RADIATION SCHEMES

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1. Introduction:

The object of any parameterization of atmospheric radiation is to provide a simple, accurate and computationally fast method that is capable of calculating the required radiation properties in terms of host model variables.

There are three main types of radiation parameterization schemes used in conjuction with GCM's (and other types of models). They are:

(i) Empirical schemes: these schemes express the relevant and desired radiation properties as empirical functions of some model variable(s) (application of the Newtonian cooling approximation, for example).

(ii) Physical schemes: these schemes employ the governing equation of radiative transfer as a means of deriving the required radiation properties and the parameters inherent to this equation are provided directly by the model. These schemes rarely exist in practice since most models cannot specify the atmosphere to the level required of such a scheme .

(iii) Hybrid empirical-physical schemes: the great majority of schemes are actually a hybrid of empirically based methods and physical schemes. The extent of the empiricism varies from scheme to scheme. We can further classify these as: Type I - where empirical functions are directly introduced into the radiative transfer equation thus creating a "new" equation, and Type II - where the physics of radiation transfer is preserved and one or several parameters contained within that equation are parameterized in terms of model variables.

The advantage of purely empirical schemes lies in their ultimate simplicity and thus computational speed. The disadvantage is their lack of flexibility. They cannot be applied under conditions that extend beyond the range of those used in their original development. Physical schemes are far

more robust and thus more portable. That is, they can be applied to models of different phenomena and, if designed properly, can readily include additional radiative processes. The major disadvantage is in their computational inefficiency. Type I hybrid schemes attempt to overcome the disadvantage of purely physical schemes but in so doing become inflexibile. Type II hybrid schemes attempt to improve the efficiency of the physical methods while at the same time retaining flexibility and physical purity. It is clear that this type of parameterization philosophy is most desirable and most new schemes that have emerged over the past few years are usually of this type.

The following discussions will focus on hybrid schemes as the pure physically scheme do not exist in practice. The accuracy of the different schemes will not be addressed directly as that is a topic central to the following lecture.

2. The nature of the problem:

Given the usual assumption of horizontal uniformity (the validity of which was discussed in the last lecture) and by identifying the different radiative processes considered important to the given problem, then the transfer of radiation is well defined by an appropriate form of the radiative transfer equation. Thus the nature of the parameterization problem is to simplify the solution of this equation. The hurdles that must be overcome are best illustrated by considering equations appropriate for longwave flux.

$$F^{\uparrow}(z) = \int_0^{\infty} \pi B_{\nu}(z = 0)\tau_{\nu}^{f}(z, z = 0)d\nu + \int_0^{\infty}\int_0^{z} \pi B_{\nu}(z')\frac{d\tau_{\nu}^{f}}{dz'}(z, z')dz'd\nu, \qquad (1)$$

$$F^{\downarrow}(z) = \int_0^{\infty}\int_z^{\infty} \pi B_{\nu}(z')\frac{d\tau_{\nu}^{f}}{dz'}(z, z')dz'd\nu, \qquad (2)$$

where $F^{\uparrow}(z)$... and ..$F^{\downarrow}(z)$. are the upward and downward longwave fluxes through level z, B_{ν}.. is Planck function and .τ_{ν}^{f}.... the diffuse transmission function defined by the hemispheric integral

$$\tau_{\nu}^{f}(z, z') = 2\int_0^{1} \tau_{\nu}(z, z', \mu)\mu d\mu, \qquad (3)$$

where μ (=cosθ) is the cosine of the zenith angle and

$$\tau_\nu(z, z', \mu) = \exp\left[-\frac{1}{\mu}\int_{u(z)}^{u(z')} k_\nu(p, T)du\right], \qquad (4)$$

where $k_\nu(p, T)$ is the absorption coefficient and u is the concentration of the attenuating gas defined along the path from z to z'. The transmission function τ_ν^f is often referred to as the slab transmission function as it describes the transmission of the hemispheric radiative flux along the path u. There are several ways of defining the path quantity u and each defines a specific form of absorption coefficient (refer Stephens, 1984 for more discussion).

The solution of (1) and (2) is relatively straightforward given k_ν and the distribution of the absorber along the path dz'. Unfortunately formal integration of (1) and (2) is far too time consuming to be viable in any form of atmospheric circulation model. Thus some simplification is necessary. Towards this end, it is appropriate to observe the 4 integrals that are either explicit to or implied in (1) and (2). They are

(i) summation of contributions over all zenith angles (refer (3))

(ii) contibution of flux at z due to the surrounding atmosphere (dz')

(iii) summation over all spectral intervals

(iv) integration of absorption over the path u (in (4)).

The first of these is adequately approximated using a "diffusivity" factor β.. coupled to the path function u (i.e. the integration of the function (3) is approximated by that same function with a modified argument). The second integral is very much inseparable from the host model for which the parameterization is to be developed and also from the very nature of the problem under study (whether or not path discontinuties exist, such as a temperature inversion). The remaining two integrals are central to most problems of parameterizing radiative transfer in the "clear sky". They also loom as significant steps which must also be addressed in the problem of parameterizing the transfer of solar radiation although the latter has additional complications.

2.1 Parameterizing the integration over path

The integration of absorption over optical path is complicated by the

dependence of k_ν on both pressure and temperature. This path dependence of absorption introduces a twofold problem. The first is that atmospheric paths along which the fluxes are to be determined generally involve large variations in both pressure and temperature, and it is not appropriate to assume constant values of k_ν. The second is that most absorption data are collected in the laboratory at fixed pressures and tempertures and one is left with the problem of applying these data to atmospheric absorption problems. This latter problem is alleviated to some extent by measuring the absorption data at pressures and temperatures which are not greatly different than those expected in the atmosphere.

The two commonly used approximations employed to overcome these problems both assume that the absorption along a nonhomogeneous path can be approximated by the absorption along some homogeneous path with some appropriately adjusted value of p and T (we denote these hereafter as $\tilde{p}$ and $\tilde{T}$). These approximations are known as the scaling approximation and the Van de Hulst-Curtis-Goodson approximations VCG). We will now briefly review the first of these since it is the simplest, while the second, which is generally believed to be more accurate, has been described elsewhere (in any general text on atmospheric radiation).

The necessary assumption for the existance of the scaling approximation involves a separation of variables

$$k_\nu(p,T) = g(\nu)\ \phi\ (p,T) \tag{5}$$

from which it is possible to approximate the integral in (4) as

$$\int k_\nu(p,T)du \approx k_\nu(p_0, T_0) \int \frac{\phi(p, T)}{\phi(p_0, T_0)} du \approx k_\nu(p_0, T_0)\tilde{u}, \tag{6}$$

where p_o and T_o are the reference pressure and temperature. Thus only $\tilde{u}$ is required to determine the optical path and, together with $k_\nu(p_o,T_o)$ all other absorptive properties. The usual form of $\tilde{u}$ is

$$\tilde{u} = \int \left(\frac{p}{p_0}\right)^n \left(\frac{T_0}{T}\right)^m du. \tag{7}$$

which is based on our knowledge of the variation of k_ν with p.

TABLE 1 Generally accepted values of n and m for various absorbing species.

Gas	Spectral region	n	m
Water vapor		0.9–1	0.45
Carbon dioxide	Shortwave	1.75	11–8
Ozone		0	0
Water vapor	Longwave	0.5–0.9	0.45
Carbon dioxide		1.75	11–8
Ozone		0.4	0.2

Table 1 lists the currently accepted values of m and n applicable to the major absorption bands of both long- and short-wave absorption spectra.

While this scaling approximation is perhaps common to the majority of parameterization schemes, the separation of variables has at first sight, no real justification and seems somewhat empirical. However a closer look at the problem reveals some physical basis for the method (apart from the fact that it seems to work well). In fact the factorization in (5) can be shown to apply for frequencies far from the line center (and much of the radiant energy exchange perhaps occurs in the wings of strong lines).

2.2 Integration of absorption over frequency

The problem of simplifying the integration over frequency in (1) and (2) is unfortunately more complex than the rather simple and obvious task of averaging k_ν over some broad interval $\Delta\nu$. Complications arise because there are four different and distinguishable frequency (or wavelength) scales contained with the integral. If we superimpose cloud effects then there is, in principle, a fifth scale that must also be accounted for. (Examples are schematically illustrated in Fig 1A for longwave and 1B for shortwave.) The first scale refers to the relatively slow variation of the Planck function B_ν with frequency (upper panel). The second scale is that of the unresolved contour of the absorption band (second panel). For gases other than water vapor, the Planck function can be treated as a constant with frequency for each band. However, for water vapor it is necessary to subdivide the absorption bands into smaller sub-intervals. The next scale is that associated with the separation of individual lines (exploded scale in Fig. 1) while the finest frequency scale is that on which Lambert's law of absorption

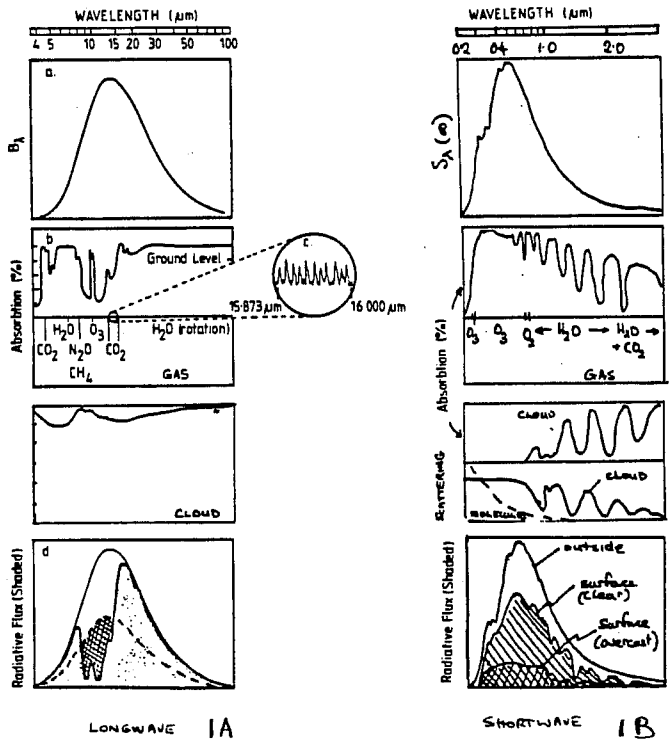

Fig. 1
Schematic of the various frequency scales encountered in the calculation of long-shortwave fluxes.

applies [i.e., a sub-line scale on which the exponential transmission function and hence on which (1) and (2) are valid]. The total broadband longwave flux then results from the convolution of the very fine frequency scale of line absorption with the coarser Planck scale (this flux is shown schematically as the shaded area in the lower panel of Fig. 1).

Clouds introduce a generally small effect in the spectral regions outside the atmospheric window region (for high cloud example shown in Fig. 1) . The absorption by cloud "fills" the absorption spectrum in the atmospheric window region (shown schematically as the additional cross hatched region: the total flux then is the sum of the two regions shaded in Fig. 1).

The integration of the absorption of solar radiation in some respects is less complex. The source function (upper curve of Fig. 1B) does not vary from level to level in the atmosphere as for the longwave problem although it is still necessary to superimpose the absorption spectrum of water vapor, (CO_2 and O_3 (second panel). In the presence of cloud, the absorption by droplets must also be overlapped somehow and the total absorption is then represented graphically as some combination of panels 2 and 3. The problem is further

complicated by the scattering of the atmosphere's molecules and by cloud particles. With the combination of all these processes, the flux of solar radiation at the ground (say) may have the spectral variation of the curve that defines the shaded region on the bottom panel of Fig. 1B.

There are basically two philosophically different ways of calculating this integrated flux quantity. The first is to resolve the variation of the emission function (i.e., the upper curves in Fig 1a and 1b) directly by dividing the spectrum into a number of discrete intervals and then to define the mean absorption characteristics for each interval. The second method involves the absorption and Planck functions and integrates this quantity over the entire longwave(and shortwave) spectra. While the first approach is more time consuming than the second, it does have several advantages,some of which are highlighted below. One particular advantage is that it is possible to choose one (or more) discrete band(s) to correspond to certain of the satellite radiometer channels (such as the 0.5-0.7 μ m channel of the TIROS N satellite). Satellite measurements provide a convenient way of calibrating the radiation parameterization. This is naturally achieved at the expense of computational efficiency and one must assess the trade-off between decreased computational speed and increased information. We will now discuss some of these approaches in terms of the parameterization philosophies identified in the introduction.

3. Broadband flux integration

3.1 The broadband longwave flux emissivity method (Type I)

It is obviously a great practical advantage to reduce the number of spectral intervals required to evaluate the boardband longwave fluxes and cooling rates to an absolute minimum. As we reduce the number of spectral intervals taken to divide the long-(or short-)wave absorption spectrum, then the transmission function becomes less exponential in character and our basic governing equations (e.g. (1) and (2)) are no longer applicable. Thus we then need to derive some modified version of this equation for our

parameterization. An example of this Type I parameterization is the so called "emissivity method" based on an "emissivity" function. This function is derived by integrating the absorption over frequency and weighting it in some way with the Planck function. This "integrated" absorption is expressed in terms of one parameter, u (or $\tilde{u}$ for nonhomogeneous paths). This function therefore has the obvious advantage of removing once and for all, the frequency integral in the longwave flux equations but does so at the expense of computation accuracy.

a) The longwave flux equations in the broadband emittance form

We can rewrite the flux equations (1) and (2) in the form

$$F^{\uparrow}(z) = \int_0^{\infty} B_{\nu}[1 - A_{\nu}(z, 0)]d\nu + \int_0^{\infty} \int_0^{z} \pi B_{\nu}(z') \frac{dA_{\nu}}{dz'}(z, z')dz'd\nu \tag{8}$$

for upward flux and a similar equation for downward flux

$$F^{\downarrow}(z) = \int_0^{\infty} \int_z^{\infty} \frac{dA_{\nu}}{dz'}(z, z')\pi B_{\nu}(z')dz'd\nu, \tag{9}$$

where A_{ν} $(=1 - \tau_{\nu})$ .. is the absorptivity of the gas. If we define the quantity ϵ along the path $z \rightarrow z'$ as

$$\epsilon(z, z') = \frac{1}{\sigma T^4} \int_0^{\infty} A_{\nu}(z, z')\pi B_{\nu}(T)d\nu, \tag{10}$$

then the flux equations follow as

$$F^{\uparrow}(z) = \sigma T_g^4(1 - \epsilon(z, 0)) + \int_0^{z} \sigma T^4(z') \frac{d\epsilon}{dz'}(z, z')dz' \qquad F^{\downarrow}(z) = \int_z^{\infty} \frac{d\epsilon}{dz}(z, z')\sigma T^4(z')dz \tag{11}$$

For homogeneous paths, (10) is the classical definition of emissivity relating the emission of a column of gas to that of a blackbody at the same temperature. This quantity has tangible physical significance and can, in principle, be measured although these measurements are very much ambiguous, often involving overlapping emission of other gases.

b) Alternate forms of "emissivity"

While $\epsilon(z,z')$ is a quantity most generally understood as emissivity, it is perhaps not always the most appropriate when applied to the calculations of

atmospheric fluxes since it is generally difficult to obtain expressions of $\epsilon(z,z')$ as a function of u that are accurate enough to evaluate $d\epsilon(z,z')/du$ [or equivalently $d\epsilon(z,z')/dz'$] with reasonable precision. This problem is overcome if we integrate (11) by parts to produce

$$\left.\begin{aligned} F^{\downarrow}(z) &= \int_0^{\infty} B_\nu(z=0)d\nu \\ &\quad + \int_0^{\infty}\int_0^{z} A_\nu(z,z')\frac{dB_\nu}{dz'}(z')dz'd\nu \\ F^{\downarrow}(z) &= \int_0^{\infty}\int_z^{\infty} A_\nu(z,z')\frac{dB_\nu}{dz'}(z')dz'd\nu \end{aligned}\right\}. \tag{12}$$

It is therefore appropriate to define quantities like

$$\epsilon'(z,z') = \frac{R(z,z')}{4\sigma T^3} = \int_0^{\infty} A_\nu(z,z')\frac{dB_\nu}{d\sigma T^4(z')}d\nu, \tag{13}$$

where $A_\nu(z,z')$ is the mean absorptivity parameter as defined by Elsasser and Culbertson (1960) The flux equations then follow as

$$\left.\begin{aligned} F^{\downarrow}(z) &= \int_z^{\infty} \epsilon'(z,z')\frac{d\sigma T^4(z')}{dz'}dz' \\ F^{\uparrow}(z) &= \sigma T_g^4 + \int_0^{z} \epsilon'(z,z')\frac{d\sigma T^4(z')}{dz'}dz' \end{aligned}\right\}. \tag{14}$$

While ϵ' bears no direct relationship to the actual emissivity ϵ, it does not differ greatly from it since the shape of dB_ν/dT is similar to that of B_ν. This aspect is shown in Fig. 2 in which both ϵ and ϵ', taken from a variety of sources, are shown as a function of water vapor path u for various specified temperatures.

The relationship between ϵ' and ϵ has significant practical implications. It is necessary in most schemes to truncate the top of the model at some level other than $Z=\infty$ in which case the blackbody function is nonzero. As a result, there will be some source of downward longwave flux at the top of the atmosphere. It is therefore necessary to include an additional term in the downward flux equation that allows for this radiation. The relevant flux equation is

$$F^{\downarrow}(z) = \sigma T^4(z_T)\epsilon(z_T,z) + \int_z^{z_T} \epsilon'(z,z')\frac{d\sigma T^4(z')}{dz'}dz'$$

where z_T refers to the top of the model and $T(\epsilon_T)$ is the characteristic radiation temperature of the atmosphere above the model. The calculation of longwave flux using the formulation thus requires the use of both ϵ and ϵ'. Ramanathan et al(1983) define a relationship between these as

$$\epsilon = \frac{\epsilon'}{0.847u^{0.222}}$$

thus requiring only the specification of ϵ' in the algorithm.

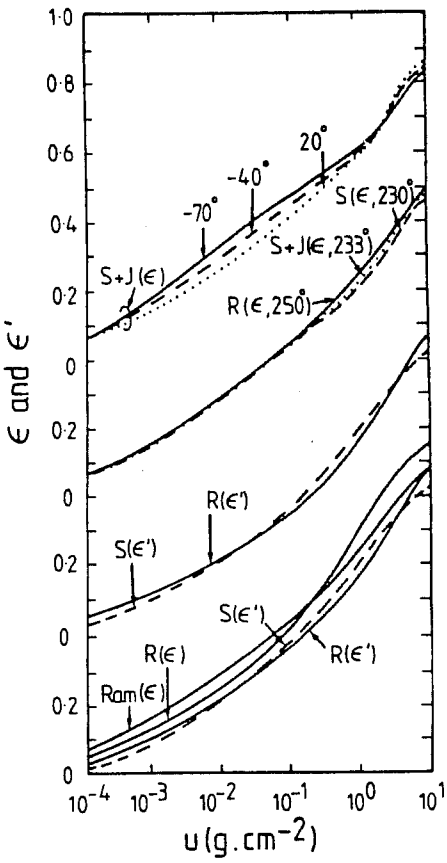

FIG. 2 The emissivity ϵ and the modified emissivity ϵ' as a function of water vapor path u. These relationships are taken from a variety of sources. R—Rodgers (1967), S & J—Staley and Jurica (1970), RAM—Ramanathan *et al.* (1983), S—Sasamori (1968). The upper curves show ϵ for three different temperatures and the lower curves show comparisons of ϵ and ϵ'.

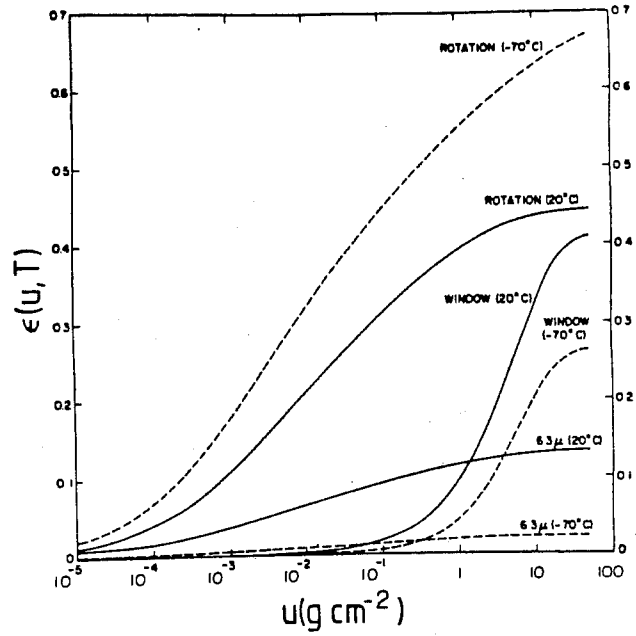

FIG. 3 The contributions to the total water vapor greybody emissivity by three broad spectral regions which include the water vapor rotation band, 6.3 μm band and the atmospheric window (excluding e type absorption). These contributions are shown as a function of u (or $\tilde{u}$ as the case may be) for two different temperatures (from Staley and Jurica, 1970).

While it is convenient, economically, to consider only one broad spectral interval, many applications require the breakdown of ϵ and ϵ' into smaller intervals in order to accomodate overlapping absorption by other gases for example. Typical of all Type I methods, once some modification from the original equations is sought, certain problems and inflexibilities arise. For example, Fig. 3 is included to illustrate the important, practical aspect that

a temperature factor is required when either $\in$ or $\in'$ is defined for a narrow spectral interval. Shown on this diagram are the contributions to by the individual absorption bands. These are strongly temperature dependent as a result of a variation of $B_\nu/\sigma T^4$ with temperature. It is rather fortuitous that the sum total is, more or less, independent of temperature as the ratio of $B_\nu/\sigma T^4$ for wavelengths in the rotation band increases with decreasing temperature and this incerase approximately cancels the reverse trend for wavelengths in the vibration band. For this reason, it is most inconvenient to calculate the longwave radiative flux for a few discrete spectral bands using the emissivity method.

Another complication arises with the emissivity method when we attempt to treat the effects of spectrally overlapping absorbers. For moleculer absorption, the most important overlap is that of H_2O and CO_2 abosrption in the spectral region from 580-750cm^{-1} . With an emissivity approach the treatment of this overlap has been empirically derived as

$$\epsilon_{\Delta\nu}(u_1{:}u_2, p, T) = \epsilon_{\Delta\nu}(u_1, p, T) + \epsilon_{\Delta\nu}(u_2, p, T) \quad - \Delta\epsilon_{\Delta\nu}(u_1, u_2, p, T) \qquad (15)$$

where the final term provides for an "overlap" correction. This empiricism and its associated error are not likely to be large for heating rates. However the application of similar approaches for clouds are likely to provide substantial errors.

3.2 Parameterization of shortwave gaeous absorption

If we neglect scattering, the transfer of broadband solar radiation is described by the following equation

$$S^{\downarrow}(z, \mu_0) = \mu_0 \int_0^\infty S_\nu(\infty)\tau_\nu(z, \infty, \mu_0)d\nu, \qquad (16)$$

where $S^{\downarrow}(z,\mu_0)$ is the downward irradiance through level z for a collimated beam of solar irradiance $S_\nu(\infty)$ at the top of the atmosphere inclined at a zenith angle μ_0. The monochromatic transmittance function for this collimated beam is

$$\tau_\nu(z, \infty, \mu_0) = \exp\left(-\frac{1}{\mu_0}\int_z^\infty k_\nu du\right). \qquad (17)$$

The transfer of solar radiation defined by (16) is less complex than the longwave counterpart as it is not necessary to consider the complicated problem of simultaneous absorption and emission from layer to layer in the atmosphere. While equivalent problems are enountered in the path and frequency integrals, the latter is also less involved as the spectral variation of the solar flux (i.e., of $S_\nu(\infty)$) can be defined a priori and the pressure and temperature effects on τ_ν only complicate matters for water vapor absorption. It thus remains to define the mean transmission function

$$\tau_i(z, \infty, \mu_0) = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp\left(-m_r(\mu_0)\int_z^\infty k_\nu du\right)d\nu, \tag{18}$$

where we introduce the relative air mass factor $m_r(\mu_0)$, in place of $1/\mu_0$ in (18). This factor is different only from $1/\mu_0$ for large solar zenith angles ($\theta_0 \gtrsim 75°$) when it is necessary to account for the effect of earth's curvature and atmospheric refraction. In most cases $m_r = 1/\mu_0$ except for ozone for which

$$m_r = 35\mu_0(1224\mu_0^2 + 1)^{-1/2} \tag{19}$$

should be used (Rodgers, 1967).

The clear-sky downward solar flux transmitted to level z along θ_0 is

$$S^\downarrow(z) = \mu_0 \sum_{i=1}^{N} S_i(\infty)\tau_{\nu i}(u), \tag{20}$$

where the integral in (16) is replaced by the summation over N subintervals and where u is used for the path length from ∞ to z along θ_0. The upward solar flux received at level z by reflection from the ground is similarly defined as

$$S^\uparrow(z) = \mu_0 \sum_{i}^{N} \alpha_{gi} S_i(\infty)\tau_{\nu i}(u^*), \tag{21}$$

where α_{gi} is the surface albedo for the ith subinterval. The path length u* is the effective total absorber amount traversed by the diffusely reflected radiation and this can be approximated usefully by (Lacis and Hansen, 1974)

$$u^* = m_r(\mu_0)u_0 + (u_0 - u)\bar{m}. \tag{22}$$

In this formula u_o is the total absorber amount of the vertical atmospheric column and $\bar{m}$ is an effective magnification factor for diffuse radiation analogous to β. Lacis and Hansen suggest that $\bar{m} \simeq 1.9$ is appropriate for ozone and 5/3 for water vapor.

The mean transmittance is commonly defined as a convolution of the transmission fuction and the extraterrestrial solar flux $S_\nu(\infty)$. One can then define a transmittance function averaged over the entire solar spectrum as

$$\bar{\tau}(z, \infty, \mu_0) = \frac{1}{S(\infty)} \int_0^\infty S_\nu(\infty)\tau_\nu(z, \infty, \mu_0)d\nu \tag{23}$$

in which case the downward solar flux at level z is defined by

$$S^\downarrow(z) = \mu_0 S(\infty)\bar{\tau}(z, \infty, \mu_0). \tag{24}$$

The use of (23) and (24) has proved to be the most popular method of calculating solar fluxes within the atmosphere since it is only necessary to perform the solar flux calculations once to obtain total broadband solar fluxes (in the same way that greybody emissivity reduced the complexity of the longwave flux calculations). However, there are a number of reasons why the discrete band approach of (20) is preferable. The major disadvantage of (24) is that different parameterizations of $\bar{\tau}$ for a given gas can vary with the choice of the extraterrestrial solar flux $S(\infty)$ even when the same absorption data are used. Therefore, each new estimate of the solar constant $S(\infty)$ requires a new parameterization of $\bar{\tau}$ As we will see below this is not a trivial disadvantage of the method.

(a) Broadband transmittance function for water vapor

Absorption of solar radiation by water vapor is the major source of solar heating in the lower atmosphere. The parameterization of this absorption is more complicated than that of ozone beacuse 1) the absorption coefficient of water vapor is more strongly frequency dependent and accurate absorption data are not available for the entire spectrum, ii) the absorption is complicated by the pressure and temperature dependencies and iii) both water vapor and liquid water absorption by clouds overlap thus precluding the simple reflecting schemes discussed below for ozone absorption.

There are numerous formulations for water vapor absorption. Fig. 4 displays the absorption $\bar{A}$ obtained from a number of these sources as a function of the water vapor path u. The difference between some of the formulations can be explained by the choice of $S(\infty)$ used to define $\tilde{\tau}$ (this certainly applies to the relations of Lacis and Hansen, 1974, compared to those of Fouquardt and Bonnel, 1980, or to the relationships of Yamamoto, 1962, compared to McDonald, 1960, for example). This diagram shows large discrepancies in the slope of the $\bar{A}$ versus the curves. Differences of solar heating are illustrated in Fig. 5. This diagram presents the clear-sky water vapor heating rate profiles with $\theta_0 = 60°$, $d_g = 0.7$ for the two model atmospheres of McClatchey et al. (1972) with $u_0 = 2.95$ cm (midlatitude summer) and $u_0 = 0.5$ cm (subarctic winter). For example, McDonald's absorptivities clearly underestimate the heating by 20-30%, those of Yamamoto and Korb et al. are inconsistent for middle to high altitude even though both use the data of Howard et al. The comparisons shown in Fig. 5

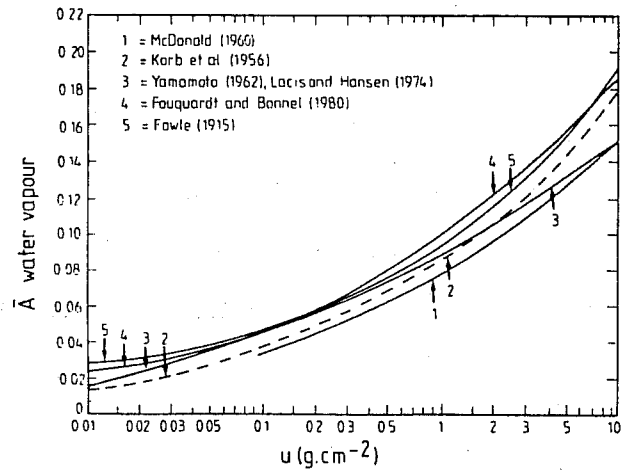

FIG. 4 A comparison of some different relationships between total water vapor absorptance and water vapor path u based on work cited.

demonstrate the importance of defining the absorptivity for relatively small values of $\tilde{u}$. The different methods for extrapolating the absorption in this small $\tilde{u}$ region could account for the discrepancies in the upper troposphere and lower stratosphere for the moist midlatitude summer atmosphere or for the discrepancies throughout the troposphere in the drier subartic winter atmosphere.

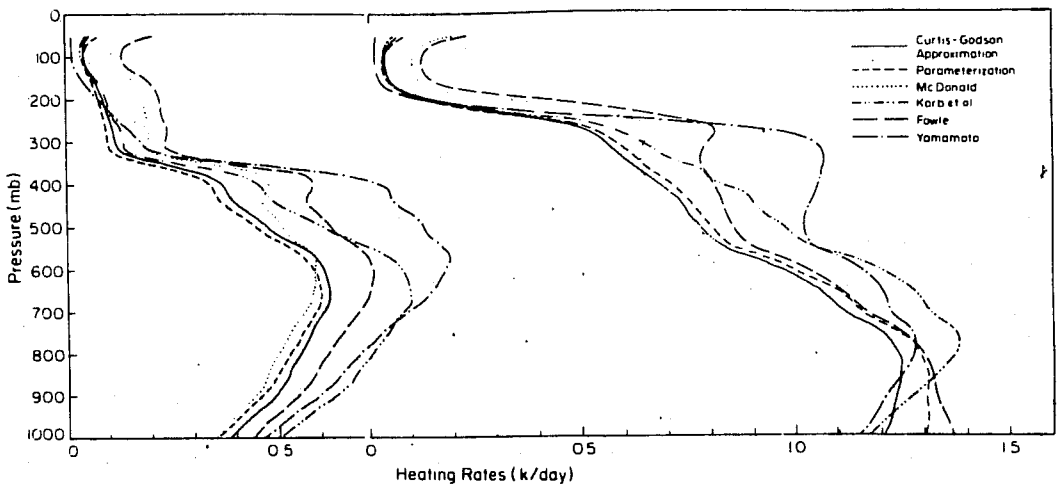

FIG. 5 Clear sky solar heating rate profiles due to water vapor absorption in model midlatitude summer and subarctic winter atmospheres. The profiles were calculated for $\theta_0 = 60°$ and $\alpha_g = 0.07$ for a variety of different absorption parameterizations which use either different absorption data and/or different extraterrestrial solar fluxes (refer Table 4 and discussion in text) (from Wang, 1976).

(b) Broadband transmittance functions for ozone

The absorption by ozone occurs at those wavelengths where molecular scattering is significant. Fortunately, this absorption occurs high in the atmosphere where there is little scattering owing to the low molecular density. Most parameterizations are therefore generally concerned with the downward direct solar beam although the absorption of the diffusely scattered and reflected upwelling flux cannot be totally excluded. The parameterization of this latter effect is readily achieved by invoking (24) and augmenting the "surface" albedo by a component that includes the combined effects of the atmosphere and surface (this is discussed below in more detail). Some examples of the parameterization of $\bar{A}$ for ozone are shown in Fig. 6

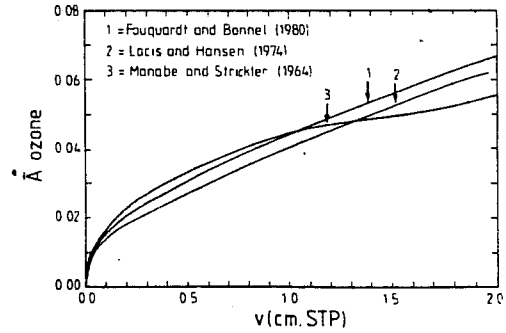

FIG. 6 A comparison of some different relationships between total ozone absorption as a function of ozone amount.

While it is fortunate that the absorption by ozone and that of water vapor occur in differnt regions of the atmosphere, it is also fortuitous that these absorptions occur in different regions of the solar absorption spectrum. Thus the combined absorption by ozone and water vapor is simply additive.

(c) Broadband transmittance functions for uniformly mixed gases.

The absorption of solar radiation by the uniformly mixed gases is small. Introduction of this absorption highlights the problems that are typical of the above described Type I parameterization - we cannot overlap the absorptions that occur in the same spectral region meaningfully. This is not likely to be a major problem in this case as the absorptions are small.

4. Type II schemes for molecular absorption

Rather than parameterize the transmission (or absorption) functions directly, Type II schemes implicitly parameterize of the optical depth which is defined in this context as

$$\delta_{\nu} = \int k_{\nu}(p,T)du = g(u) \qquad (25)$$

where g (u) is the required parameterization. In this way, the broadband transmittance approximates to

$$\bar{\tau}_{\Delta\nu} = \frac{1}{\Delta\nu} \int e^{-\delta_{\nu}} d\nu \simeq \exp(-\bar{g}(u)) \qquad (26)$$

where $\bar{g}$ is the broadband average of δ_{ν}. The transmission therefore maintains a purely exponential form. The presentation of this exponential form of transmission is important for several reasons (some mentioned above):

(i) The basic form of the longwave flux equations (1) and (2), and for that matter, the shortwave flux equations (16), rely on the assumption of Lambertain extinction (which simply states that transmission is exponential).

(ii) Overlapping absorption by the constituents, such as aerosol, cloud and other gases, in principal, is straighforward (provided scattering is

negligible).

(iii) Even when the combined effects of scattering and molecular absorption are important (such as for solar radiation transfer), then an exponential form of transmission allows the incorporation of these effects into the governing radiative transfer equation for scattering.

We have already remarked that the transmission averaged over broad spectral intervals becomes less exponential as the interval width increases. Thus Type II approaches necessarily involve the incorporation of several intervals and therefore not as computationally efficient as Type I schemes. Some examples are

(a) Band Models

The concept of a band model is to consider the absorption spectra in terms of some defined statistical function which enables the definition of average absorption properties over some given interval. Not all band models represent a Type II philosophy. An example of one that does, and perhaps the best known, is the Goody band model which yields the parameterization.

$$g(u) = \frac{\bar{S}u\beta}{\bar{d}}\left[1 + \frac{\bar{S}u\beta}{\pi\bar{\alpha}}\right]^{1/2} \qquad (27)$$

where $\bar{S}$, $\bar{\alpha}$ and $\bar{d}$ are the "mean" line intensity, half width and spacing for that given interval.

Using this approach, it seems that 15-25 or so discrete intervals in optimum for the integration over the longwave absorption spectra. One particular advantage of this approach is that the path integral approximation can be introduced in several different ways (via a single parameter scaling of u or via two or more scaled parmeters (e.g., VCG).

(b) The k distribution method.

An alternative approach has been employed more recently to approximate the frequency integration in the flux equations is the k distribution method. The method has two major advantages (i) it is demonstrably fast and accurate and (ii) it is straightforward to treat molecular absorption and the

scattering by cloud droplets in a self-consistent fashion.

The method makes use of the fact that for a homogeneous atmosphere, the transmission within a relatively wide spectral interval is independent of the ordering of the value of k_ν with respect to ν but depends only on the fraction of the interval that is associated with a particular value of k_ν. This fraction is just a probability density function defined such that $f(k)$ is the fraction of the frequency interval for which the absorption coefficient lies between k and $k+dk$. The basic idea of grouping frequency intervals according to line strengths dates back at least to Ambartsumian (1936). It has been used more recently by Arking and Grossman (1972), by Chou and Arking (1980, 1981) and Hansen et al. (1983) among others.

The k distribution is formally related to the transmission function for a homogeneous path via

$$\tau_i(z, z') = \frac{1}{\Delta\nu}\int_{\Delta\nu} e^{-k_\nu u}d\nu = \int_0^\infty f(k)e^{-ku}dk. \qquad (28)$$

Chou and Arking (1980) use this technique to calculate the radiative cooling by the major water vapor bands and employ a relationship like (7) to account for pressure variations along the absorption path. Hansen et al. demonstrated that the k distribution provides an alternate and more accurate treatment of vertical inhomogeneity of the atmosphere by noting that the k distributions at all altitudes are correlated in frequency space, i.e., the strongest (and weakest) absorption occurs at the same frequencies at all altitudes. The pressure effects are then included explicitly in the choice of k used to evaluate the k integral in (13).

Figure 7 shows a schematic illustration of the k distribution method. Panel (a) is a schematic of an absorption line spectrum at two different pressures. The greater the pressure (p_2), the broader the spectral features. Panel (b) illustrates examples of the two probability density functions f(k) associated with (a). The shaded area depicts the strongest absorption which refers to the same spectral interval. These two panels

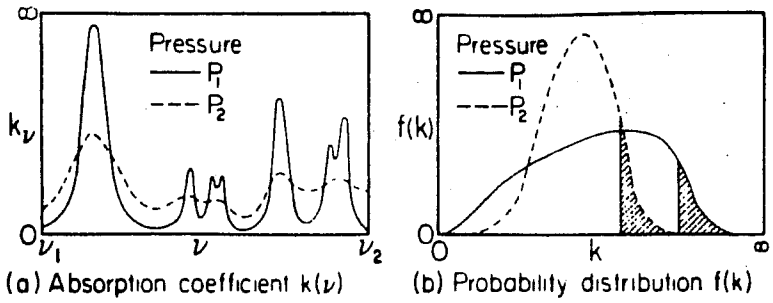

FIG.7. A schematic illustration portraying the essence of the *k*-distribution method. A schematic of absorption line spectra at two different pressures are shown in (a). In (b) the two probability density functions $f(k)$ associated with (a) are illustrated. The shaded area depicts the strongest absorption (i.e., largest k) for the same spectral interval (i.e., $f(k)$ for different pressures are correlated). Integration of $f(k)$ over k replaces the integration of k_ν over ν (modified from Hansen *et al.*, 1985).

illustrate the essence of the k distribution approach; the integration over ν space of a complex function k_ν is replaced by the integration of a more smoothly varying function over k space.

We approximate the integral in (28) in the following manner

$$\tau_{\Delta\nu}(\tilde{u}) = \int_{\Delta\nu} f(k)e^{-k\tilde{u}}dk \approx \sum_{n=1}^{N} f(k_n)e^{-k_n\tilde{u}}. \tag{29}$$

The sum of exponentials representation is sometimes called the "picket fence" approach since the distribution function $f(k_n)$ is taken to be only nonzero at discrete "n" values.

Thus the calculation of broadband flux involving (1) or (2) (or (16)) for some interval $\Delta\nu$ reduces to solving N "pseudo" monochromatic problems. In treating the problem of overlapping two different absorbers, then (29) is modified to

$$\tau(u_1,u_2) = \sum_{1=1}^{N}\sum_{j=1}^{M} F_{ij}\, e^{-(k_i u_1 + k_j u_2)} \tag{30}$$

where u_1 is the path associated with absorber 1 and u_2 is the absorber 2. We now require NxM solutions to our equation in order to obtain the broadband fluxes. However, it may be that the matrix $\tilde{F}$ of elements F_{ij} is primarily diagonal and some sort of linerization of $(\tilde{F})$ may be possible. In addition, it may be possible to provide a suitable approximation to $\tau(u_1,u_2)$ using only small numbers of "intervals" N and M. In any event, (30) has rarely been used with the notable exception of Feigleson (1970) who introduced a form similar to (30) for the transmission along a path in which water vapor,

CO_2 and liquid water absorption overlap.

5. Longwave cloud parameterization: The absorption approximation

The relevance of the interaction of longwave radiation with clouds for the different radiation budgets quantities and atmospheric cooling was discussed in the previous lecture. In that lecture, we derived the emissivity of a cloud in terms of the absorption approximation (i.e. we neglected scattering),

$$\varepsilon = A = 1-\exp(-\delta_\nu) \tag{31}$$

where

$$\delta_\nu = \frac{1}{\mu}\int a(z'')dz'' \tag{32}$$

and where $a(z'')$ is the absorption coefficient which is integrated over path dz". (In the last lecture we assumed, for convenience, that a was independent of z" - we now relax that assumption).

In the above context, α_ν is the volume absorption coefficient which is defined as

$$\alpha_\nu(z'') = \pi \int_0^\infty n(r, z'')r^2 Q_{abs}(\nu, r)dr, \tag{33}$$

where r is the radius of the water droplet, n(r) is the droplet's size distribution and Q_{abs} is the absorption efficiency of a droplet of radius. This efficiency is obtained from Mie theory for spherical water droplets (see Van de Hulst, 1957, for an excellent discussion of Mie theory).

The above formulae are not of great practical use since the droplet absorption coefficient a varies along the path z" (through the dependence of $n(r)$ on z") and contains the particle size distribution which is a quantity that is not readily (or may never be) supplied by general circulation models. Thus we need to establish some way of approximating the path integral. We encountered this same problem in relationship to molecular absorption in which the absorption coefficient varied as a function of path variables (in that case pressure and temperature).

Consider the following change of variable: we can convert a to a mass absorption coefficient via

$$a = \frac{K}{w} = \frac{3}{4}\frac{\int_0^\infty n(r, z'')r^2 Q_{abs}(\nu, r)dr}{\int_0^\infty n(r, z'')r^3 dr}, \tag{34}$$

where w is the liquid water content of the cloud at z". It is well known (e.g., Van de Hulst, 1957, p. 113) that Q_{abs} increases linearly with particle radius when the size parameter $x\,(=2\pi r/\lambda)$ is small. That is

$$Q_{abs}(\nu, r) = c(\nu)r \quad \text{for} \quad r < r_m, \tag{35}$$

where c is some frequency factor and r_m is the radius limit to which this linear relation is applicable. An example of such a relationship is shown in Fig. 8 for $\lambda = 9.5\ \mu m$ (i.e., $\nu = 1053\ cm^{-1}$). for which $r_m \approx 12.5 \mu m$.

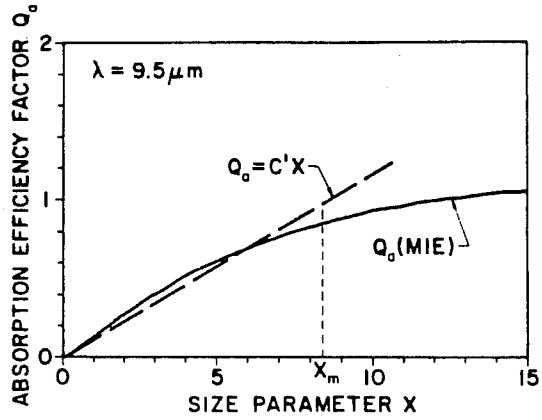

FIG. 8 The Mie absorption efficiency Q_a as a function of size parameter $x = (2\pi r/\lambda)$ and thus, of particle size for $\lambda = 9.5\ \mu m$. x_m corresponds to $2\pi r_m/\lambda$ where r_m is referred to in the text (from Pinnick *et al.*, 1979). ($Q_a = Q_{abs}$ in text)

It is indeed fortunate that such an approximation applies since (34) then reduces to

$$K = \frac{3}{4}c(\nu), \tag{36}$$

and the absorptivity follows as

$$A_\nu(z, z') = 1 - \exp(-k_\nu^c W(z, z')\mu^{-1}), \tag{37}$$

where $W(z, z')$ is the liquid water path along the geometric path and defined as

$$W(z, z') = \int_z^{z'} w(z'')dz'', \tag{38}$$

where the limits of the integral are usually defined from cloud top (or base) to the depth z' under consideration. This expression has been derived by a number of workers who all made use of the relationship between Q_{abs} and r (e.g., Platt 1976; Chylek 1978; Pinnick et al., 1979; among others).

Thus we see again the role of the separation of variables (ν and r in this case and ν and p/T for molecular absorption) in establishing an approximation to the path integral.

Equation (37) is only valid for clouds composed of droplets that are smaller than r_m . Despite the fact that real clouds contain some droplets that exceed this size limit, it has been illustrated repeatedly (both from observations and rigorous theory) that (37) describes the bulk absorption characteristics of a wide range of water cloud

The application of (37) and the assumptions contained within are not appropriate for those clouds composed chiefly of large particles (e.g., precipitating and ice crystal clouds) for which some other form of parameterization is required.

At this stage it is relevant to remark that the above is a Type II parameterization as we have merely provided some convenient way of parameterizing the (absorption) optical thickness of the cloud which could be incorporate into a scheme like that outlined by (30). This optical thickness does not change greatly with wavelength (i.e. cloud droplet absorption is "grey") thus simplifying the incorporation of droplet absorption with modecular absorption.

If we now consider the calculation of longwave fluxes through an atmosphere containing cloud and make the standard blackbody assumption for clouds in the IR, then the calculation of longwave flux through the entire atmosphere is not overly complicated by their presence. Black clouds simply add extra boundaries to the clear sky flux equations; one at the cloud base represneting an absorbing lid to the upwelling radiation from below and the other at cloud top forming another surface which radiates to space. The calculation of clear sky longwave flux then proceeds in the usual manner.

Unfortunately, we encounter difficulties when the cloud is only "partially" black (i.e. $\in < 1$) when we employ the emissivity method. These problems arise from the empirical nature of this Type I parameterization

that is from the need to define a transmittance along paths that have sections in both clear and cloudy sky. If we employ the customary greybody approach for both gaseous and cloud absorption, then the emission along a path with overlapped absorption is often approximated by

$$\epsilon_{overlap} \approx 1 - (1 - \epsilon_{cloud})(1 - \epsilon_{gas}) \tag{39}$$

which is the equivalent of (15). This overlapped emissivity was employed by Griffith and Cox (1979) and is implied in the formulas of Ramanathan et al. (1983) among others. Eq. (39) is, at best, an empirical approximation to the problem of absorption overlap and the errors associated with its applications have not been assessed. If these errors prove to be intolerable, then some alternate method of overlapping cloud and gaseous absorption is required (e.g., such as that of Fiegelson, 1970, or that described as follows in relation to shortwave parameterization).

As a final comment, we addressed the issue of partial cloudiness and the empirical nature in which we attempt to overcome this problem. This will not be discussed further in this lecture.

6. Parameterization of the shortwave properties of cloud, a Type II approach

The interaction of clouds with solar radiation cannot be described by the simple absorption models discussed above for longwave flux transfer since the attenuation includes important contributions by scattering and absorption of the cloud particles as well as by gas absorption. The problem of solar transfer is thus complex and one is formally faced with the twofold problem of not only characterizing the cloud microphysics but also solving a complex radiative transfer equation.

In the last lecture, we introduced the two flow version of the general radiative transfer equation to provide some insight into relative transfer processes in clouds. There we identified the three major optical profiles which, together, enable the solution of this two flow model in a closed form.

This model provides an idea basis for the development of a parameterization scheme. The heart of any parameterization based on such a model then rests with the suitable description of these optical properties.

6.1 Cloud optical thickness

We can formally express the cloud optical depth as

$$\delta = \int_0^{\Delta z} \underbrace{\int_0^{\infty} n(r)\pi r^2 Q_{ext}\left(x = \frac{2\pi r}{\lambda}, m_\lambda\right) dr}\, dz. \qquad (40)$$

where the inner underlined integral defines the volume attenuation coefficient (α) as used in the previous lecture.

The difficulty in evaluating (40) lies in the specification of both n(r) and the efficiency factor for extinction Q_{ext}. The latter quantity is defined as the ratio of the extinction to geometric cross-sectional areas of the particles. For spherical droplets, Q_{ext} is obtained from Mie theory and is a function of particle size through the Mie size parameter $x = 2\pi r/\lambda$ and of the refractive index of the particle $m\lambda$. For large x, i.e. for shortwave radiation, Q_{ext} asymptotes to a value of approximately 2. This feature, together with the definition

$$r_e = \frac{\int_0^{\infty} n(r)r^3 dr}{\int_0^{\infty} n(r)r^2 dr} \qquad (41)$$

was employed by Stephens (1978b) to yield

$$\delta \approx \frac{3}{2} W r_e^{-1}, \qquad (42)$$

where W is the liquid water path ($g.m^{-2}$). This dependence of δ on r_e (or α on r_e) was illustrated in the previous lecture (refer Fig.16b). The parameterization contained in (42) is not practical as r_e is a microphysical property of the cloud and is not likely to be provided by a GCM. However, we expect that r_e is related to cloud liquid water; the greater the water content the larger is r_e, although

examples can be forwarded that contradict this (such as discussed in the previous lecture). If we assume that r_e is more or less a unique function of cloud liquid water than cloud optical depth can be described solely in terms of liquid water path. Some assessment of the variability of the r_e - W relation for different cloud types is therefore required in order to validate this single parameter parameterization.

Stephens (1978) used eight different cloud droplet size distributions to deduce such a relationship. An example of such a relation is shown in Fig. 9 from the spectral range $0.75 \leq \lambda \leq 4.0 \mu m$.

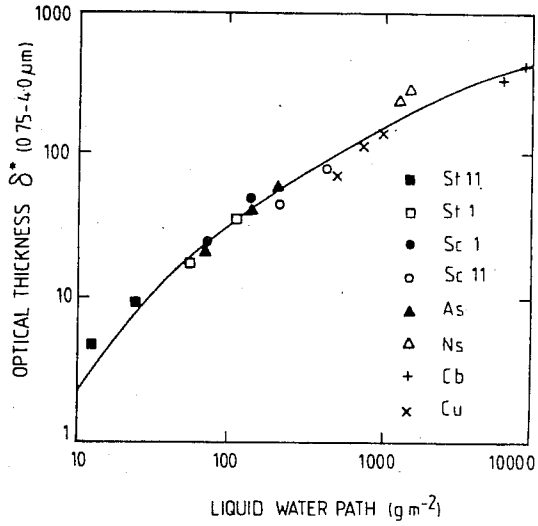

FIG. 9 . The optical thickness δ of various cloud types averaged over the range $0.75 < \lambda < 4.0$ μm as a function of cloud liquid water path based on a variety of cloud types (from Stephens, 1978b).

6.2 Single scatter albedo

The key to the evaluation of cloud absorption using the two stream method is to specify $\tilde{\omega}_o$ accurately. Despite this, relatively few explicit parameterizations of this property can be found in the literature. We observed from the discussion of the previous lecture that $\tilde{\omega}_o$ as a function of r_e Fouquardt and Bonnel derived the following relationship:

$$\tilde{\omega}_0 = 0.9989 - 0.0004e^{-0.15\delta} \tag{43}$$

by tuning their calculations to a more accurate reference and claim that it is representative (within 5%) for $\delta \leq 20$.

Stephens tuned a two stream model to a more accurate set of calculations and tabulated values of $\tilde{\omega}_o$ as a function of liquid water path W and solar zenith angle μ_o.

6.3 Asymmetry parameter g

The relative invariance of g with differing cloud type and wavelength suggest that this parameter may be set as a constant. This assumption can be supported for water clouds, but it is more dubious for non spherical ice cloud particles.

6.4 Droplet and vapour absorption

We saw in the last lecture that both droplet and vapour absorption spectrally overlap. Unfortunately both are of similar magnitude and one cannot be neglected for the sake of the other (it is often assumed in very simple schemes that water vapour absorption is dominant). Droplet and vapour absorptions also have different functional dependences on solar zenith angle so it is not possible to parameterize one by modifying the parameterization of the other.

The basic problem in deriving shortwave absorption is that scattering complicates the issue. In order to include vapour absorption in simple scattering models such as described last lecture (or even more complex ones) we require that the transmission function for vapour is exponential (i.e. absorption is Lambertian) such that we can simply add the optical depth by vapour to cloud optical depth. We thus encounter the same problems that arose in developing a method of calculating broadband clear sky fluxes. For that problem we introduced the k distribution method (or sum of exponentials) such that

$$\tau_{\Delta\nu}(\tilde{u}) = \int_{\Delta\nu} f(k)e^{-k\tilde{u}}dk \approx \sum_{n=1}^{N} f(k_n)e^{-k_n\tilde{u}}. \tag{44}$$

Thus if one considers a frequency interval $\Delta\nu$ over which the cloud properties δ and $\tilde{\omega}_o$ are to be taken as constant, then we use the new properties

$$\left.\begin{aligned}\delta_n &= \delta + k_n\tilde{u}\\ \tilde{\omega}_n &= \tilde{\omega}_0\delta/\delta_n\end{aligned}\right\} \tag{45}$$

which replace δ and $\tilde{\omega}_o$ in the formal two stream solution. The broadband solar fluxes $S_{\Delta\nu}$ are then obtained from the individual solution for "S_n" and then combined by

$$S_{\Delta\nu} = \sum_{n=1}^{N} f(k_n)S_n(k_n). \tag{46}$$

Thus the problem of molecular absorption for the interval $\Delta\nu$ in a scattering cloud layer reduced to one of N "pseudo" monochromatic multiple scattering problems. The above procedure has been employed in many more complex multiple scattering problems as well as with the simpler two stream methods described before (e.g., Lacis and Hansen, 1974; Wiscombe and Evans, 1977; Liou and Sasamori, 1975, among others).

6.5 The transfer of solar radiation in a scattering, multilayered atmosphere

The two stream solutions described in the previous lecture apply only to a single homogeneous cloud layer with fixed values of δ, $\tilde{\omega}_o$ and g. The problem remains to combine several layers each with different values of these optical properties. It is useful to do this in a way that makes it relatively straight forward to include the variable effects of a number of different factors (such as different cloud and haze types). The simplest and often used approach is to consider only the depletion of the directly transmitted solar beam as in (16) or (24) and define an appropriate form of transmittance function. This Type I approach, while computationally simple, suffers from the drawback that it precludes the effects of multiple scattering between layers and that it is difficult to define a transmission function that correctly overlaps the combined effects of gas absorption, droplet absorption and droplet scattering not only in a single layer but also through several adjacent layers. A more precise and consistent approach which overcomes these problems is the so called adding method. This method forms the basis for the solution of a majority of the more sophisticated multiple scattering problems and has the advantage in that it is simple in concept and easily reduced in

complexity for the purpose of operational models. Despite these obvious advantages, the approach is rarely used in this simple context, (three examples are Lacis and Hansen, 1974; Stephens and Webster, 1979; Fouquardt and Bonnel, 1980).

An outline of the procedure can be best described by reference to a simple two layer atmosphere. If we employ the subscripts a and b to refer to the top and bottom layers respectively, then the reflection and transmission at the boundaries of the atmosphere for the combined layers are

$$\left.\begin{aligned} \alpha_{ab} &= \alpha_a + \tau_a\alpha_b\tau_a/(1-\alpha_a^*\alpha_b) \\ \tau_{ab} &= \tau_a\tau_b/(1-\alpha_a^*\alpha_b) \end{aligned}\right\}. \tag{47}$$

There are three important factors in (47) that sets the parameterization apart from the standard technique described by (20). These are:

(i) The multiplication of transmittance through the combined layers requires the transmission in a form consistent with (4).

(ii) Differentiating between the reflections from above (α_a) and below (α_b), (often it can be assumed $\alpha_a = \alpha_b$). It is important to provide such a distinction as the reflection of a layer illuminated by a diffuse source is different from the reflectance of layer illuminated by a collimated beam.

(iii) The propagator factor (i.e., the denominator) which accounts for the multiple reflections between layers. This factor is especially significant when the reflection between layers is large and this tyically occurs when a relatively dense cloud overlies a bright surface (such as another cloud or a snow covered surface).

It is a simple matter to extend (47) to several layers. The reflection and transmission of individual cloud layers can be determined using the simple recursive relations which can be found in either of the above three cited references.

7. Summary

In this lecture, the various different methods for calculating the transfer of radiation in the atmosphere were reviewed. These methods were categorized into 3 classes: empirical, physical and hybrid. Only the

latter class of techniques were considered. In most cases, radiation schemes mix empiricism with theory in order to gain computational speed. This invariably introduces further complexities when there is a need to incorporate other processes into the parameterization. This is highlighted in the discussion of the emissivity method which is perhaps the most popular way of calculating longwave flux but creates several problems when we try to overlap cloud absorption (say) with molecular absorption.

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