SEMI-IMPLICIT SEMI-LAGRANGIAN FULLY ELASTIC NONHYDROSTATIC MODEL FORMULATION

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Abstract: A review is made of the formulation of fully elastic nonhydrostatic models solved with semi-implicit and semi-Lagrangian numerical methods. Examples of simulations using this approach will be presented covering a wide range of scales. It will be shown that hydrostatic pressure can be used advantageously as vertical coordinate of nonhydrostatic models, thus permitting to retain a close correspondence with the formulation of hydrostatic models. Current streams of research to solve some outstanding difficulties will also be described briefly, including the treatment of stationary forcings such as orography.

1. INTRODUCTION

With the advent of ever faster computers, model resolution increases continuously. This is one recognised way to decrease model simulation errors. Indeed as resolution is increased, truncation errors associated with approximations of numerical discretisation of the continuous equations are expected to decrease too. Furthermore with higher resolution a larger spectrum of the actual flow is resolved explicitly by the model, and hence the "turbulent" interaction between scales is expected to be better handled.

When model grids become sufficiently fine, nonhydrostatic effects eventually become resolved by the model and the traditional "primitive equation" hydrostatic formulation is inadequate. At micro- and meso-scale the anelastic approximation has been used traditionally with success. This approximation however is not deemed appropriate for large-scale flow. So for global high-resolution models, the unapproximated equations appear to be the only alternative. The exact equations however contain terms that give rise to very rapid oscillations. This is a problem for a large- or meso-scale model because these rapid oscillations would severely limit the length of the maximum timestep that can be used for stable integration of the model using explicit Eulerian numerics. Because however that these rapid oscillations are not deemed important for their own sake, save for very fine micro-scales, there are numerical ways of treating them so that they do not interfere with the efficiency of the model.

In what follows, a formulation of the exact fully elastic nonhydrostatic Euler field equations solved with an efficient semi-implicit and semi-Lagrangian numerical marching method will be described. Special attention will be given to the vertical coordinate system in an attempt to closely parallel the formalism of existing operational hydrostatic models. Current avenues of research using this approach will be briefly described as well as some outstanding problems relating to the treatment of stationary forcings, including topography, in this formulation.

2. THE "UNIVERSAL" MODEL FORMULATION

2.1 The Tanguay-Robert-Laprise hypothesis

In addition to producing reliable simulations of the atmosphere, atmospheric models should ideally have two other qualities: (1) to be computationally efficient, that is to be able to perform a given integration in the least amount of time, or conversely to be able to achieve the most detailed simulation (resolution, complexity) in a given amount of computer time, and (2) to be based on a formalism (dynamics, parameterisation) sufficiently general to permit their application on a wide range of physical situations, thus avoiding the need for developing and maintaining more than one model. Semi-implicit (SI) and semi-Lagrangian (SL) marching schemes were developed with computational efficiency in mind. The implicit treatment of terms responsible for gravity waves in primitive equation (PE) hydrostatic models has lead to an increased efficiency without notable degradation in the integration of such models at large scale. Most hydrostatic models today employ SI algorithms. An increasing number of models are also using SL transport scheme which can result in substantial saving, at the same time as reducing numerical noise present in some variants of Eulerian advection models. Hence SI-SL hydrostatic models are very efficient.

The thesis advanced by Tanguay-Robert-Laprise (1990, henceforth referred to as TRL) is that, within the context of a model based on SI-SL numerics, the generalisation to remove the hydrostatic approximation by reverting to the unapproximated fully elastic (FE) nonhydrostatic Euler equations is simple and does not introduce appreciable computational overhead to the model. The idea consists in treating the terms responsible for gravity and elastic disturbances in an implicit fashion; this is a generalisation of the approach used at large scale in hydrostatic models whereby the terms responsible for gravity waves are treated implicitly, and of that used at meso-scale by Carpenter (1979) in which only the terms responsible for elastic disturbances were treated implicitly. The advantages of the TRL approach compared to that of Carpenter is that the resulting model retains its efficiency when used at large-scale. Indeed when applied at large scale there is essentially no difference (neither in results nor in performance) compared to integrating the hydrostatic equations (as ought to be expected). Such a model however can then also be applied at much smaller scale where the hydrostatic approximation does not hold; under standard atmospheric conditions this occurs at horizontal scales of the order of 10 km.

As demonstrated by A. Robert and his students and colleagues for applications at micro- and meso-scales (see Laprise et al. 1997 for a detailed review), such SI-SL FE model has demonstrated to be very competitive compared with time-celebrated Eulerian anelastic formulations. The advantage of FE models however is that the same dynamical kernel of model can be used at all scales, thus permitting to anticipate the event of a universal model one day when physical parameterisations can be made suitably versatile to adapt for scales.

It may be useful to remind here the philosophy underlying the SI-SL marching scheme. Because the numerical method is (almost) unconditionally stable, any (reasonable) timestep can be used for integrating the model. However it is up to the user to choose the timestep on the basis of the required accuracy for the problem that is addressed. Hence the timestep is dictated by accuracy rather than by stability as is the case with the explicit Eulerian approach. This means typically that at large-scale the timestep is chosen to produce good Rossby waves and advection, but that it may be too large to simulate with fidelity gravity or elastic waves; in general however these do not have serious repercussions on large-scale simulations. Similarly, at meso-scale the timestep would be reduced to produce good internal gravity waves simulations, but external gravity and elastic waves may be seriously contaminated, but these are not overly important in general at these scales. When a simulation requires the whole spectrum of waves to be accurately simulated, then the timestep must be small enough to resolve the fastest moving disturbances, generally the elastic waves moving approximately at the speed of sound.

In Canada, this FE SI-SL dynamical-numerical approach has been at the basis of three streams of research: the Cooperative Centre for Research in Mesometeorology (CCRM) model (Laprise et al. 1997), the Mesoscale Community Model (MC2, Benoit et al. 1997) and the Canadian Regional Climate Model (CRCM, Caya and Laprise 1998, Laprise et al. 1998) are based on the same FE SI-SL dynamical kernel. The CCRM model uses cloud microphysics, the MC2 uses large- and meso-scale forecast physics, and the CRCM uses large-scale climate physics parameterisations.

2.2 Hydrostatic-pressure (π) coordinate FE model

An important consideration in choosing a formulation is the ease with which changes can be applied to existing models to cast them into the desired form. The experience of TRL has been that the most substantial changes required to develop a FE SI-SL model from an existing PE SI-SL model were associated to the change of vertical coordinate: PE models are generally cast in terrain-following pressure-type coordinate while FE and anelastic nonhydrostatic models are generally cast in terrain-following height-type coordinate.

While it is possible to envisage employing pressure coordinate for nonhydrostatic models, such coordinate runs the risk of becoming singular in cases of severe vertical accelerations. The pressure-coordinate framework can be generalised to make it applicable to FE models as demonstrated by Laprise (1992). The hydrostatic-pressure coordinate, which henceforth will be referred to as the π-coordinate, leads to formulation that closely parallels that of hydrostatic pressure-coordinate large-scale models while avoiding the potential singularity of (true) pressure coordinate.

In hydrostatic-pressure coordinate the (exact) mass continuity equation takes the form of a diagnostic relation:

$$\vec{\nabla}_\pi \cdot \vec{V} + \frac{\partial \dot{\pi}}{\partial \pi} = 0, \qquad \dot{\pi} \equiv \frac{d\pi}{dt}$$

where π satisfies

$$\frac{\partial \pi}{\partial z} = -\rho g$$

i.e. π satisfies the following definition:

$$\pi(x,y,z,t) = \pi_T + \int_z^{z_T} \rho(x,y,z',t)\, g\, dz', \quad \pi_T \equiv \pi(x,y,z_T,t)$$

The usual pressure-coordinate used in primitive equation large-scale models is a special case where $\pi = p$.

2.2.1 Terrain-following hydrostatic-pressure coordinate

Consider a generalisation of the hybrid pressure-type vertical coordinate η used in most PE models:

$$\pi(x,y,\eta,t) = A(\eta) + B(\eta)\, \pi_s(x,y,t)$$

where π_S is the hydrostatic surface pressure, i.e. the weight of an entire atmospheric column of unit area above the surface of the earth. The nonhydrostatic equations into terrain-following hydrostatic-pressure coordinate are as follows (additional terms due to the nonhydrostatic nature of the equations have been put in boxes):

Horizontal (x,y) momentum: $$\frac{d}{dt}\vec{V} + RT\, \vec{\nabla}_\eta \ln p + \left(\frac{\partial p/\partial \eta}{\partial \pi/\partial \eta}\right) \vec{\nabla}_\eta \phi = \vec{F}$$

Vertical momentum: $$\boxed{\frac{dw}{dt}} + g\left(1 - \frac{\partial p/\partial \eta}{\partial \pi/\partial \eta}\right) = \boxed{F_z}$$

Thermodynamic: $$\frac{dT}{dt} - \frac{RT}{C_p}\frac{d \ln p}{d t} = \frac{Q}{C_p}$$

State law: $$\alpha = \frac{RT}{p}$$

Definition of hydrostatic pressure: $$\phi = \phi_s + \int_\eta^{\eta_s} \alpha \frac{\partial \pi}{\partial \eta} d\eta'$$

Mass continuity and thermodynamic: $$\boxed{\frac{d\ln p}{dt} + \frac{C_p}{C_v} D_3 = \frac{Q}{C_v T}}$$

The mass continuity: $$\left(\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial \eta}\right)\right)_\eta + \overrightarrow{\nabla_\eta}\cdot\left(\vec{V}\frac{\partial \pi}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial \pi}{\partial \eta}\right) = 0$$

This equation, combined with suitable boundary conditions, collapses to a single level equation:

The total column mass: $$\frac{\partial}{\partial t}\ln \pi_s + \frac{1}{\pi_s}\vec{\nabla}\cdot\int_{\eta_T}^{\eta_s}\vec{V}\frac{\partial \pi}{\partial \eta}\,d\eta' = 0$$

Other definitions include:

The substantial derivative: $$(\dot{\ }) \equiv \frac{d}{dt} \equiv \left(\frac{\partial}{\partial t}\right)_\eta + \vec{V}\cdot\overrightarrow{\nabla_\eta} + \dot{\eta}\frac{\partial}{\partial \eta}$$

The three-dimensional flow divergence:

$$D_3 \equiv \overrightarrow{\nabla_\eta}\cdot\vec{V} + \frac{\rho}{(\partial\pi/\partial\eta)}\left(\overrightarrow{\nabla_\eta}\phi\right)\cdot\left(\frac{\partial\vec{V}}{\partial\eta}\right) - \frac{\rho g}{(\partial\pi/\partial\eta)}\frac{\partial w}{\partial \eta}.$$

The terrain-following, hydrostatic-pressure coordinate field equations used in all GCM and NWP models are a subset of these (exact) non-hydrostatic field equations. In the hydrostatic framework, (1) $\pi = p$ and hence the ratio $(\partial p/\partial\eta)/(\partial\pi/\partial\eta)$ equals unity, (2) the vertical momentum equation collapses to a diagnostic hypsometric relation for the vertical distribution of mass, and (3) the pressure tendency equation for p (resulting from a blend of thermodynamic energy and mass continuity equations) is redundant with the mass continuity equation for π and hence is not used. In fact the p-equation would be the one used to recover the true vertical velocity w in a diagnostic fashion if w were needed in a hydrostatic model cast in pressure-type vertical coordinate.

In summary, within the PE context there are 3 multi-level prognostic equations and one single-level prognostic for surface pressure, which is coherent with the existence of internal Rossby (single sign phase speed) and gravity (paired sign phase speed) waves and an external Lamb wave; in the FE context there are 5 multi-level prognostic equations and one single-level prognostic for total column weight, which is coherent with the existence of internal Rossby (single), gravity (pair) and elastic (pair) waves and an external Lamb wave.

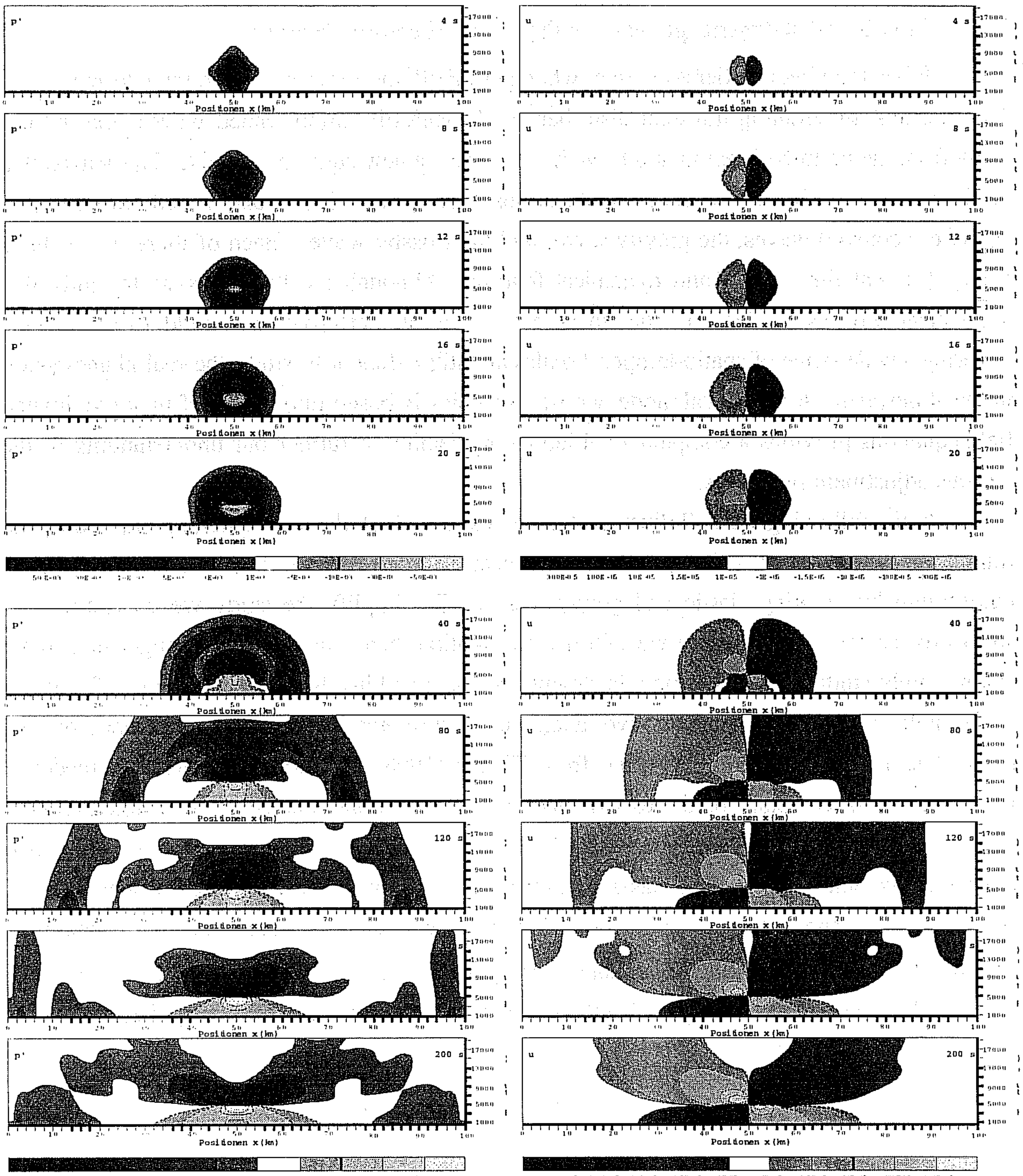
This hydrostatic-pressure coordinate approach has been successfully applied in several atmospheric models, e. g. in ALADIN in France (Bubnova, Hello, Bénard and Geleyn, 1994 MWR) and in the ETA regional model in USA (Rancic, 1994 QJRMS). Work is also in progress to implement this coordinate in the recently developed GEM in Canada (Côté and Staniforth, in progress).

3. ON-GOING RESEARCH

3.1 Elastic adjustment process, subject to thermal forcing

It has long been understood that, when perturbed, the atmosphere attempts to restore an "equilibrium" by emitting transient disturbances (henceforth simply called waves) that radiate away from the perturbed region and leave in their wake a new equilibrated state. The waves that participate to this adjustment process in the atmosphere are the elastic (also called compression, sound or acoustic) waves, the gravity waves and the Rossby waves. Each of these waves have vastly different time scales and dynamical features. Although the theory exists to study this adjustment process (at least in the linear case of weak perturbations), field measurements covering a wide range of spatio-temporal scales are still deficient to study the real phenomenon in the atmosphere. A numerical model whose dynamics is based upon a set of unapproximated field equations provides a computational laboratory facility to further our understanding of the various adjustment processes.

In the following we will illustrate the process by which the atmosphere, perturbed by the impulsive switch-on of a localised elevated heat source, tries to reach a state of quasi-equilibrium by emitting elastic and gravity waves. To simplify the study we concentrate on short time scales, and therefore we will neglect rotation; hence the classical, longer time scale (more amply studied) geostrophic adjustment process resulting from the emission of (mostly horizontally propagating) gravity waves and generation of Rossby wave will be absent from the scene. The following results are taken from Thurre (1998) to whom the interested reader is referred to for more details on the experimental set-up and more detailed physical interpretation of the results. The following figure shows the time evolution of pressure perturbation and horizontal velocity, in a plane passing through the maximum heating region which is of spherical bell-shape with half width of 500 m located at 5 km altitude, and with maximum heating of 7.2 ^{o}C/h. Note that the model was integrated in full three dimensions, but because of the cylindrical symmetry, a single slice through the centre is sufficient to display the results.

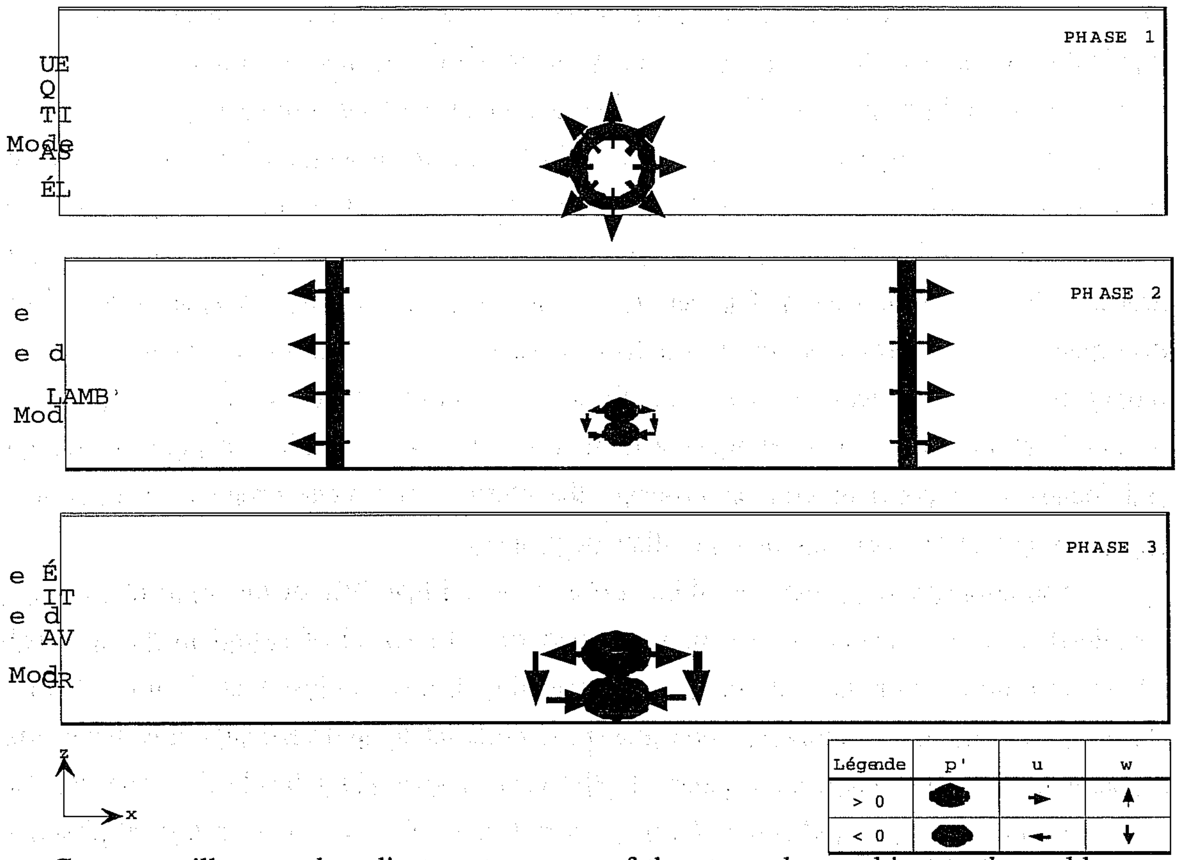


Vertical cross-sections (through the maximum heating region) of pressure perturbation (left column) and horizontal velocity (right column) at various times, from 4 to 20 s at 4 s intervals, then from 40 to 200 s at 40 s intervals (adapted from Thurre, 1998).

Subject to the sudden application of a localised elevated heat source, temperature and pressure increase locally in the region of heating while density remains constant. The pressure gradient resulting from the locally higher pressure in the heated region, induces an acceleration of outward flow in a spherical "front" zone surrounding the heated region. This flow has two effects: (1) mass divergence in the zone located just inside the front tends to reduce the pressure in the heated region and the associated adiabatic cooling reduces the temperature elevation due to the heat source, and (2) the mass convergence in the zone located just outside the front tends to heat by adiabatic compression. The net effect is the radially outward propagation of an elastic disturbance that contributes to re-establish pressure equilibrium in the heated region by propagating an elastic wave moving radially at (approximately) the speed of sound. The net result of the emission of such internal elastic waves is that, while heating occurs instantaneously and locally as a constant volume process, the elastic adjustment process restores a state approaching that of a constant pressure diabatic process.

The atmosphere is relatively thin: suffice to recall that 99% of the atmospheric mass is contained in a layer about 30 km thick. Considering the speed of sound in the atmosphere (about 350 m/s), an elastic disturbance can cover this distance in less than 2 min. Given that elastic waves propagate (almost) at the same speed horizontally and vertically, this means that in the same time that the whole atmospheric depth has been covered by the elastic wave, the lateral propagation has also covered about 30 km, a short distance for most meteorological phenomena (this is smaller than the size of a mesoscale convective complex for example). Hence the lateral propagation will proceed typically for periods much longer than the characteristic time necessary to achieve propagation and adjustment in the vertical. Beyond this characteristic time, the waves responsible for the adjustment are different from the radially propagating internal elastic waves dominating the scene initially: the second phase of the adjustment is characterised by horizontally propagating external Lamb disturbances. The behaviour of these disturbances is somewhat similar to the aforementioned (internal) elastic waves except for the fact that they only contain horizontal flow perturbations and propagate horizontally.

For yet longer time scale, behind the passage of Lamb disturbances, a thermally driven flow develops, characteristic of forced internal gravity waves. This regime is characterised by upward motion in the region of heating (this upward motion results in adiabatic cooling which almost cancels the diabatic heating), with weak compensating subsidence over a broader region surrounding the heated region. Compensating low-level convergence and upper-level divergence result in a nearly anelastic (non-divergent) flow. This is the type of circulation typical for example in the Inter Tropical Convergence Zone near the equator on average.



Cartoon to illustrate the adjustment process of the atmosphere subject to the sudden switch-on of an elevated heat source. The three panels show the dominant disturbance at various stages: initially the radially propagating internal elastic disturbance, then the horizontally propagating Lamb disturbance and later the stationary thermally forced internal quasi anelastic gravity disturbance (taken from Thurre, 1998).

For yet longer time scales, the earth rotation would have to be taken into account, resulting in the geostrophic adjustment process. This later aspect will not be pursued further here.

3.2 Treatment of diabatic term

3.2.1 Empirical formulations of diabatic forcing

We have seen in section 1.2 that a FE SI-SL model integrated with small timestep can reproduce adequately all types of atmospheric waves. In large- and meso-scale applications however, the model's timestep is chosen to be fairly large for efficiency reasons, and in general the elastic waves are then poorly reproduced. The classical numerical analysis of SI-SL scheme reveals that what happens then is a reduction of the phase speed of the fastest moving disturbances. In the simplest case the frequency is reduced as follows:

$$\omega_{SI} = \frac{\operatorname{atan}\left(\omega\, \Delta t\right)}{\Delta t}$$

where ω is the true frequency and ω_{SI} is the numerically simulated frequency. Given the severe contamination of elastic waves for large timestep, it is appropriate to ask oneself what the impact of the severe retardation can have on the simulation, and whether is would not be preferable to "filter out" the elastic disturbances from the simulation in such case.

In the following a number of empirically derived approximations to the treatment of the diabatic heating have been tested. These are labelled quasi anelastic and quasi hydrostatic approximations to the diabatic heating because they produce results analogous to those obtained with correspondingly approximated dynamical models, when using short timestep. For reference, recall the exact form of the thermodynamic and continuity equations in the form that they are usually employed:

$$\frac{dT}{dt} - \frac{RT}{C_p}\frac{d\ln p}{d\,t} = F_1$$

$$\frac{d\ln p}{d\,t} + \frac{C_p}{C_v} D_3 = F_2$$

The exact, quasi anelastic and quasi hydrostatic approximations to the diabatic heating correspond to replacing the F_i terms as follows:

	F_1	F_2
Exact	$\frac{Q}{C_p}$	$\frac{Q}{C_v T}$
Quasi hydrostatic	$\frac{Q}{C_p} + \frac{RT}{C_p} F_2$	$\frac{g}{RC_v} \int_{z_z}^{z} \frac{Q}{T^2} dz'$
Quasi anelastic	$\frac{Q}{C_p}$	0

The interested reader is referred to Thurre (1998) for further details on how these approximations were empirically derived.

The results show that the model, when submitted impulsively to a localised heat source and integrated with a small timestep, performs very similarly to an anelastic or hydrostatic model when employing the so-called quasi anelastic and quasi hydrostatic approximations to diabatic heating, respectively. It is important to reiterate that these results were obtained despite the fact that only the handling of the diabatic forcing has been modified, and that the dynamical equations are kept unapproximated. On the other hand, when the model is integrated with a complete physical parameterisation and a long timestep typical of large-scale applications, all three versions perform nearly identically to one another. This is additional indirect evidence of the negligible role played by elastic disturbances at large scale.

3.2.2 Numerical treatment of diabatic terms

Any comprehensive numerical model is composed of 2 parts: a dynamical kernel solving for the fluid mechanical field equations and a physical parameterisation package to account for the ensemble effect of subgrid-scale processes upon the resolved scales of the model. There are a number of techniques that can be used for combining the dynamics and physics contributions in a semi-implicit model. One such method called time-splitting is used in some models because of its simplicity and stability property: in this context time-splitting consists in solving the dynamical part of the model equations with semi-implicit scheme, without the physical contribution to the tendencies, and then adding physics contributions as a second, correction step. The more traditional alternative is to add physics tendencies together with nonlinear tendencies as part of the explicit component of the SI scheme.

To better understand the difference between the two methods, let us consider the following canonical equation for a dependent variable ψ:

$$\frac{d\psi}{dt} = L(\psi) + R(\psi) + P(\psi)$$

where L and R represent the linear and nonlinear dynamical contributions to the tendencies, respectively, and P represent the physical contribution to the tendencies. Integrating this equation with a three-time-level semi-implicit time scheme, the traditional way of solving the SI system uses a single step as follows:

$$\frac{\psi(t+\Delta t) - \psi(t-\Delta t)}{2\Delta t} = L\left(\frac{\psi(t+\Delta t) + \psi(t-\Delta t)}{2}\right) + R(\psi(t)) + P(\psi)$$

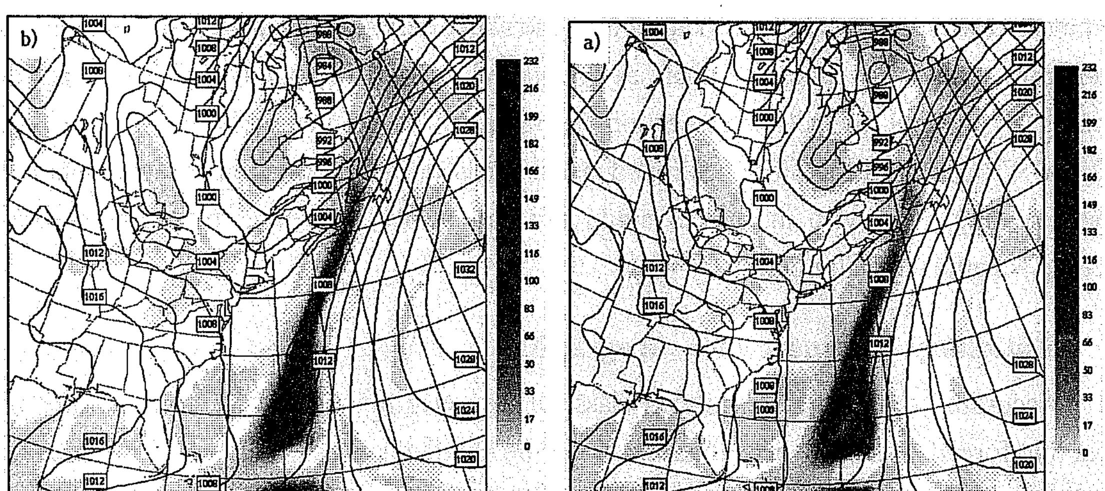
whereas the time-splitting method operates in two steps:

Step 1 (dynamics): $$\frac{\psi^*(t+\Delta t) - \psi(t-\Delta t)}{2\Delta t} = L\left(\frac{\psi^*(t+\Delta t) + \psi(t-\Delta t)}{2}\right) + R(\psi)$$

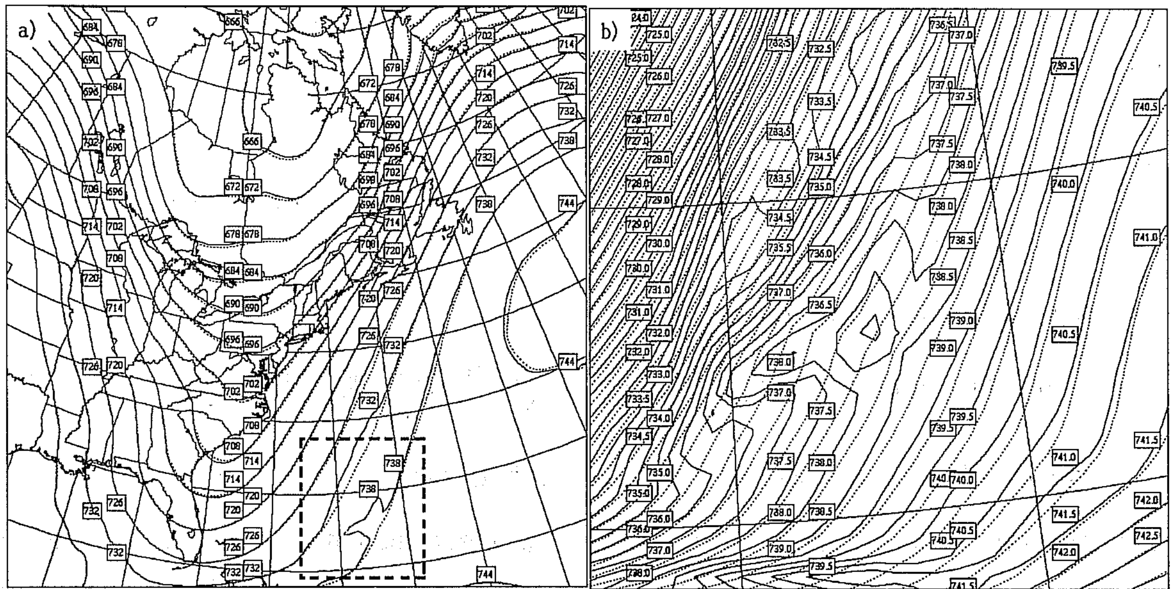
Step 2 (physics): $$\psi(t+\Delta t) = \psi^*(t+\Delta t) + 2\Delta t P(\psi^*)$$

where $\psi^*(t+\Delta t)$ is an interim result of the first, adiabatic time step (note that to retain full generality, no specific time level has been associated to the P term as different parts of the physics use different time levels).

To show the effect of splitting in a SI model, we present results obtained with a 45-km version of the Canadian Regional Climate Model (CRCM) over a region covering the East Coast of North America, nested with Canadian GCMII simulated data for a January case during which there was an episode of strong coastal development, a common event in winter over the area.



6-h average precipitation rate (shades of grey, units are mm/da) and mean sea level pressure (thin lines, units are hPa and contour interval is 4 hPa) on a January day after 54 h of CRCM simulation for the cases without splitting (left) and with splitting (right).



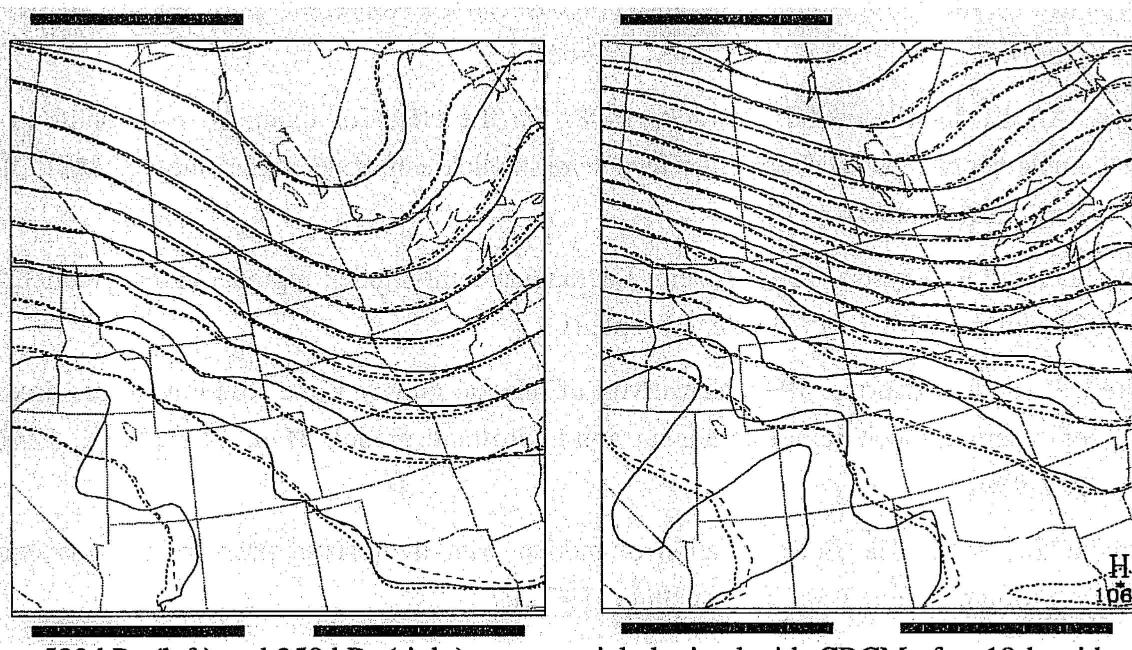
(Left) 400 hPa geopotential height fields on a January day after 54 h of CRCM simulation with splitting (full line) and without splitting (dotted line). Units are dam and contour interval is 6 dam. (Right) Zoom in a southern portion of left figure. Contour interval is 0.5 dam.

While low-level flow fields and precipitations are very similar, the model version using the splitting method develops an abnormally intense ridging over the area of strong release of latent heat associated with the precipitation. These results were taken from Caya et al. (1998) who have shown that the splitting method may introduce serious truncation errors when used in conjunction with long timesteps that are permitted by semi-implicit and semi-Lagrangian marching algorithms. In fact the truncation errors are so severe that early versions of CCRM, MC2 and CRCM used the aforementioned quasi anelastic approximation for the treatment of diabatic heating, otherwise the model could not run stably with long timesteps that SI-SL marching scheme normally permits.

3.3 Stationary forcings, incl. orography

According to the aforementioned classical response function of SI scheme, one would expect that forced stationary perturbations would be handled perfectly. However it turns out that forced perturbations solutions obtained with SI-SL scheme, with long timesteps permitted by stability considerations, under substantial flows can be highly erroneous. The following figure taken from a short simulation of the Canadian Regional Climate Model (see Caya and Laprise 1998 and Laprise et al. 1998 for a complete description) run at 45-km resolution over the Rocky Mountains with timesteps of 5 and 15 min. It can be seen that the run using 15-min timestep exhibits very suspicious noise over and in the lee of the Rocky Mountains. This noise is absent in the shorter timestep run.

Various modifications to traditional SI-SL scheme have been suggested to reduce this noise (see Héreil and Laprise 1996 for a review). These modifications usually involve some form of off-centring of the time-averaging at the core of the SI scheme. In this model the off-centring is controlled by the parameter ε; it can be seen in the figure that $\varepsilon = 0.10$ permits to achieve a solution close to that obtained with a small timestep in this case. However as shown also in that article none of the proposed alternatives really cure entirely the problem. In fact for a mesoscale mountain wave case, the proposed alternatives are just as bad as the original scheme.



500 hPa (left) and 250 hPa (right) geopotential obtained with CRCM after 18 h with 15 min. time step and $\varepsilon = 0.01$ (full), with 15 min. time step and $\varepsilon = 0.10$ (dashed), and with 5 min. time step and $\varepsilon = 0.01$ (dotted). Here ε is the uncentering parameter.

It is worth noting that this problem with the treatment of forced stationary perturbations is not specific to FE models and that PE models also suffer from it. This outstanding problem severely limits taking fully advantage of the efficiency of SI-SL models and hence requires attention. Various approaches are currently being investigated to cure this problem (Geleyn, personal communication). Laprise (1998) also proposed an alternative that still remain to be tested.

4. CONCLUSIONS

A general formalism for a dynamical kernel of atmospheric model has been presented, based on the unapproximated field equations solved by semi-implicit and semi-Lagrangian numerical method. The advantage of the approach resides in its universality: a unique model can be applied efficiently at all scales, from the global- to the micro-scales. The timestep in this formulation is a user-adjustable parameter, based on the required accuracy. It may anticipated that all operational atmospheric models will soon be based on variants of such formulation.

5. REFERENCES

Benoit, R., M. Desgagné, P. Pellerin, S. Pellerin, S. Desjardins and Y. Chartier, 1997: The Canadian MC2: A semi-Lagrangian, semi-implicit wide-band atmospheric model suited for fine-scale process studies and simulation. *Mon. Wea. Rev.*, **125**(10), 2382-2415.

Carpenter, K. M., 1979. An Experimental Forecast using a Non-Hydrostatic Mesoscale Model. *Quart. J. Roy. Meteor. Soc.*, **105**, 629-655.

Caya, A., R. Laprise and P. Zwack, 1998: On the effect of using process splitting for implementing physical forcings in a semi-implicit semi-Lagrangian model. *Mon. Wea. Rev.* **126**(6), 1707-1713.

Caya, D., and R. Laprise, 1998: A semi-Lagrangian semi-implicit regional climate model: The Canadian RCM. *J. of Climate,* (accepted).

Héreil, P., and R. Laprise, 1996: Sensitivity of internal gravity wave solutions to the timestep of a semi-implicit semi-Lagrangian non-hydrostatic model. *Mon. Wea. Rev.*, **124**(4), 972-999.

Laprise, R., 1992: The Euler equation of motion with hydrostatic pressure as independent coordinate. *Mon. Wea. Rev.*, **120**(1), 197-207.

Laprise, R., D. Caya, G. Bergeron and M. Giguère, 1997: The formulation of André Robert MC2 (Mesoscale Compressible Community) model. *The André J. Robert Memorial Volume* (C. Lin, R. Laprise and H. Ritchie, Eds.), companion volume to *Atmos.-Ocean*, **35**(1), 195-220.

Laprise, R., D. Caya, M. Giguère, G. Bergeron, H. Côté, J.-P. Blanchet, G. J. Boer and N. A. McFarlane, 1998: Climate and climate change in Western Canada as simulated by the Canadian Regional Climate Model. *Atmos.-Ocean,* (accepted).

Laprise, R., 1998: Stationary forcing / orographic resonance. Section in chapter on Numerical methods. NATO Advanced Study Institute, Il Ciocco, Italy, May 25 - June 5 1998.

Tanguay, M., A. Robert and R. Laprise, 1990: A semi-implicit semi-Lagrangian fully compressible regional forecast model. *Mon. Wea. Rev.*, **118**(10), 1970-1980.

Thurre, C. 1998: Étude de l'ajustement hydrostatique suite à un forçage diabatique dans un modèle pleinement élastique. Ph. D. thesis in Atmospheric Sciences, Cooperative programme UQAM-McGill U., 116 pp. (submitted).