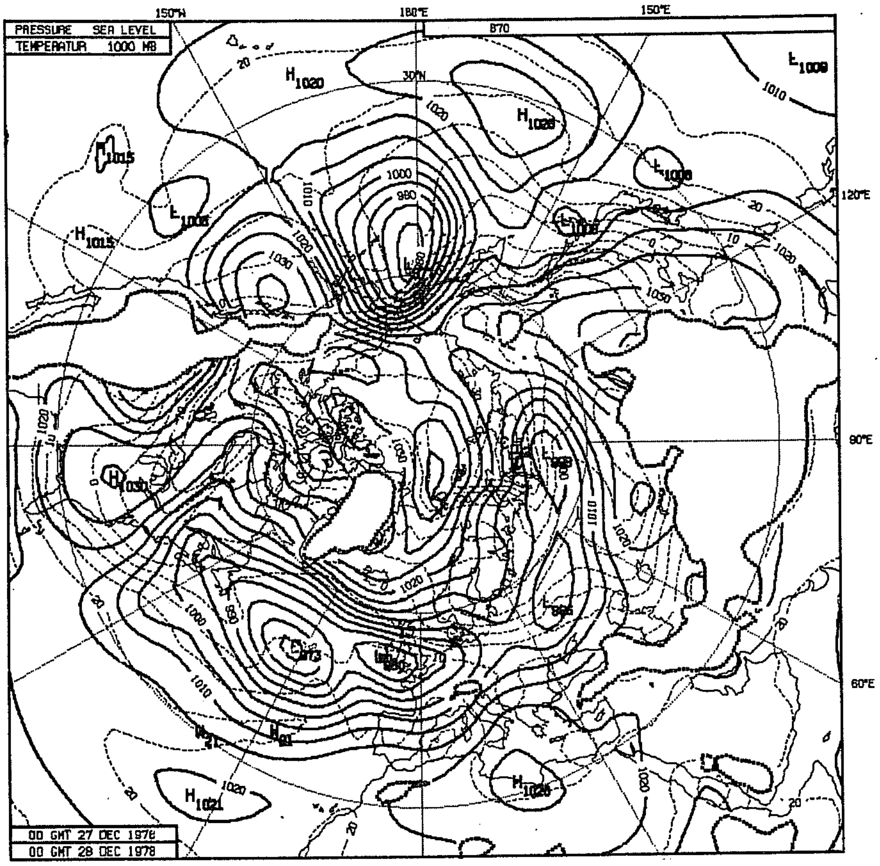
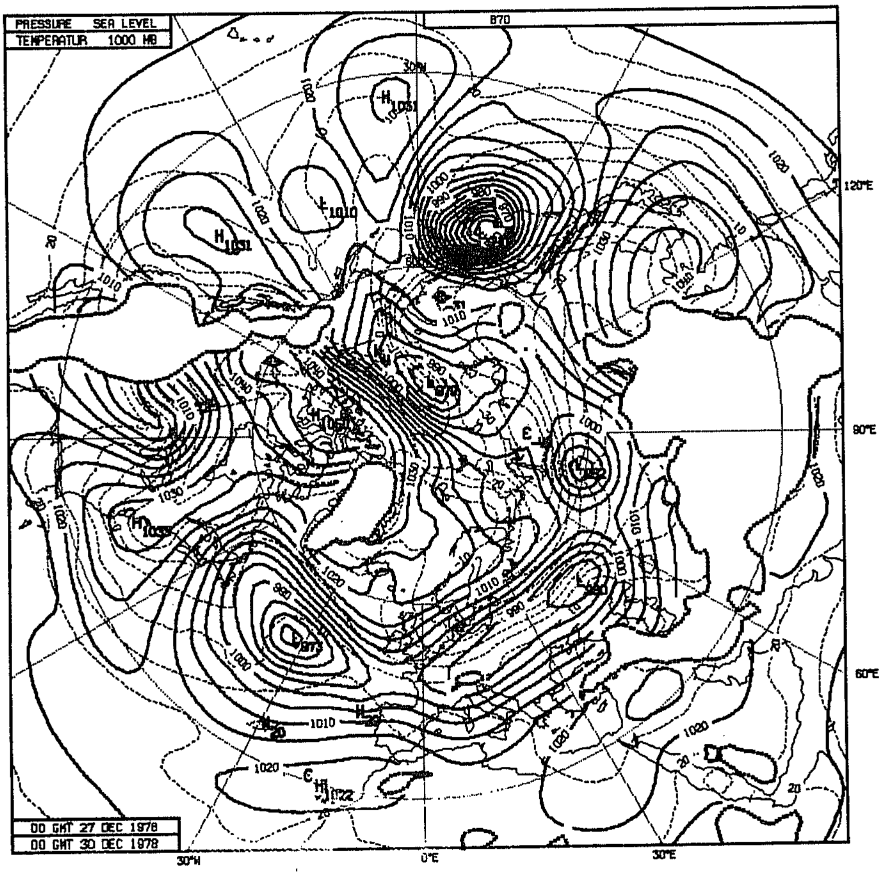


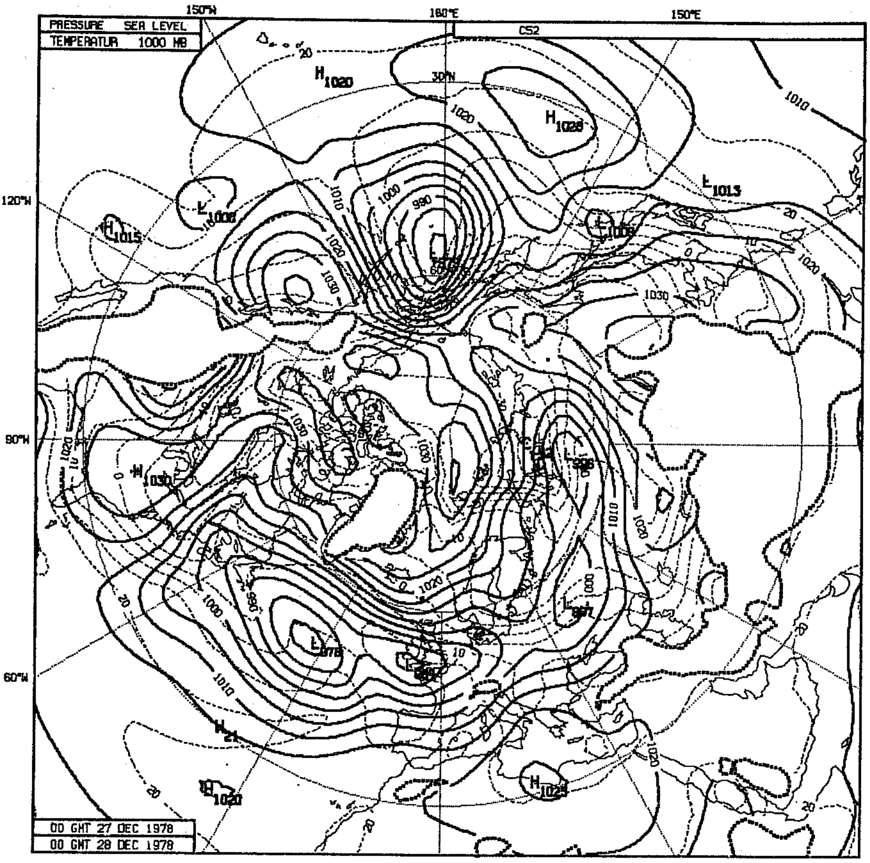
Fig. 4(a) Surface charts for day 0 (27 Dec 1978) and for days 1, 2 and 3 of the forecast with latent heat release.

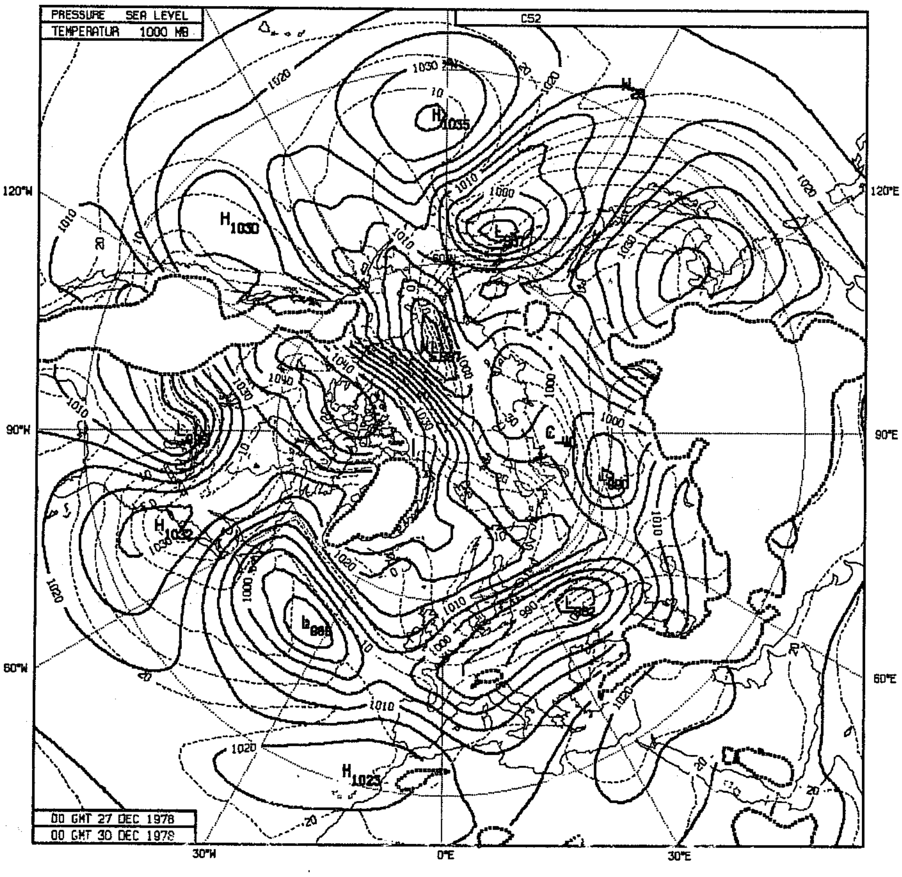


PRESSURE SEA LEVEL
TEMPERATUR 1000 MB
150°W
180°E
150°E
870
120°E
90°E
60°E
H1020
H1020
L1009
L1008
H1015
L1008
H1030
H1021
H1020
00 GMT 27 DEC 1978
00 GMT 28 DEC 1978



PRESSURE SEA LEVEL
TEMPERATUR 1000 MB
870
120°E
90°E
60°E
H1031
L1010
H1031
H1035
H1022
00 GMT 27 DEC 1978
00 GMT 30 DEC 1978
30°W
0°E
30°E


PRESSURE SEA LEVEL
TEMPERATUR 1000 MB
CS2
150°W
180°E
150°E
120°W
90°W
60°W
30°N
60°N
H1020
H1028
L1013
L1008
H1015
L1008
H1030
L978
H1024
H1020
00 GMT 27 DEC 1978
00 GMT 28 DEC 1978


PRESSURE SEA LEVEL
TEMPERATUR 1000 MB
CS2
120°W
90°W
60°W
120°E
90°E
60°E
30°W
0°E
30°E
H1035
H1030
L987
H1032
L988
H1023
00 GMT 27 DEC 1978
00 GMT 30 DEC 1978

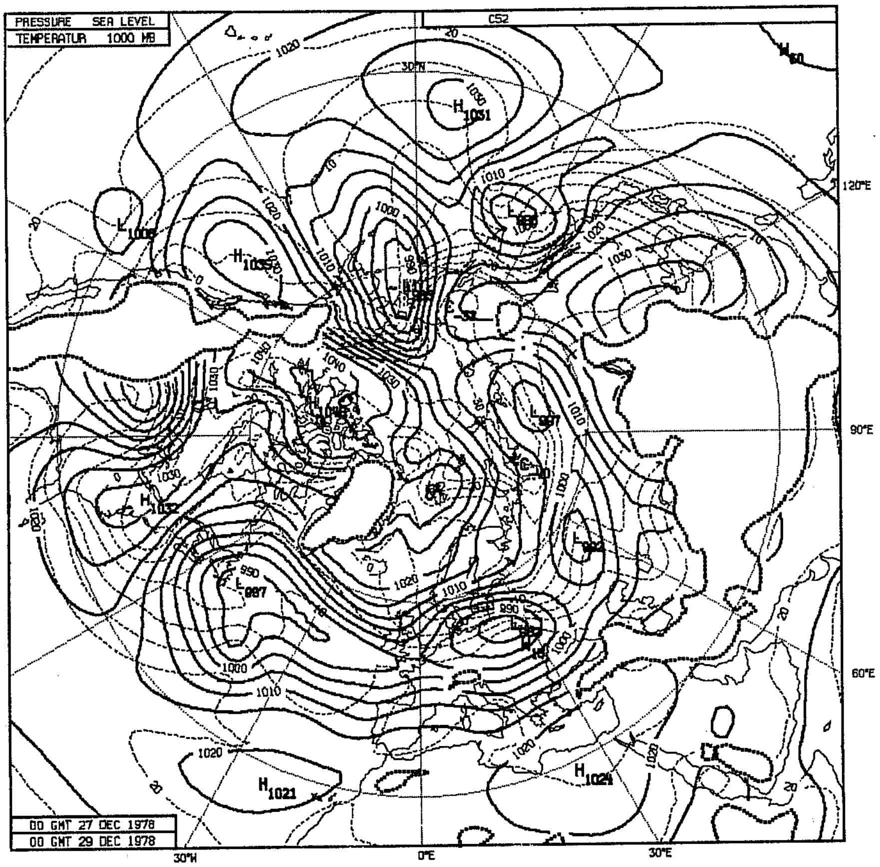


Fig. 4(b) Surface charts for days 1, 2 and 3 of the forecast without latent heat release.

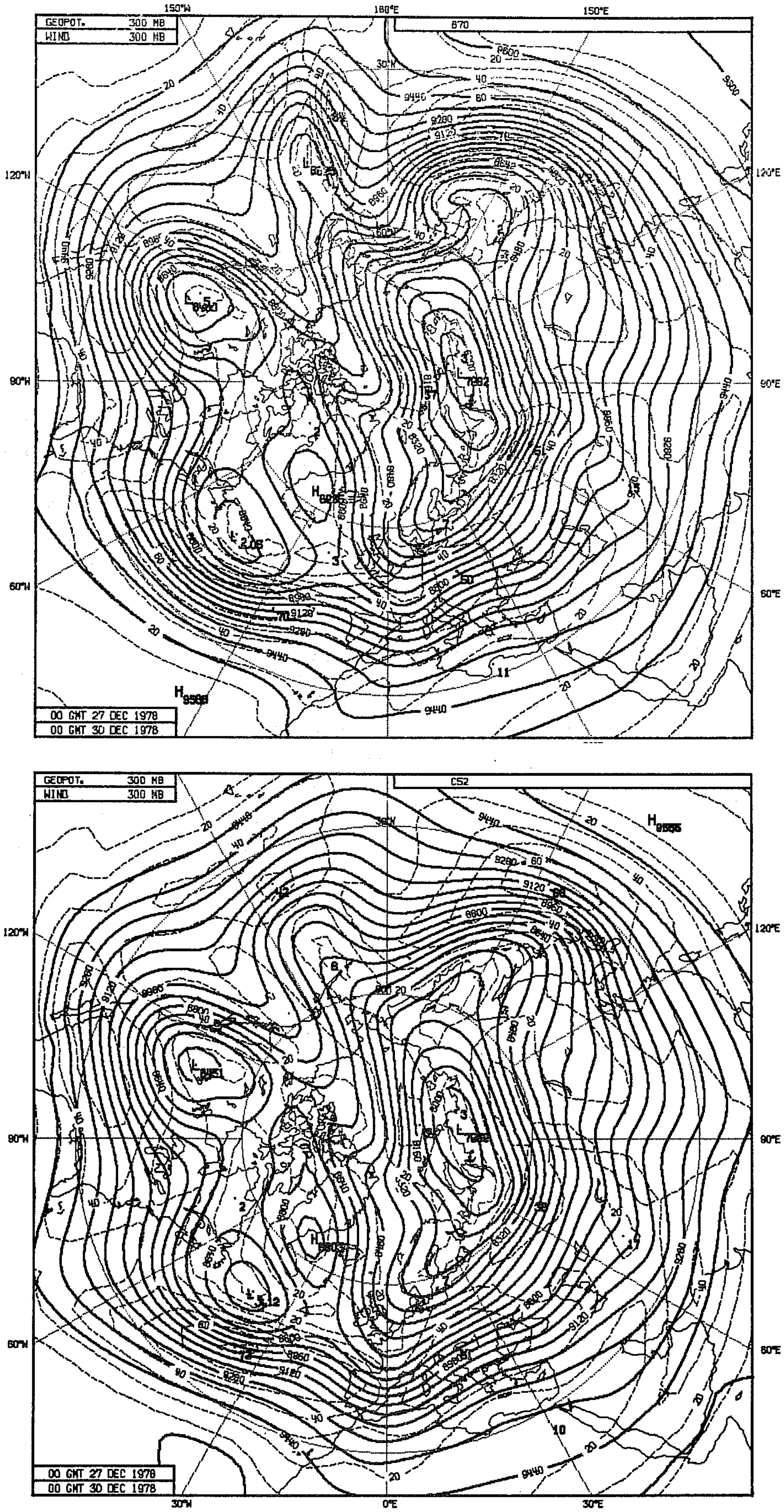


Fig. 4(c) 300 mb charts at day 3 for the two forecasts. The upper chart is for the forecast with latent heat release.

3.3 Decay

Diabatic processes can be crucial in the mature and decaying stages of transient waves. However the present state of knowledge allows little to be said on the subject.

4. A FORECAST EXAMPLE

To give an example where latent heat release was crucial in one system but not in others, two forecasts were made at ECMWF from initial data for 00 GMT 27 December 1978. The full forecast verified quite well at day 3. In particular it captured the spectacular development of a depression off the east coast of Asia. Surface pressure maps for days 0-3 are shown in Fig. 4a. The surface pressure minima at days 0,1,2,3 were 1016, 1009, 984, and 940 mb respectively. The second forecast was identical except that the value for the latent heat parameter was set to zero. At day 3 the forecasts were very similar almost everywhere except off the east coast of Asia. The corresponding system now attained 1009, 996, and 987 mb at days 1,2 and 3.

The surface pressure maps for the zero latent heat release forecast are also shown in Fig. 4b. At 300 mb, also, the forecasts were very similar except that, as expected from our previous arguments, the upper level ridge in the development region shows large amplification for the case with latent heat release (Fig. 4c). The absolute vorticity is near zero over a sizeable area.

Acknowledgement

It is a pleasure to record the help of Adrian Simmons with the forecasts described in Section 4 and Mike Revell with the preparation of Fig. 1.

References

Barcilon, V. 1964 Role of Ekman layers in the stability of the symmetric regime obtained in a rotating annulus. J.Atmos.Sci., 21, 291-299.

Gall, R. 1976 The effects of released latent heat in growing baroclinic waves. J.Atmos.Sci., 33, 1686-1701.

Hoskins, B.J., I. Draghici and H.C. Davies 1978 A new look at the ω-equation. Quart.J.Roy.Met.Soc., 104, 31-38.

Tokioka, T. 1973 A stability study of the medium scale disturbances with inclusion of convective effects. J.Met.Soc.Japan, 51, 1-9.

Tracton, M.S. 1973 The role of cumulus convection in the development of extratropical cyclones. Mon.Wea.Rev., 101, 573-592.