TECHNICAL MEMORANDUM

165

Description of the Radiation scheme in the ECMWF model

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CONTENTS

	Page
1. INTRODUCTION	1
2. LONGWAVE RADIATION	3
2.1. Vertical integration	4
2.2. Spectral integration	5
2.3. Incorporation of the effects of clouds	8
3. SHORTWAVE RADIATION	10
3.1. Spectral integration	11
3.2. Vertical integration	14
3.2.1. Cloudy fraction of the layers	14
3.2.2. Clear-sky fraction of the layers	19
3.3. Multiple reflections between layers	21
3.4 Cloud shortwave optical properties	23
REFERENCES	24

1. INTRODUCTION

This report describes in details the radiation scheme which, since 2 May 1989, is part of the package of parametrizations for the physical processes in the ECMWF forecast model (ECMWF Research Manual 3). This radiation package calculates the heating/cooling rate due to absorption-emission of longwave radiation and reflection, scattering and absorption of solar radiation by the earth's atmosphere and surfaces. Outline of the code, in the format of Stephens's (1984) paper, is given in Table 1. The longwave and shortwave radiation parts of the scheme are described in details in sections 2 and 3, respectively.

Table 1: Summary of the ECMWF operational radiation code

a. Clear-sky

(i) Shortwave: Two-stream formulation employed together with photon path distribution method (Fouquart and Bonnel, 1980) in 2 spectral intervals (0.25-0.68 and 0.68-4.0 μm).

Rayleigh scattering	Parametric expression of the Rayleigh optical thickness
Aerosol scattering and absorption	Mie parameters for 5 types of aerosols based on climatological models (WMO-ICSU, 1984)
Gas absorption	from AFGL 1982 compilation of line parameters (Rothman et al., 1983)
H_2O	1 interval
Uniformly mixed gases	1 interval
O_3	2 intervals

(ii) Longwave: Broad band flux emissivity method with 6 intervals covering the spectrum between 0 and 2620 cm^{-1}. Temperature and pressure dependence of absorption following Morcrette et al. (1986). Absorption coefficients fitted from AFGL 1982.

H_2O	6 spectral intervals, e- and p-type continuum absorption included between 350 and 1250 cm^{-1}
CO_2	Overlap between 500 and 1250 cm^{-1} in 3 intervals by multiplication of transmission
O_3	Overlap between 970 and 1110 cm^{-1}
Aerosols	Absorption effects using an emissivity formulation

b. Cloudy sky

(i) Shortwave

Droplet absorption and scattering	Employs a delta-Eddington method with τ and ω determined from LWP, and preset g and r_e
Gas absorption	Included separately through the photon path distribution method

(ii) Longwave

Scattering	Neglected
Droplet absorption	From LWP using an emissivity formulation
Gas absorption	as in (a.ii)

Timing: 8 ms for a column computation for a 19-level model on a CRAY XMP-48

2. LONGWAVE RADIATION

The rate of atmospheric cooling by emission-absorption of longwave radiation is

$$\frac{dT}{dt} = \frac{g}{C_p} \frac{dF}{dp} \qquad (2.1)$$

where F is the net total longwave flux.

Assuming a non-scattering atmosphere in local thermodynamic equilibrium, F is given

$$F = \int_{-1}^{+1} \mu \, d\mu \int_0^{\infty} d\nu \left[L_\nu(p_s,\mu) \; t_\nu(p_s,p,\mu) + \int_{p_s}^{0} L_\nu(p',\mu) \; dt_\nu \right] \qquad (2.2)$$

where $L_\nu(p,\mu)$ is the monochromatic radiance of wavenumber ν at level p propagating in a direction such as μ is the cosine of the angle that this direction makes with the vertical, and $t_\nu(p,p',\mu)$ is the monochromatic transmission through a layer whose limits are at p and p' seen under the same angle ϑ.

After separating the upward and downward components, and integrating by parts, we obtain the radiation transfer equation as it is actually estimated in the radiation code

$$F_\nu^+(p) = \left[B_\nu(T_s) - B_\nu(T_{0+}) \right] \; t_\nu(p_s,p;r) + B_\nu(T_p) + \int_{p_s}^{p} t_\nu(p,p';r) \;\; dB_\nu \qquad (2.3)$$

$$F_\nu^-(p) = \left[B_\nu(T_t)-B_\nu(T_\infty)\right] \ t_\nu(p,0;r) \ + B_\nu(T_p)$$

$$+ \int_p^0 t_\nu(p',p;r) \ dB_\nu$$

where, taking benefit of the isotropic nature of the longwave radiation, the radiance L_ν of (2.2) is replaced by Planck function $B_\nu(T)$ in unit of flux, Wm^{-2} (hereafter B_ν always includes the π factor). T_s is the surface temperature, T_{0+} that of the air just above the surface, T_p is the temperature at level of pressure p, T_t that at the top of the atmospheric model. The transmission t_ν is evaluated as the radiance transmission in a direction θ to the vertical such that $r = \sec\vartheta$ is the diffusivity factor (Elsasser, 1942). Such an approximation for the integration over the angle is usual in radiative transfer calculations, and tests on the validity of this approximation have been presented by Rodgers and Walshaw (1966) among others. The use of the diffusivity factor gives cooling rates within 2 % of those obtained with a 4-point Gaussian quadrature.

2.1 Vertical integration

The integrals in (2.3) are evaluated numerically, after discretization over the vertical grid, considering the atmosphere as a pile of homogeneous layers. As the cooling rate is strongly dependent on local conditions of temperature and pressure, and energy is mainly exchanged with the layers adjacent to the level where fluxes are calculated, the contribution of the distant layers is simply computed using a trapezoidal rule integration, but the contribution of the adjacent layers is evaluated with a 2-point Gaussian quadrature, thus

$$\int_{p_s}^{p_i} t_\nu(p,p';r) = \sum_{l=1}^{2} dB_\nu(l) \ w_l \ t_\nu(p_i,p_l;r)$$

$$+ \frac{1}{2}\sum_{j=1}^{i-2} dB_\nu(j) \ [t_\nu(p_i,p_j,r) + t_\nu(p_i,p_{j-1},r)] \qquad (2.4)$$

where p_l and w_l are the pressure corresponding to the gaussian root and the gaussian weight, respectively. $dB_\nu(j)$ and $dB_\nu(l)$ are the Planck function gradients calculated between two interfaces, and between mid-layer and interface, respectively.

2.2 Spectral integration

The integration over wavenumber ν is performed using a band emissivity method, as first discussed by Rodgers (1967). The longwave spectrum is divided into six spectral regions

1. 0 - 350 cm^{-1} + 1450 - 1880 cm^{-1}
2. 500 - 800 cm^{-1}
3. 800 - 970 cm^{-1} + 1110 - 1250 cm^{-1}
4. 970 - 1110 cm^{-1}
5. 350 - 500 cm^{-1}
6. 1250 - 1450 cm^{-1} + 1880 - 2820 cm^{-1}

corresponding to the centers of the rotation and vibration-rotation bands of H_2O, the 15 micron band of CO_2, the atmospheric window, the 9.6 micron band of O_3, the 25 micron "window" region, and the wings of the vibration-rotation band of H_2O, respectively. Over these spectral regions, band fluxes are evaluated with the help of band transmissivities precalculated from the narrow-band model of Morcrette and Fouquart (1985) -- See Appendix of Morcrette et al. (1986) for details --.

Integration of (2.3) over wavenumber ν within the k-th spectral region gives the upward and downward fluxes as

$$F_k^+(p) = [B_k(T_s) - B_k(T_{0+})]\, t_{B_k}(ru(p_s,p),\; T_u(p_s,p)) \quad + \; B_k(T_p)$$
$$+ \int_{p_s}^{p} t_{dB_k}(ru(p,p'),\; T_u(p,p'))\, dB_k \qquad (2.5a)$$

$$F_k^-(p) = [B_k(T_0) - B_k(T_\infty)] \quad t_{B_k}(ru(p,0)\ ,\ T_u(p,0)\ ) \quad - \ B_k(T_p) - \int_p^0 t_{dB_k}(ru(p',p),\ T_u(p',p))\ dB_k \tag{2.5b}$$

The formulation accounts for the different temperature dependences involved in atmospheric flux calculations, namely that on T_p, the temperature at the level where fluxes are calculated, and that on T_u, the temperature that governs the transmission through the temperature dependence of the intensities and half-widths of the lines absorbing in the concerned spectral region. The band transmissivities are non-isothermal accounting for the temperature dependence that arises from the wavenumber integration of the product of the monochromatic absorption and the Planck function. Two normalized band transmissivities are used for each absorber in a given spectral region: the first one for calculating the first r.h.s. term in (2.3), involving the boundaries; it corresponds to the weighted average of the transmission function by the Planck function

$$t_B(\bar{up}, T_p, T_u) = \frac{\int_{\nu_1}^{\nu_2} B_\nu(T_p)\ t_\nu(\bar{up}, T_u)\ d\nu}{\int_{\nu_1}^{\nu_2} B_\nu(T_p)\ d\nu} \tag{2.6a}$$

the second one for calculating the integral terms in (2.3) is the weighted average of the transmission function by the derivative of the Planck function

$$t_{dB}(\bar{up}, T_p, T_u) = \frac{\int_{\nu_1}^{\nu_2} dB_\nu(T_p)/dT \quad t_\nu(\bar{up}, T_u)\ d\nu}{\int_{\nu_1}^{\nu_2} dB_\nu(T_p)/dT\ d\nu} \tag{2.6b}$$

where $\bar{up}$ is the pressure weighted amount of absorber.

In the scheme, the actual dependence on T_p is carried out explicitly in the Planck functions integrated over the spectral regions. Although normalized relative to $B(T_p)$ (or $dB(T_p)/dT$), the transmissivities still depend on T_u, both through Wien's displacement of the maximum of the Planck function with temperature and through the temperature dependence of the absorption coefficients. For computational efficiency, the transmissivities have been developed into Padé approximants

$$t(\bar{up}, T_u) = \frac{\sum_{i=0}^{2} C_i \, u_{eff}^{i/2}}{\sum_{j=0}^{2} D_j \, u_{eff}^{j/2}} \qquad (2.7)$$

where $u_{eff} = r \; \bar{up} \; f(T_u, \bar{up})$ is an effective amount of absorber which incorporates the diffusivity factor r, the weighting of the absorber amount by pressure, $\bar{up}$, and the temperature dependence of the absorption coefficients, with

$$f(T_u, \bar{up}) = \exp\,[a(\bar{up})\ (T_u - 250) + b(\bar{up})\ (T_u - 250)^2] \qquad (2.8)$$

The temperature dependence due to Wien's law is incorporated although there is no explicit variation of the coefficients C_i and D_j with temperature. These coefficients have been computed for temperatures between 187.5 and 312.5 K with a 12.5 K step, and transmissivities corresponding to the reference temperature the closest to the pressure weighted temperature T_u are actually used in the scheme.

2.3 Incorporation of the effects of clouds

The incorporation of the effects of clouds on the longwave fluxes follows the treatment discussed by Washington and Williamson (1977). Whatever the state of cloudiness of the atmosphere, the scheme starts by calculating the fluxes corresponding to a clear-sky atmosphere and stores the terms of the energy exchange between the different levels (the integrals in (2.3)). Let F_0^+ (i) and F_0^- (i) be the upward and downward clear-sky fluxes. For any cloud layer actually present in the atmosphere, the scheme then evaluates the fluxes assuming a unique overcast cloud of unity emissivity. Let F_n^+ (i) and F_n^- (i) the upward and downward fluxes when such a cloud is present in the n-th layer of the atmosphere. Downward fluxes above the cloud and upward fluxes below it have kept their clear-sky values

$$F_n^+ (i) = F_0^+ (i) \quad \text{for } i \le n$$
$$F_n^- (i) = F_0^- (i) \quad \text{for } i > n \tag{2.9}$$

Upward fluxes above the cloud (F_n^+ (k) for k ≥ n+1) and downward fluxes below it (F_n^- (k) for k < n) can be expressed with expressions similar to (2.3) provided the boundary terms are now replaced by terms corresponding to possible temperature discontinuities between the cloud and the surrounding air.

$$F_n^+(k) = [F_{cld}^+ - B(n+1)]\ t(p_k,p_{n+1};r) + B(k) + \int_{p_{n+1}}^{p_k} t(p_k,p';r)\ dB$$
$$\tag{2.10}$$
$$F_n^-(k) = [F_{cld}^- - B(n)\ ]\ \ t(p_k,p_n\ ;\ r) + B(k) + \int_{p_k}^{p_n} t(p',p_k;r)\ dB$$

where B(i) is now the total Planck function (integrated over the whole longwave spectrum) at level i, and F_{cld}^+ and F_{cld}^- are the fluxes at the upper and lower boundaries of the cloud. Terms under the integrals correspond to exchange of energy between layers in clear-sky atmosphere and have already been computed in the first step of the calculations. This step is repeated for all cloudy layers. The fluxes for the actual atmosphere (with semi-transparent, fractional and/or multi-layered clouds) are derived from a linear combination of the fluxes calculated at the previous steps with some cloud overlap assumption in the case of clouds present in several layers. Let

N be the index of the layer containing the highest cloud, C_i the fractional cloud cover in layer i, with $C_0 = 1$ for the upward flux at the surface, and with $C_{N+1} = 1$ and $F^-_{N+1} = F^-_0$ to have the right boundary condition for downward fluxes above the highest cloud. The cloudy upward (F^+) and downward (F^-) fluxes are obtained as

$$F^+(i) = F^+_0(i) \qquad \text{for } i=1$$

$$F^+(i) = C_{i-1} F^+_{i-1}(i) + \sum_{n=0}^{i-2} C_n F^+_n(i) \prod_{l=n+1}^{i-1} (1 - C_l) \qquad \text{for } 2 \le i \le N+1$$

$$F^+(i) = C_N F^+_N(i) + \sum_{n=0}^{N-1} C_n F^+_n(i) \prod_{l=n+1}^{N} (1 - C_l) \qquad \text{for } i \ge N+2$$

(2.11)

In case of semi-transparent clouds, the fractional cloudiness entering the calculations is an effective cloud cover equal to the product of the emissivity by the horizontal coverage of the cloud layer, with the emissivity related to the cloud liquid water amount by

$$\varepsilon_{cld} = 1 - \exp(-K_{abs}\, u_{LWP}) \qquad (2.12)$$

where K_{abs} is the liquid water mass absorption coefficient set to 158 m^2kg^{-1} according to Stephens (1978, 1979).

3. SHORTWAVE RADIATION

The rate of atmospheric heating by absorption and scattering of shortwave radiation is

$$\frac{dT}{dt} = \frac{g}{C_p}\frac{dF}{dp} \qquad (3.1)$$

where F is the net total shortwave flux

$$F(\delta) = \int_0^{\infty} d\nu \int_0^{2\pi} d\phi \int_{-1}^{+1} \mu\, L_\nu\, (\delta\, ,\mu,\phi)\, d\mu\, d\phi \qquad (3.2)$$

L_ν is the diffuse radiance at wavenumber ν, in a direction given by ϕ the azimuth angle and $\mu = \cos\vartheta$, with ϑ the zenith angle. In (3.2), we assume a plane parallel atmosphere, and the vertical coordinate is the optical depth δ a convenient variable when the energy source is outside the medium

$$\delta(p) = \int_p^0 \beta_\nu(p)\, dp \qquad (3.3)$$

$\beta_\nu^{ext}(p)$ is the extinction coefficient equal to the sum of the scattering coefficient β_ν^{sca}, of the aerosol or cloud particle absorption coefficient β_ν^{abs}, and of the purely molecular absorption coefficient k_ν. The diffuse radiance L_ν is governed by the radiation transfer equation

$$\mu\, \frac{dL_\nu(\delta,\mu,\phi)}{d\delta} = L_\nu(\delta,\mu,\phi) - \frac{\bar{\omega}_\nu(\delta)}{4}\, P_\nu(\delta,\mu,\phi,\mu_o,\phi_o)\, E_\nu^o\, e^{-\frac{\delta}{\mu_o}}$$

$$- \frac{\bar{\omega}_\nu(\delta)}{4} \int_0^{2\pi}\int_{-1}^{+1} P_\nu(\delta,\mu,\phi,\mu',\phi')\, L_\nu(\delta,\mu',\phi')\, d\mu' d\phi' \qquad (3.4)$$

E_ν^o is the incident solar irradiance in the direction $\mu_o = \cos\vartheta_o$, $\bar{\omega}_\nu$ is the single scattering albedo ($= \beta_\nu^{sca} / K_\nu$) and $P(\delta,\mu,\phi,\mu',\phi')$ is the scattering phase function which defines the probability that radiation coming from direction (μ', ϕ') is scattered in direction (μ, ϕ). The shortwave part of the scheme, originally developed by Fouquart and Bonnel (1980) solves the radiation transfer equation and integrates the fluxes over the whole shortwave spectrum between 0.2 and 4 microns. Upward and downward fluxes are obtained from the reflectances and transmittances of the layers, and the photon path distribution method allows to separate the parametrization of the scattering processes from that of the molecular absorption.

3.1 Spectral integration

Solar radiation is attenuated by absorbing gases, mainly water vapor, carbon dioxide, oxygen and ozone, and scattered by molecules (Rayleigh scattering), aerosols and cloud particles. Since scattering and molecular absorption occur simultaneously, the exact amount of absorber along the photon path length is unknown, and band models of the transmission function cannot be used directly as in longwave radiation transfer (see 2.1) The approach of the photon path distribution method is to calculate the probability p(U) dU that a photon contributing to the flux F_c in the conservative case (i.e., no absorption, $\bar{\omega}_\nu = 1$, $k_\nu = 0$) has encountered an absorber amount between U and U + dU. With this distribution, the radiative flux at wavenumber ν is related to F_c by

$$F_\nu = F_c \int_0^\infty p(U) \exp(- k_\nu U) \, dU \qquad (3.5)$$

and the flux averaged over the spectral interval $\Delta\nu$ can then be calculated with the help of any band model of the transmission function $t_{\Delta\nu}$

$$F = \frac{1}{\Delta\nu} \int_{\Delta\nu} F_\nu \, d\nu = F_c \int_0^\infty p(U) \; t_{\Delta\nu}(U) \, d\nu \qquad (3.6)$$

To find the distribution function p(U), the scattering problem is solved first, by any method, for a set of arbitrarily fixed absorption coefficients k_1, thus giving a set of simulated fluxes F_{k_1}. An inverse Laplace transform is then performed on (3.5) to get p(U) (Fouquart, 1974). The main advantage of the method is that the actual distribution p(U) is smooth enough that (3.5) gives accurate results even if p(U) itself is not known accurately. In fact, p(U) needs not be calculated explicitly as the spectrally integrated fluxes are

$$F = F_c \, t_{\Delta\nu}(\langle U \rangle) \quad \text{in the limiting case of weak absorption}$$
$$F = F_c \, t_{\Delta\nu}(\langle U^{1/2} \rangle) \quad \text{in the limiting case of strong absorption} \tag{3.7}$$

where $\langle U \rangle = \int_0^\infty p(U)\, U\, dU$ and $\langle U^{1/2} \rangle = \int_0^\infty p(U)\, U^{1/2}\, dU$.

The atmospheric absorption in the water vapor bands is generally strong, and the scheme determines an effective absorber amount U_e between $\langle U \rangle$ and $\langle U^{1/2} \rangle$ derived from

$$U_e = \ln\left(F_{k_e} / F_c\right) / k_e \tag{3.8}$$

where k_e is an absorption coefficient chosen to approximate the spectrally averaged transmission of the clear-sky atmosphere

$$k_e = \frac{1}{U_{tot} / \mu_o} \ln\left(t_{\Delta\nu}(U_{tot} / \mu_o)\right) \tag{3.9}$$

where U_{tot} is the total amount of absorber in a vertical column and μ_o = cos ϑ_o . Once the effective absorber amounts of H_2O and uniformly mixed gases are found, the transmission functions are computed using Padé approximants

$$t_{\Delta\nu}(U) = \frac{\sum_{i=0}^{N} a_i\, U^{i-1}}{\sum_{j=0}^{N} b_j\, U^{j-1}} \tag{3.10}$$

Absorption by ozone is also taken into account, but since ozone is located at low pressure levels for which molecular scattering is small and Mie scattering is negligible, interactions between scattering processes and ozone absorption are neglected. Transmission through ozone is computed using (3.10) where U_{O_3}, the amount of ozone is

$$U_{O_3}^{d} = M \int_{p}^{0} dU_{O_3}$$ for the downward transmission of the direct solar beam,

$$U_{O_3}^{u} = r \int_{p_s}^{p} dU_{O_3} + U_{O_3}^{d}(p_s)$$ for the upward transmission of the diffuse radiation;

r = 1.66 is the diffusivity factor (see 3.), and M is the magnification factor (Rodgers, 1967) used instead of μ_o to account for the sphericity of the atmosphere at very small solar elevations

$$M = 35 / \sqrt{1224\, \mu_o^2 + 1} \qquad (3.11)$$

To perform the spectral integration, it is convenient to discretize the solar spectral interval into subintervals in which the surface reflectance can be considered as constant. Since the main cause of the important spectral variation of the surface albedo is the sharp increase in the reflectivity of the vegetation in the near infrared, and since water vapor does not absorb below 0.68 µm, the shortwave scheme considers two spectral intervals, one for the visible (0.2 - 0.68 µm), one for the near infrared (0.68 - 4.0 µm) parts of the solar spectrum. This cut-off at 0.68 µm also makes the scheme more computationally efficient, inasmuch as the interactions between gaseous absorption (by water vapor and uniformly mixed gases) and scattering processes are accounted for only in the near-infrared interval.

3.2 Vertical integration

Contrarily to the scheme of Geleyn and Hollingsworth (1979), the fluxes are not obtained through the solution of a system of linear equations in a matrix form. Rather, assuming an atmosphere divided into N homogeneous layers, the upward and downward fluxes at a given layer interface j are given by

$$F^{-}(j) = F_0 \prod_{k=j}^{N} T_b(k) \qquad (3.12)$$

$$F^{+}(j) = F^{-}(j)\, R_t(j-1)$$

where $R_t(j)$ and $T_b(j)$ are the reflectance at the top and the transmittance at the bottom of the j-th layer. Computations of R_t's start at the surface and work upward, whereas those of T_b's start at the top of the atmosphere and work downward. R_t and T_b account for the presence of cloud in the layer

$$R_t = C\, R_{cdy} + (1 - C)\, R_{clr}$$

$$T_b = C\, T_{cdy} + (1 - C)\, T_{clr} \qquad (3.13)$$

clr and cdy respectively refer to the clear-sky and cloudy fractions of the layer, and C is the cloud fractional coverage.

3.2.1 Cloudy fraction of the layers

$R_{t_{cdy}}$ and $T_{b_{cdy}}$ are the reflectance at the top and transmittance at the bottom of the cloudy fraction of the layer calculated with the Delta-Eddington Approximation. Given δ_c, δ_a, and δ_g, the optical thicknesses for the cloud, the aerosol and the molecular absorption (= k_e U), and g_c and g_a the cloud and aerosol asymmetry factors, $R_{t_{cdy}}$ and $T_{b_{cdy}}$ are calculated as functions of the total optical thickness of the layer

$$\delta = \delta_c + \delta_a + \delta_g$$

of the total single scattering albedo

$$\omega^{*} = \frac{\delta_c + \delta_a}{\delta_c + \delta_a + \delta_g} \qquad (3.14)$$

of the total asymmetry factor

$$g^{*} = \frac{\delta_c}{\delta_c + \delta_a} g_c + \frac{\delta_a}{\delta_c + \delta_a} g_a$$

of the reflectance R_ of the underlying medium (surface or layers below the j-th interface), and of an effective solar zenith angle $u_e(j)$ which accounts for the decrease of the direct solar beam and the corresponding increase of the diffuse part of the downward radiation by the upper scattering layers.

$$u_e(j) = [\ (1 - C^{al}(j)) \ / \ \mu + r\ C^{al}(j)\ ]^{-1} \qquad (3.15)$$

with

$$C^{al}(j) = 1 - \prod_{i=j+1}^{N} (\ 1 - C(i)\ E(i)\ )$$

and

$$E(i) = 1 - \exp \left[- \frac{(\ 1 - \omega_c(i)\ g_c(i)^2)\ \delta_c(i)}{\mu} \right] \qquad (3.16)$$

$\delta_c(i)$, $\omega_c(i)$ and $g_c(i)$ are the optical thickness, single scattering albedo and asymmetry factor of the cloud in the i-th layer, and r is the diffusivity factor. The scheme follows the Eddington Approximation, first proposed by Shettle and Weinman (1970), then modified by Joseph et al. (1976) to account more accurately for the large fraction of radiation directly transmitted in the forward scattering peak in case of highly asymmetric phase functions. Eddington's approximation assumes that, in a scattering medium of optical thickness to*, of single scattering albedo w, and of asymmetry factor g, the radiance L entering (3.4) can be written as

$$L(\delta,\mu) = L_0(\delta) + \mu\, L_1(\delta) \tag{3.17}$$

In that case, when the phase function is expanded as a series of associated Legendre functions, all terms of order greater than one vanish when (3.4) is integrated over μ and ϕ . The phase function is therefore given by

$$P(\theta) = 1 + \beta_1(\theta) \cos\theta$$

where θ is the angle between incident and scattered radiances. The integral in (3.4) thus becomes

$$\int_0^{2\pi}\int_{-1}^{+1} P(\mu,\phi,\mu',\phi')\, L(\mu',\phi')\, d\mu' d\phi' = 4\pi\,(L_0 + \pi L_1) \tag{3.18}$$

where

$$g = \frac{\beta_1}{3} = \frac{1}{2}\int_{-1}^{+1} P(\theta) \cos\theta\, d(\cos\theta)$$

is the asymmetry factor.

Using (3.18) in (3.4) after integrating over μ and dividing by 2π,we get

$$\mu \frac{d(L_0+\mu L_1)}{d\delta} = -\,(L_0+\mu L_1) + \omega\,(L_0 + g\,\mu\, L_1) + \frac{1}{4}\,\omega\, F_0 \exp(-\delta/\mu_0)\,(1+3g\,\mu_0\mu) \tag{3.19}$$

We obtain a pair of equations for L_0 and L_1 by integrating (3.19) over μ

$$\frac{d\,L_0}{d\,\delta} = -3\,(1-\omega)\,L_0 + \frac{3}{4}\omega\,F_0\,\exp\,(-\delta\,/\,\mu_0) \qquad (3.20)$$

$$\frac{d\,L_1}{d\,\delta} = -\,(1-\omega g)\,L_1 + \frac{3}{4}\omega\,g\,\mu_0\,F_0\,\exp\,(-\delta\,/\,\mu_0) \qquad (3.21)$$

For the cloudy layer assumed non-conservative ($\omega < 1$), the solutions to (3.20) and (3.21), for $0 \le \delta \le \delta^*$, are

$$L_0(\delta) = C_1\,\exp(-k\delta) + C_2\,\exp(+k\delta) - \alpha\,\exp(-\delta/\mu_0)$$

(3.22)

$$L_1(\delta) = p\,(C_1\,\exp(-k\delta) - C_2\,\exp(+k\delta)) - \beta\,\exp(-\delta/\mu_0)$$

where

$$k = [\,3\,(1-\omega)\,(1-\omega g)]^{1/2}$$

$$p = [\,3\,(1-\omega)/(1-\omega g)]^{1/2}$$

$$\alpha = 3\,\omega\,F_0\,\mu_0^2\,[\,1 + g\,(1-\omega)]\,/\,4\,(1-k^2\,\mu_0^2)$$

$$\beta = 3\,\omega\,F_0\,\mu_0\,[\,1 + 3g\,(1-\omega)\,\mu_0^2]\,/\,4\,(1-k^2\,\mu_0^2\,)$$

The two boundary conditions allow to solve the system for C_1 and C_2; the downward directed diffuse flux at the top of the layer is zero, i.e.,

$$F^-(0) = [\,L_0(0) + \frac{2}{3}L_1(0)\,] = 0$$

which translates into

$$(1 + 2p/3)\ C_1 + (1 - 2p/3)\ C_2 = \alpha + 2\beta/3 \tag{3.23}$$

the upward directed flux at the bottom of the layer is equal to the product of the downward directed diffuse and direct fluxes and the corresponding diffuse and direct reflectances (R_d and $R\_$, respectively) of the underlying medium

$$F^+(\delta^*) = [\ L_0(\delta^*) - \frac{2}{3} L_1(\delta^*)\ ]$$

$$= R\_[L_0(\delta^*) + \frac{2}{3} L_1(\delta^*)] + R_d\ \mu_0\ F_0\ \exp\ (-\delta^*/\mu_0)$$

which translates into

$$\begin{aligned} &(1 - R\_ - 2\ (1 + R\_)\ p\ /\ 3)\ C_1\ \exp(-k\ \delta^*) \\ &+ (1 - R\_ + 2\ (1 + R\_)\ p\ /\ 3)\ C_2\ \exp(+k\ \delta^*) \\ &= (\ (1 - R\_)\ \alpha - 2\ (1 + R\_)\ \beta\ /\ 3 + R_d\ \mu_0\ F_0\ )\ \exp(-\delta^*/\mu_0) \end{aligned} \tag{3.24}$$

In the Delta-Eddington approximation, the phase function is approximated by a Dirac delta function forward scatter peak and a two-term expansion of the phase function

$$P(\theta) = 2\ f\ (1-\cos\ \theta) + (1-f)\ (1 + 3g'\ \cos\ \theta)$$

where f is the fractional scattering into the forward peak and g' the asymmetry factor of the truncated phase function. As shown by Joseph et al. (1976), these parameters are

$$\begin{aligned} &f = g^2 \\ &g' = g\ /\ (\ 1+g) \end{aligned} \tag{3.25}$$

The solution of the Eddington's equations remains the same provided that the total optical thickness, single scattering albedo and asymmetry factor entering (3.19)-(3.24) take their transformed values

$$\delta^{*'} = (1 - \omega f) \delta^{*}$$

$$\omega' = \frac{(1 - f) \omega}{1 - \omega f} \qquad (3.26)$$

Practically, the optical thickness, single scattering albedo, asymmetry factor, and solar zenith angle entering (3.23)-(3.26) are δ^{*}, ω^{*}, g^{*} and u_e defined in (3.14) and (3.15).

3.2.2 Clear-sky fraction of the layers

In the clear-sky fraction of the layers, the shortwave scheme accounts for scattering and absorption by molecules and aerosols. As the optical thickness for both Rayleigh and aerosol scattering is small, $R_{clr}(j-1)$ and $T_{clr}(j)$, the reflectance at the top and transmittance at the bottom of the j-th layer can be calculated using respectively a first and a second-order expansion of the analytical solutions of the two-stream equations similar to that of Coakley and Chylek (1975). For Rayleigh scattering, the optical thickness, single scattering albedo and asymmetry factor are respectively δ_R, $\omega_R = 1$, and $g_R = 0$, so that

$$R_R = \frac{\delta_R}{(2\mu + \delta_R)} \qquad (3.27)$$

$$T_R = \frac{2 \mu}{(2\mu + \delta_R)}$$

The optical thickness tor of an atmospheric layer is simply

$$\delta_R = \delta_R^{*} (p(j) - p(j-1)) / p_{surf}$$

where δ_R^* is the Rayleigh optical thickness of the whole atmosphere parametrized as a function of the solar zenith angle (Deschamps et al., 1983)

$$\delta_R^* = \sum_{i=0}^{5} a_i \, \mu_0^{i-1}$$

For aerosol scattering and absorption, the optical thickness, single scattering albedo and asymmetry factor are respectively δ_a , ω_a (with $1-\omega_a \ll 1$) and g_a, so that

$$\begin{aligned} den = 1 &+ (1 - \omega_a + back(\mu_e) \, \omega_a) \, \delta_a / \mu_e \\ &+ (1 - \omega_a) (1 - \omega_a + 2 \, back(\mu_e) \, \omega_a) \, \delta_a^2 / \mu_a^2 \end{aligned}$$

$$R(\mu_e) = \frac{back(\mu_e) \, \omega_a \, \delta_a / \mu_e}{den} \qquad T(\mu_e) = \frac{1}{den} \tag{3.28}$$

where $back(\mu_e) = (2 - 3 \, \mu_e \, g_a) / 4$ is the backscattering factor.

Practically, R_{clr} and T_{clr} are computed using (3.28) and the combined effect of aerosol and Rayleigh scattering comes from using modified parameters corresponding to the addition of the two scatterers with provision for the highly asymmetric aerosol phase function through a Delta-approximation of the forward scattering peak (as in (3.25)-(3.26))

$$\delta^+ = \delta_R + \delta_a \, (1 - \omega_a \, g_a^2)$$

$$g^+ = \frac{g_a}{1 + g_a} \left(\frac{\delta_a}{\delta_R + \delta_a} \right) \tag{3.29}$$

$$\omega^+ = \frac{\delta_R}{\delta_R + \delta_a} \omega_R + \frac{\delta_a}{\delta_R + \delta_a} \frac{\omega_a (1 - g_a^2)}{1 - \omega_a g_a^2}$$

As for their cloudy counterparts, R_{clr} and T_{clr} must account for the multiple reflections due to the layers underneath

$$R_{clr} = R(\mu_e) + R\_ \, T(\mu_e) \,/\, (1 - R^* R\_)$$

$$T_{clr} = T(\mu_e) \,/\, (1 - R^* R\_) \tag{3.30}$$

with $R^* = R\ (1/r)$

$T^* = T\ (1/r)$

and R_ is the reflectance of the underlying medium (R_ = R_t (j-1)) and r is the diffusivity factor.

Since interactions between molecular absorption and Rayleigh and aerosol scattering are negligible, the radiative fluxes in a clear-sky atmosphere are simply those calculated from (3.12) and (3.30) attenuated by the gaseous transmissions (3.10).

3.3 Multiple reflections between layers

To deal properly with the multiple reflections between the surface and the cloud layers, it should be necessary to separate the contribution of each individual reflecting surface to the layer reflectances and transmittances inasmuch as each such surface gives rise to a particular distribution of absorber amount. In case of an atmosphere including N cloud layers, the reflected light above the highest cloud consists of photons directly reflected by the highest cloud without interaction with the underlying atmosphere, and of photons that have passed through this cloud layer and undergone at least one reflection on the underlying atmosphere. In fact, (3.6) should be written

$$F = \sum_{l=0}^{N} F_{cl} \int_0^\infty p_l(U)\, t_{\Delta\nu}(U)\, d\nu \tag{3.31}$$

where F_{cl} and $p_l(U)$ are the conservative fluxes and the distributions of absorber amount corresponding to the different reflecting surfaces.

Fouquart and Bonnel (1980) have shown that a very good approximation to this problem is obtained by evaluating the reflectance and transmittance of each layer (using (3.24) and (3.30)) assuming successively a non-reflecting underlying medium ($R_- = 0$), then a reflecting underlying medium ($R_- \neq 0$). First calculations provide the contribution to reflectance and transmittance of those photons interacting only with the layer into consideration, whereas the second ones give the contribution of the photons with interactions also outside the layer itself.

From these two sets of layer reflectances and transmittances (R_{t0}, T_{b0}) and ($R_{t\neq}$, $T_{b\neq}$) respectively, effective absorber amounts to be applied to computing the transmission functions for upward and downward fluxes are then derived using (3.8) and starting from the surface and working the formulas upward

$$\begin{aligned} U^-_{e0} &= \ln (T_{b0} / T_{bc}) / k_e \\ U^-_{e\neq} &= \ln (T_{b\neq} / T_{bc}) / k_e \\ U^+_{e0} &= \ln (R_{t0} / R_{tc}) / k_e \\ U^+_{e\neq} &= \ln (R_{t\neq} / R_{tc}) / k_e \end{aligned} \qquad (3.32)$$

where R_{tc} and T_{bc} are the layer reflectance and transmittance corresponding to a conservative scattering medium.

Finally the upward and downward fluxes are obtained as

$$F^+(j) = F_0 \left[R_{t0}\, t_{\Delta\nu}(U^+_{e0}) + (R_{t\neq} - R_{t0})\, t_{\Delta\nu}(U^+_{e\neq})\right] \qquad (3.33a)$$

$$F^-(j) = F_0 \left[T_{b0}\, t_{\Delta\nu}(U^-_{e0}) + (T_{b\neq} - T_{b0})\, t_{\Delta\nu}(U^-_{e\neq})\right] \qquad (3.33b)$$

3.4 Cloud shortwave optical properties

As seen in section 3.2.1, the cloud radiative properties depend on three different parameters: the optical thickness δ_c , the asymmetry factor g_c, and the single scattering albedo ω_c .

δ_c is related to the cloud liquid water amount u_{LWP} by

$$\delta_c = \frac{3\, u_{LWP}}{2\, r_e} \qquad (3.34)$$

where r_e is the mean effective radius of the size distribution of the cloud droplets. Presently r_e is fixed to 15 μm, but this radius may vary with height from 5 μm in the planetary boundary layer to 40 μm at 100 hPa, in an empirical attempt at dealing with the variation of cloud type with height. Smaller water droplets are observed in low-level stratiform clouds whereas larger particles are found in cumuliform and cirriform clouds.

In the two spectral intervals of the shortwave radiation scheme, g_c is fixed to 0.865 and 0.910 , respectively, and ω_c is given as a function of δ_c following Fouquart (1987)

$$\omega_{c1} = 0.9999 - 5 \times 10^{-4} \exp(-0.5\, \delta_c)$$

$$\omega_{c2} = 0.9988 - 2.5 \times 10^{-3} \exp(-0.05\, \delta_c) \qquad (3.35)$$

These cloud shortwave radiative parameters have been fitted to *in situ* measurements of stratocumulus clouds (Bonnel et al., 1983).

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