TECHNICAL MEMORANDUM

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Error estimation of Buoy, Satellite and Model Wave height Data

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Abstract

Triple collocation is a powerful method to estimate from three collocated data sets the rms error in each of them, provided the errors are not correlated. Wave height analyses from the operational ECMWF (European Centre for Medium-Range Weather Forecasts) wave forecasting system over a two year period are compared with independent buoy data and dependent ERS (European Remote Sensing) -2 Altimeter wave height data, which have been used in the wave analysis. In order to apply the triple collocation method, a fourth, independent data set is obtained from a wave model hindcast without assimilation of Altimeter wave observations. The seasonal dependence of the respective errors is discussed, while in agreement with the properties of the analysis scheme, the wave height analysis is found to have the smallest error.

In this comparison the Altimeter wave height data have been obtained from an average over N individual observations. By comparing model wave height with the Altimeter superobservations for different values of N, an alternative estimate of the Altimeter error and model error is obtained. There is only agreement with the estimates from the triple collocation when correlation between individual Altimeter observations is taken into account.

The collocation method is also applied to estimate the error in ENVISAT (Environmental Satellite) and ERS-2 Altimeter wave height, buoy wave height, model first-guess and analysis. It is shown that there is a high correlation between ENVISAT and ERS-2 wave height error, while the quality of ENVISAT Altimeter wave height is high.

1 Introduction.

Satellite observations have resulted in considerable improvements of weather and wave forecasting. Because these observatations have such a large potential value, it is important to validate them. A common procedure to do this is as follows. As soon as the satellite is launched and the instruments on board are performing in a stable manner, the observed products are compared with analyzed fields to check on gross errors and, if needed, to retune geophysical algoritms. The advantages of a comparison against analyses are that the quality of an analyses is fairly well-known and that in a relatively short period many collocations between observed quantities and the analysed counterparts are available. Thus, a rapid assesment of the quality of satellite observations may be given. Also, the collocation between analysis and observations is of vital importance to develop geophysical algorithms such as CMOD4 (Stoffelen and Anderson, 1997) and NSCAT (Wentz and Smith, 1999) for C-band and Ku-band Scatterometers. Nevertheless, a check against in-situ observations is an important addition to the quality assurence of satellite products, although the number of collocations is lower by typically two orders of magnitude.

However, when comparing several types of data it is desirable to have an idea about the size of the errors. By the way, these errors consist of several components. The instrumental, measurement error usually only gives a small contribution to the total error. More significant are representativeness errors and errors caused by the finite distance and time between two observations.

For example, when calibrating one instrument against another it is important to know their error because the calibration constants depend on them. The example of linear regression is discussed by Marsden (1999), see also Tolman (1998).

Furthermore, data assimilation requires knowledge of the weights given to the data and to the first-guess field. These weights depend on the ratio of the first-guess error and the observation error. In wave forecasting these errors are usually not known, and one assumes, as is done in the Optimum Interpolation (OI) scheme of the ECMWF wave forecasting system, that the errors are equal. Hence first-guess and observations get equal weight during the analysis.

The need for estimates of errors of different data sources was realized by Stoffelen (1998). He proposed to use a triple collocation method to calibrate observations of winds from a Scatterometer using winds from buoys, a model analysis and the ERS-1 Scatterometer. In his approach it was assumed that error and truth were not correlated. In a similar vein, Caires and Sterl (2003) applied a triple collocation method to estimate and calibrate analysed winds and wave heights from the ERA-40 analysis effort. Quilfen et al (2001) followed a different approach proposed by Freilich and Vanhoff (1999) to estimate and calibrate ERS Scatterometer wind measurements over the period 1992 to 1998. However, in this methodology the true wind speed was assumed to be Weibull distributed and the data sets were not independent because, through the data assimilation, the analysed wind depends on both buoy winds and Scatterometer winds. In a somewhat different context, Tokmakian and Challenor (1999) estimated errors in model and ERS-2 and Topex/Poseidon satellite mean sea level anomalies using a method that only assumes that there is no correlation between the respective errors. However, a calibration is then not possible.

It is straightforward to show that with three data sets which have uncorrelated errors, the error of each data type can be estimated from the variances and covariances of the data sets. However, unless additional assumptions are being made, it is not possible to perform a calibration among the data sets, simply because there are not enough equations. A possible way out of this dilemma is to use a minimization procedure. We therefore propose the following method to estimate errors and to calibrate data sets. We assume that the errors are not correlated and that the errors of the three data sets are estimated using the triple collocation method. Given these estimated errors, calibration is then performed using the neutral regression approach of Deming (Mandel, 1964, see also Marsden, 1999), which is based on the minimization of the error in both variates. Using the calibrated data sets a new estimate of the errors may be obtained resulting in new calibration constants. Thus, an iteration procedure is started and continued until convergence of the results is obtained. This is discussed in some detail in section 2.

In section 3 we apply this approach to the estimation of the wave height error of the ECMWF wave analysis. Operationally, we have available joint estimates of the true state from buoys, ERS-2 Altimeter and the wave analysis. However, these data sources are not independent because the wave analysis uses Altimeter data. One would expect, and this is common practice in meteorological data assimilation, that the first-guess field and Altimeter observations may be regarded as independent. But we argue and show that this is not an appropriate assumption in the case of ERS-2 Altimeter data. For this reason we generated a fourth independent data set by rerunning the wave forecasting system with ECMWF analyzed winds but without the assimilation of ERS-2 wave height data (this is called a hindcast). With four data sets, in which there is one independent triplet, all relevant variances and error covariances may be obtained. Monthly root mean square (rms) errors are obtained over a two year period and we discuss seasonal variations in the errors and calibration constants for buoys, ERS-2 satellite and wave analysis. Most probably because of representativeness errors the buoy errors are found to be the largests, followed by the Altimeter error while the wave analysis has the smallest error. This follows in a straightforward manner from the properties of the OI scheme which results in analysis errors being the smaller of the first-guess error and the observation error.

In a collocation study, the representativeness error is a serious issue which needs to be addressed. Usually, instruments and model refer to different scales of the truth, and in order to reduce problems with representativeness, averaging of the observations towards the scales as seen by the model is required. Superobservations for buoys are obtained by time averaging 5 hourly observations. The Altimeter superobservations are obtained from an average of N individual observations where for ERS-2, $N = 30$. It is, of course, of interest to investigate the dependence of the comparison results between model and Altimeter superobservations as function of N. It turns out that this then gives another estimate for the model error and the Altimeter error. Agreement with the results from the triple collocation is only obtained when correlation between the individual Altimeter measurements is taken into account. For ERS-2 we typically find a correlation length scale of 70 km, in agreement with the practice to smooth the tracking results over 10 consecutive observations.

Finally, during the ENVISAT commissioning phase, ENVISAT and ERS-2 are in almost identical polar orbits. As a consequence, there are 5 collocated data sets, namely from the ENVISAT and ERS-2 Altimeters, from buoys, from model first-guess and analysis. Note that during the commissioning phase ENVISAT data are not assimilated. It is shown that there are correlations between ENVISAT and ERS-2 Altimeter wave height errors because results from a triple collocation of ENVISAT, ERS-2 and buoys are not consistent with results from a triple collocation of ENVISAT, model first-guess and buoy data. When correlated errors are taken into account we find that the relative errors in wave height are 6%, 7%, 9%, and 5% for respectively ENVISAT, ERS-2, buoys and wave analysis. The errors for ERS-2, buoy and analysis are in fair agreement with the operational results.

2 On error estimation.

In this section a description is given of the method how to determine from three independent estimates of the truth their respective errors and how to calibrate the data by minimization. Note that it is essential that assumptions have to be made regarding the relation between model and observations on the one hand and the truth on the other hand. At the same time this gives an implicit definition of the error. Because of an assumed relation between observation and truth it follows that in case that this relation is incorrect the error has both a systematic and random component. Therefore, the assumption of uncorrelated errors is by no means evident, and should, if possible, be tested.

Suppose we have three estimates of the truth, denoted by X, Y, and Z, obtained from observations or from simulations of the truth by means of a forecasting system. In the following all these estimates of the truth will be referred to as measurements. Furthermore, it is assumed that the measurements depend on the truth T in a linear fashion

$$\begin{aligned} X &= \beta_X T + e_X, \\ Y &= \beta_Y T + e_Y, \\ Z &= \beta_Z T + e_Z, \end{aligned} \tag{1}$$

where e_X, e_Y, and e_Z denote the errors in the measurements X, Y, and Z, while β_X, β_Y, and β_Z are the calibration constants. Since we are estimating wave height, which is a quantity that is positive definite, no intercept is included in the model for the measurements. A finite intercept (such as used by Caires and Sterl, 2003) gives rise to negative values of either the mean value of the truth or of the measurement, which physically does not make sense.

It is emphasized that the linear dependence of the measurement on the truth is an assumption which needs not to be true and, therefore, one cannot assume that the errors are random. For example, if actually there is a nonlinear relation between measurement and truth but one would take the linear model (1) instead, the error will have a random ànd a systematic component. Furthermore, if two types of measurents have a similar nonlinear relation with the truth, then in the context of the linear model (1) there is now the possibility of correlated errors. This may be the case when intercomparing two Altimeters which share the same measurement principle.

Let us now assume that the linear model (1) is valid and that the measurement results X, Y, and Z have uncorrelated errors,

$$\langle e_X e_Y \rangle = \langle e_X e_Z \rangle = \langle e_Y e_Z \rangle = 0, \tag{2}$$

where the angle brackets denote the average over a sufficiently large sample. In order to eliminate the calibration constants we introduce the new variables $X' = X/\beta_X$, $e'_X = e_X/\beta_X$, etc so that

$$\begin{aligned} X' &= T + e_{X'}, \\ Y' &= T + e_{Y'}, \\ Z' &= T + e_{Z'}, \end{aligned} \tag{3}$$

and the primed observations have uncorrelated errors as well. We eliminate now the truth to obtain

$$\begin{aligned} X' - Y' &= e_{X'} - e_{Y'}, \\ X' - Z' &= e_{X'} - e_{Z'}, \\ Y' - Z' &= e_{Y'} - e_{Z'}, \end{aligned} \tag{4}$$

Then, multiplying the first with the second equation of (4) and utilizing the assumption of independent errors (2) one immediately obtains the variance of error in X' in terms of the variance of X' and the covariances of X' and Y', X' and Z', and Y' and Z'. In a similar manner, by multiplying the first with the third equation of (4) one obtains the variance of error in Y', whilst the variance of error in Z' is obtained by multiplying the second and the third equation. Hence,

$$\begin{aligned} \langle e^2_{X'} \rangle &= \langle (X' - Y')(X' - Z') \rangle, \\ \langle e^2_{Y'} \rangle &= \langle (Y' - X')(Y' - Z') \rangle, \\ \langle e^2_{Z'} \rangle &= \langle (Z' - X')(Z' - Y') \rangle. \end{aligned} \tag{5}$$

Therefore, if errors are uncorrelated only three collocated data sets are needed to estimate the variance of the error in each of them.

The next step is to perform a calibration of the measurements. Since the truth is not known, only two of the three calibration constants can be obtained. Therefore, we arbitrarily choose X as the reference. Since the errors in the measurements are now known the calibration constants for Y and Z may be obtained using neutral regression (Marsden, 1999). As a result, the regression constant for Y becomes

$$\beta_y = \left(-B + \sqrt{(B^2 - 4AC)}\right)/2A, \tag{6}$$

where $A = \gamma\langle XY\rangle$, $\gamma = \langle e_X^2\rangle/\langle e_Y^2\rangle$, $B = \langle X^2\rangle - \gamma\langle Y^2\rangle$, and $C = -\langle XY\rangle$. Replacing Y with Z in Eq.(6) then gives the regression constant for Z.

Having performed the calibration of Y and Z it is clear that the work is not finished yet because this calibration will affect the estimation of the errors in X, Y and Z and hence the calibration constants, etc. Therefore, we adopted the following iteration procedure. We start with the initial guess $\beta_Y = 1$, $\beta_Z = 1$, we scale Y and Z with β_Y and β_Z respectively and we determine the errors using Eq.(5). A first estimate for the calibration constants follows then from Eq.(6). In the next step we scale Y and Z with the newly found estimates for β_Y and β_Z, determine the errors and the regression constant using (5-6) and we continue until convergence is achieved. By comparing results from a different number of iterations it was found that after 10 iterations an accuracy of four significant digits was achieved.

Note that only a relative calibration is possible. Nevertheless, results on errors do not depend on the chosen reference standard. This was checked by choosing Y instead of X as a reference standard: errors were identical up to four significant digits and, as expected, the calibration constant of X was the inverse of the calibration constant of Y when X was chosen as a reference.

In the remainder of this paper we use the triple collocation scheme (5-6) as a basic tool to try to understand relations and errors among collocated data sets. In Section 3 we apply this approach to the estimation of wave height error in the ECMWF wave analysis, in buoy and in ERS-2 wave heights over a two year period, while in Section 4 we use wave height results obtained during the ENVISAT-ERS-2 tandem mission and we apply the triple collocation approach to show that there are significant correlations between the Altimeters from ERS-2 and ENVISAT.

3 On errors in the operational ECMWF wave analysis.

Operationally, joint estimates of the true state from buoys, ERS-2 Altimeter (as they arrive in near real time through the GTS) and the wave analysis are available, but these data sources are not independent because the wave analysis uses Altimeter data. Therefore, another data set is needed, which is independent from the observations. A possible choice would be to use the first-guess field as an independent data set. It is common practice in data assimilation to regard first-guess field and observations as independent, because these observations have not yet been used in the analysis and forecasting scheme. This is plausible when an instrument has only random errors, but it may be problematic in case of systematic errors in the observations. For example, it is well-known that the so-called fast delivery significant wave heights from the ERS-2 Altimeter have a systematic error for low wave heights, simply because this Altimeter does not produce wave heights lower than about 60 *cm*. This is illustrated in Fig. 1 where over a two year period a comparison is shown between wave height data from the ERS-2 Altimeter and Buoy data. Note the overestimation of wave height by the Altimeter for wave heights below 1.5 *m*.

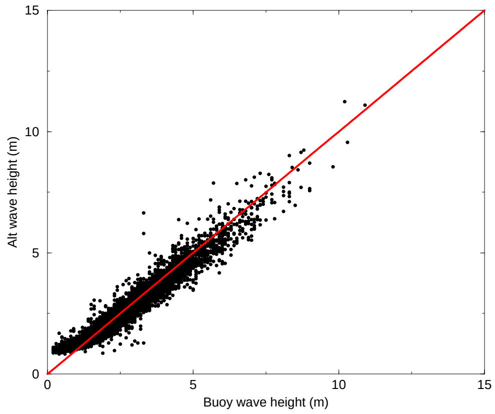

Figure 1: Comparison of Altimeter wave height data with Buoy data for the period of 1 January 2000 until 31 December 2001.

Suppose these erroneous low wave height data are assimilated in the model. As low wave heights usually correspond to swell conditions and swell has a long memory, it is likely that the following first-guess field is contaminated by the wrong data. Hence, first-guess and Altimeter data may have correlated errors.

For this reason, we generated a fourth independent data set by running the wave forecasting system with six-hourly ECMWF analysed winds over the two year period of January 2000 until December 2001 after a two-month warm-up period to eliminate any Altimeter impacts. No ERS-2 Altimeter data were assimilated so that the hindcast results are independent of Altimeter and buoy wave height data. From the operational results and the hindcast the following collocated data set was generated: X = Hindcast, Y = Altimeter, Z = Buoy, V = First-guess, and W = Analysis. We adopt model (1) for these data sets. Noting that the buoy data are not used in the analysis, we allow for error correlations between X and V, X and W, Y and V, Y and W, and finally between V and W. In other words, the covariances

$$\langle e_{X'}e_{V'}\rangle, \langle e_{X'}e_{W'}\rangle, \langle e_{Y'}e_{V'}\rangle, \langle e_{Y'}e_{W'}\rangle, \langle e_{V'}e_{W'}\rangle \tag{7}$$

are finite. Here, a prime again denotes scaling with the slope β. Together with 5 error variances this gives 10 unknowns, while by correlating the differences $X'-Y'$, $X'-Z'$, $X'-V'$, $X'-W'$ etc, there are $4+3+2+1=10$ equations, so all the unknows may be determined.

This large set of equations may be solved in a straightforward manner by realizing that the first three measurements are independent, hence the triple collocation technique may be applied and their errors follow from Eq.(5). The errors for the remaining measurement types, which have correlated errors with each other and the Altimeter and the Hindcast, follow from two steps. For example, the variance of the analysis error follows by correlating $W'-Y'$ and $W'-Z'$. Using (7) we find

$$\langle e_{W'}^2\rangle = \langle e_{Y'}e_{W'}\rangle + \langle (W'-Y')(W'-Z')\rangle, \tag{8}$$

and the correlation between W and Y follows from correlating $X'-W'$ and $Z'-Y'$. Hence

$$\langle e_{Y'}e_{W'}\rangle = \langle (X'-W')(Z'-Y')\rangle. \tag{9}$$

The error and correlation with the Altimeter for the first-guess V follows from (8)-(9) by replacing W by V.

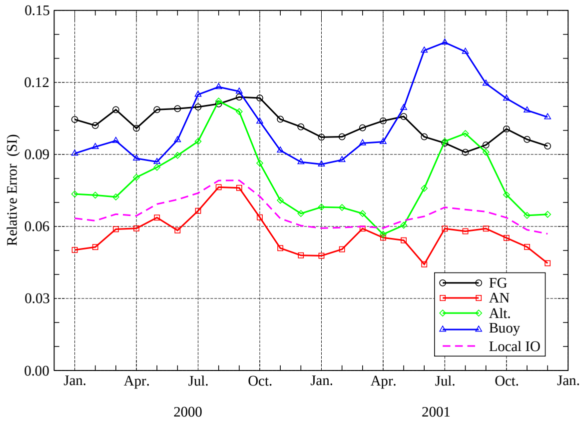

Figure 2: Monthly Relative Error of First-Guess(FG), Analyzed(AN), ERS-2 Altimeter(Alt) and Buoy wave height. Maximum Relative Collocation Difference is 5%*. For comparison the Analysis error according to a local Optimum Interpolation(OI) Scheme is shown as well.*

Before we present results on error estimation, it is important to discuss the collocation method. First, each estimate represents a different aspect of the truth. The global ECMWF wave prediction system has a spatial resolution of 55 *km*, and is forced by atmospheric winds which had in 2000-2001 a spatial resolution of 65 *km*. However, horizontal diffusion in the atmospheric model reduces activity at the short scales considerably, and also the first-order advection scheme in the wave model may give rise to smoother wave fields. Hence, in practice the model wave height fields only properly represent spatial scales larger than about 100 *km*. The ERS-2 Altimeter measures significant wave height every 7 *km*, while buoys typically produce hourly measuments which are 20 *min* averages. Clearly, instruments and model refer to different scales of the truth, and in order to reduce problems with representativeness, averaging of the observations towards the scales as seen by the model is required. Superobservations for buoys are obtained by time averaging 5 hourly observations. The Altimeter superobservations are obtained from an average of N individual observations, where operationally, $N = 30$.

Second, each estimate refers to a different location and time. In order to alleviate this problem, the wave model field is linearly interpolated in space and time towards the Altimeter and buoy observations. Hence, we deal with two model counterparts, one referring to the Altimeter observation, denoted by X_{alt}, etc., and one referring to the buoy observation, denoted by X_{buoy}, etc. In this collocation study the model value is taken as the mean of X_{alt} and X_{buoy}. Nevertheless, a collocation error between Altimeter and Buoy remains. Using the difference $X_{alt} - X_{buoy}$, the collocation error can be estimated, however. In order to ensure that the error estimation is not affected by the collocation error, only collocations satisfying a relative difference of at most 5% are considered. This corresponds to a relative collocation error, defined as the rms difference normalised with the average model wave height, of between 1% and 2%. Because of this restriction, the number of collocations reduces by 50% from about $17,000$ to 8000.

3.1 Results.

Results over the period January 2000 until December 2001 of monthly relative error for first-guess, analyzed, ERS-2 Altimeter and buoy wave height are shown in Fig. 2. Here, the relative error is defined as the ratio of rms error to the mean wave height. This ratio is usually called the Scatter Index (SI). It is striking that these errors are relatively small, with the buoy errors the largests while the analysis errors are the smallest. The reason of the high quality analysis is a consequence of the properties of the Optimum Interpolation (OI) scheme used to produce the wave analysis.

In order to see this we discuss the well-known case of the assimilation of a single measurement. Results can be generalized straightforwardly to the case of the assimilation of many observations. Let us denote the analysis by A, the observation by O, the first-guess by F and their corresponding errors by e_A, e_O and e_F. According to OI, the analysis is given by a linear combination of model first-guess and observation,

$$A = wO + (1-w)F, \tag{10}$$

and the weight w is chosen in such a way that the analysis error is minimal. By subtracting the truth and assuming no systematic errors, hence the error model (1) with $\beta = 1$ is adopted, the analysis error becomes

$$e_A = we_O + (1-w)e_F, \tag{11}$$

and assuming that there is no correlation between first-guess error and observation error, $\langle e_O e_F \rangle = 0$, the mean square analysis error becomes

$$\langle e_A^2 \rangle = w^2 \langle e_O^2 \rangle + (1-w)^2 \langle e_F^2 \rangle. \tag{12}$$

The analysis scheme is optimal when the mean square analysis error is minimal. By differentiating the mean square error with respect to the weight, one finds for w at the minimum

$$w = \frac{\langle e_F^2 \rangle}{(\langle e_O^2 \rangle + \langle e_F^2 \rangle)}, \tag{13}$$

while the analysis error in terms of observation and first-guess error becomes

$$\frac{1}{\langle e_A^2 \rangle} = \frac{1}{\langle e_O^2 \rangle} + \frac{1}{\langle e_F^2 \rangle}. \tag{14}$$

Therefore, the analysis error is the smaller of the observation and first-guess error.

Note that the result (14) is based on the assumption of no bias and no correlation between first-guess and observation error, while it is also assumed that observation error and first-guess error are known. We will see in a moment that the bias is relatively small (at least with respect to the buoys). Also, the correlation between first-guess and observation is at most 20%. However, the reason for this study is to obtain information on the first-guess and observation error, and therefore in the ECMWF wave analysis these errors are not known. Instead, for wave heights larger than 1.5 m observations and first-guess are given equal weight, therefore $w = 0.5$, while for wave heights below 1.5 m the weight w is reduced because of the known problems with the Altimeter (cf. Fig.1). In other words, the analysis error is given by Eq.(12), and the results of this expression are plotted in Fig. 2. The errors from the local OI scheme are slightly larger than the analysis error as obtained through the triple collocation method. This is most likely caused by the fact that the actual analysis is not local, i.e. the quality of the analyzed wave height benefits from remote observations as well.

By comparing Eq.(12) with given weight, say $w = 0.5$, and Eq.(14) the importance of knowing $\langle e_F^2 \rangle$ and $\langle e_O^2 \rangle$ becomes apparent. This is most easily seen by taking the extreme case of large observation error. In the OI scheme with weights given by Eq.(13) the observations get zero weight, and hence the analysis error is given by the first-guess error. In contrast, in the usual application of OI, with given weights, the analysis error is dominated by the observation error! Evidently, (14) is optimal while (12) with constant weights is not.

Let us now study some further results of our statistical analysis. In Fig.3 monthly timeseries of the regression constant β for Altimeter, first-guess and analyzed wave height are shown. As a reference we have chosen the

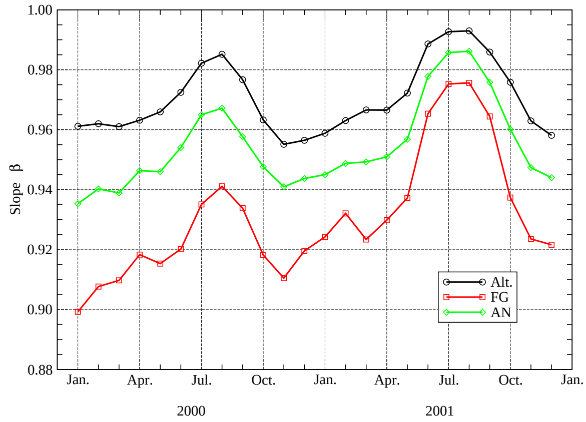

Figure 3: Monthly slopes β for Altimeter, First-guess and Analyzed wave height over the period January 2000 until December 2001. Reference is the buoy wave height.

in-situ buoy wave height data. The largest correction is required for the first-guess wave height, but over this two year period improvements are clearly visible. This change is most likely to be caused by the introduction of the T_l 511 atmospheric model together with a doubling of angular resolution of the wave spectrum on the 21rst of November 2000. In particular, the increased spatial resolution has resulted in stronger surface winds, giving higher wave heights. The smallest correction, of the order of 4%, is needed for the ERS-2 Altimeter wave height data. In an operational wave forecasting system, such as the one at ECMWF, normally fast delivery products from ESA (European Space Agency) are assimilated. The fast delivery wave heights are usually about 7%-8% lower than the buoy wave heights (Janssen, 2000). At ECMWF, the fast delivery wave height data are corrected for the non-Gaussian nature of the ocean surface resulting in increased wave height by 3%-4% (Janssen, 2000).

As noted before, in analysis schemes it is usually assumed that first-guess and observations have uncorrelated

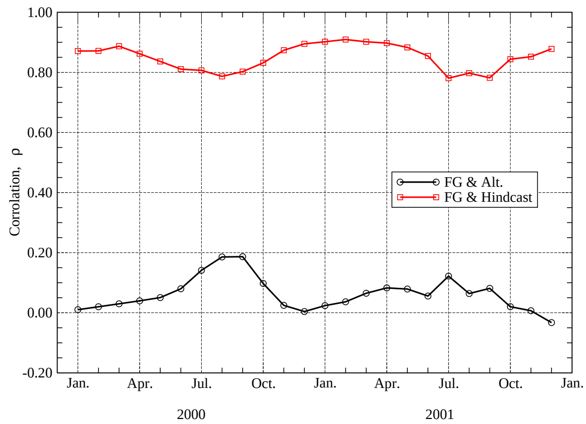

Figure 4: Monthly Correlation between First-Guess and Altimeter wave heights errors as compared to the correlation between First-Guess and Hindcast errors.

errors. With the present collocation study it is possible to determine possible correlations (cf. Eq.(9)). In Fig.4 monthly time series of the correlation between first-guess and Altimeter errors is shown and as a reference the correlation between first-guess and Hindcast error is also shown. Note that, in particular during the Northern Hemisphere summer months when low wave heights and therefore swells prevail, there is a considerable correlation between first-guess and Altimeter.

Last, it is noted that, with the exception of the first-guess error, there is a clear seasonal cycle in the statistical results. Typically, the relative error or scatter index is the largest during the summer time. This can be understood for the Altimeter data (and, as a consequence, for the analysis) because of the already mentioned problems at low wave height. However, it is not clear why buoy data have larger relative errors during the summer compared to winter time. On the other hand, during the summer time there is less need for corrections of Altimeter, first-guess and analyzed data.

3.2 Correlation between individual Altimeter observations.

An important issue to address is how the statistical results given in the previous section depend on the procedure to obtain e.g. the Altimeter superobservations. Recall that individual Altimeter wave height observations are obtained every 7 km and that operationally a superobservation involves an average over $N = 30$ observations. Hence, formally, the Altimeter superobservations refer to a larger spatial scale (210 km) than the model analysis or first-guess ($\simeq 100$ km). Therefore, the statistical results could depend on the number N of observations used to make the superobservation, but it turns out that the dependence of results on N is weaker than expected.

Assume for the moment that the individual Altimeter observations are independent from each other, that is there is no correlation between the errors. Also, assume that first-guess and Altimeter wave height errors are not correlated (this can be achieved by choosing a winter month, cf. Fig. 4). Let σ be the rms error resulting from the comparison of first-guess and Altimeter wave height, then

$$\sigma^2 = \langle e_F^2 \rangle + \frac{1}{N}\sigma_a^2, \tag{15}$$

where $\langle e_F^2 \rangle$ is the variance of the first-guess error, while σ_a is the error of an individual Altimeter wave height measurement. Note that $\langle e_A^2 \rangle = \sigma_a^2/N$ in case the individual Altimeter observations are uncorrelated.

Therefore, Eq.(15) suggests a sensitive dependence of the rms error σ on the number of observations used in the averaging. At the same time, it was realized that Eq.(15) suggests another method to estimate first-guess error and the altimeter error: do the comparison between first-guess and Altimeter data for different N and plot the results for σ as function of $1/N$. The intercept at $1/N = 0$ then gives the first-guess error while the slope gives the error of the individual Altimeter measurement. We therefore redid for the month of December 2001 the intercomparison between first-guess and altimeter wave height data for different values of N. For two areas, namely the whole Globe and the Tropics, we determined the Scatter Index as function of $1/N$. The results of this exercise are shown in Fig.5. In contrast to expectation, the Scatter Index is not increasing linearly with $1/N$, and there are clear signs of saturation when only a few observations are used in the averaging. This is an indication that the error in the individual Altimeter observations is spatially correlated.

Let us explore the consequences of spatially correlated errors. Consider a superobservation A_{sup} defined as the spatial average of a number of N individual observations a_i taken at location i, or,

$$A_{sup} = \frac{1}{N}\sum_i a_i. \tag{16}$$

Suppose that each individual observation obeys the error model (1). Then the variance of the error in the superobservation becomes

$$\langle e_A^2 \rangle = \frac{1}{N^2} \sum_{i,j} \langle e_{a_i} e_{a_j} \rangle. \tag{17}$$

In order to make progress, a simple correlation model, of the Markovian type, is assumed. Thus,

$$\langle e_{a_i} e_{a_j} \rangle = \sigma_a^2 c^{|i-j|}, \tag{18}$$

where σ_a is the error of the individual observation and c is the constant correlation coefficient between two neighbouring observations. Substitution of (18) in (17) gives

$$\langle e_A^2 \rangle = \frac{\sigma_a^2}{N^2} \left\{ N + 2(N-1)c + 2(N-2)c^2 + \ldots \right\}. \tag{19}$$

The summation can be performed and the final result is

$$\langle e_A^2 \rangle = \frac{\sigma_a^2}{\mathcal{N}}, \tag{20}$$

where $\mathcal{N}$ is the effective number of degrees of freedom,

$$\frac{1}{\mathcal{N}} = \frac{1}{N} \left[1 + \frac{2c}{1-c} \left\{ 1 + \frac{c^N - 1}{N(1-c)} \right\} \right], \tag{21}$$

which, in case of positive correlation, is less than the number of observations N used in the averaging. Hence, in case of correlated errors Eq.(15) is replaced by

$$\sigma^2 = \langle e_F^2 \rangle + \frac{1}{\mathcal{N}} \sigma_a^2. \tag{22}$$

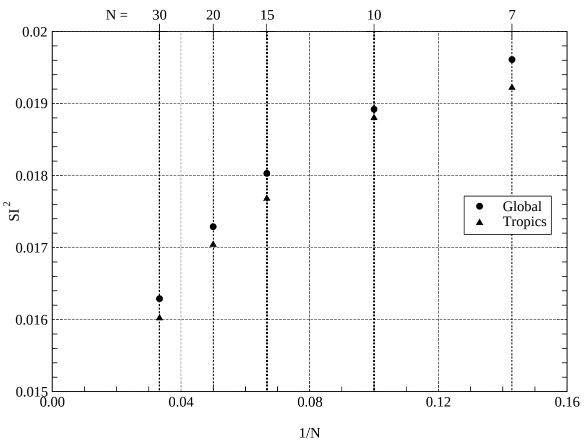

Figure 5: Comparison of ERS-2 Altimeter Superobservations with the first-guess wave heights for December 2001. Regions are Globe and Tropics. Dependence of the square of the scatter index, SI^2, on the inverse of the number N of observations used in the Altimeter superobservations. If individual Altimeter observations are independent a linear dependence is expected.

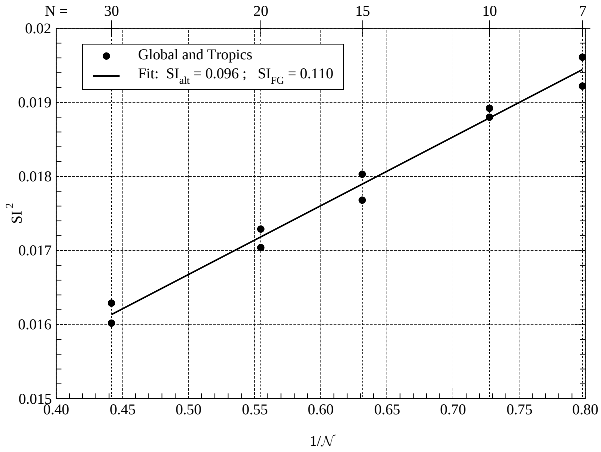

Figure 6: The same as Fig.(5) but now as a function of the effective number of degrees of freedom $\mathcal{N}$, where a correlation of 0.9 *is taken between neighbouring Altimeter observations. A best fit gives a scatter index of* 11% *for first-guess and* 9.6% *for the individual Altimeter observations.*

One still needs to determine the correlation coefficient c. This was done by trial and error insisting that, in agreement with (22) a linear relation exists between σ^2 and the inverse of the effective number of degrees of freedom, $1/\mathcal{N}$. Best results are obtained with a correlation coefficient c of 0.9, in other words, the spatial correlation scale is about 70 *km* (10 observations). Results for this choice of correlation coefficient are shown in Fig.6. Fitting the results with a linear function it is then found that the first-guess SI is 0.11, which is in fair agreement with the result from the quintuple collocation study of the previous section which gave a first-guess SI of about 0.094 for December 2001. The SI for the individual ERS-2 Altimeter measurement is found to be 0.096. In the quintuple collocation study the Altimeter superobservation consisted of an average of $N = 30$ individual observations. With a correlation of 0.9 the effective number of degrees of freedom $\mathcal{N}$ becomes about 2.3, giving a SI of the Altimeter superobservation of 0.064 which compares favourably with the result from the quintuple study where an SI of 0.065 was found. Note that results for the SI of the Altimeter superobservation are fairly insensitive to the averaging number N. In view of matching spatial scales with the model it would have been more appropriate to use $N = 15$ in the averaging of the Altimeter data. However, this only increases the SI of the Altimeter superobservation from 0.064 to 0.071. There is no need to emphasize that this weak dependence on N is caused by the significant spatial correlation between individual Altimeter observations.

It is important to try to understand why there is such a large spatial correlation between errors in the individual ERS-2 Altimeter observations. In order to be able to make a wave height observation an Altimeter needs to track the ocean surface. In case of the ERS-2 Altimeter, the tracking results are smoothed over 10 individual observations. This results in spatially correlated errors in significant wave height and makes it plausible why we find a correlation scale of about 70 *km*. It would be desirable to test whether indeed the smoothing of the tracker results causes spatial error correlation by varying the length of the smoothing filter. Now, the Altimeter on board of JASON does not average the tracker results but averages wave height results over 5 individual observations. Jean-Michel Lefèvre (private communication, 2002) studied results from the collocation between JASON Altimeter wave height data and ECMWF wave height analysis and he plotted the dependence of the variance of the total error σ^2 as function of the effective number of degrees of freedom $\mathcal{N}$. In the case of JASON he found a spatial correlation scale of about 30 *km* (5 observations).

It is concluded that the present method and the triple collocation technique give consistent results for first-guess and Altimeter superobservation provided there is a significant spatial correlation between the errors of individual Altimeter observations. The reason for the correlation is most likely the smoothing of the results, since the correlation scales for ERS-2 and JASON are found to correspond with the length of the smoothing filter.

4 A first validation of ENVISAT Altimeter wave height data.

On the 28th of February 2002 the European Space Agency (ESA) launched the ENVISAT satellite which carries on board 9 instruments, two of which are relevant for understanding ocean waves, namely the RA-2 Altimeter and the Advanced Synthetic Aperture Radar (ASAR). We discuss here some first Altimeter wave height results. The RA-2 Altimeter is a dual frequency Altimeter, but only Ku-band results will be studied.

The ENVISAT satellite was maneuvred in such a manner that during the commisioning phase it had an almost identical orbit as the ERS-2 satellite. The time difference between observations from the two satellites is, therefore, only 20 minutes. Hence, this provides a unique opportunity for validation, because we have 5 collocated data sets available, namely from the ENVISAT and ERS-2 Altimeter, from buoys, from model first-guess and analysis (quintuple collocation). The period is the 18th of July until the 17th of November 2002. During this period ERS-2 data were assimilated into the ECMWF wave forecasting system, but, for obvious reasons, ENVISAT data were not.

Let us first discuss how it was realized that errors in ENVISAT and ERS-2 Altimeter are correlated. Since there are 5 collocated data sets available there are several possibilities to apply the triple collocation method. For example, one might assume that the errors in ENVISAT and ERS-2 Altimeter wave height are not correlated and that they are not correlated with the buoy errors. Other possibilities are the triplet ENVISAT, buoy and first-guess and the triplet ENVISAT, buoy and analysis. All these triplets have ENVISAT and buoy wave height data in common. Results of the three triple collocation exercises are given in Table 1. It is evident from Table 1 that in particular the results for the relative error in the ENVISAT Altimeter wave height data are not consistent. Also, the relative error in the ERS-2 wave height data is much smaller than found during the two year period analyzed in the previous Section.

This inconsistency is plausible when errors are correlated. But, clearly, additional information is needed to determine correlations between the errors of different observations. With 5 collocated data sets and assumptions on which observation type is correlated it is just possible to obtain all relevant variances and covariances. To this end it is assumed that buoy errors are not correlated with errors of any other observation type. We have seen in the previous section that first-guess and ERS-2 errors are correlated because of the systematic problems in the ERS-2 Altimeter at low wave height. But it is unlikely that the first-guess error is correlated with the

	N	SI_{ENV}	SI_{ERS}	SI_{FG}	SI_{AN}	SI_{BUOY}
ENV-ERS-Buoy	723	2.3%	3.9%	-	-	11%
ENV-FG-Buoy	723	6.3%	-	9.5%	-	9.3%
ENV-AN-Buoy	723	5.7%	-	-	3.8%	9.7%

Table 1: Results of several triple collocations obtained during the ENVISAT commisioning phase for wave height larger than 1 *m. Here, N denotes the number of collocations and SI denotes the scatter index.*

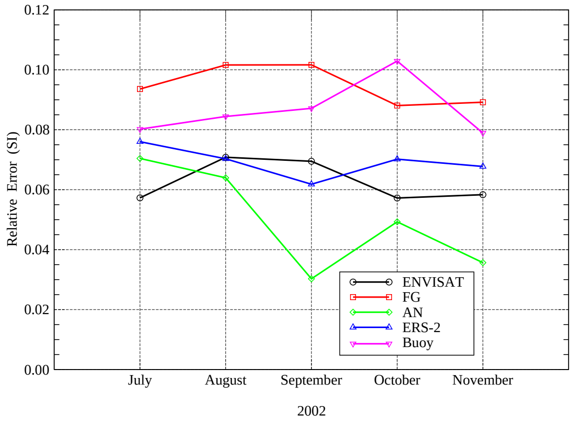

Figure 7: Monthly Relative Error of ENVISAT, first-Guess, analyzed, ERS-2 Altimeter and Buoy wave height over the period July until November 2002. The maximum relative collocation difference is 5%.

ENVISAT error because the ENVISAT Altimeter does not suffer from the ERS-2 problems at low wave height. We therefore assume that the triplet ENVISAT, buoy and model first-guess is independent, therefore their errors can be determined by means of the triple collocation method.

Furthermore, correlation between ENVISAT and ERS-2 errors is allowed. This implies that ENVISAT and analysis errors are correlated as well, because, due to the assimilation, analysis and ERS-2 errors are correlated. Last, first-guess and analysis error are evidently correlated. As a result we have 10 unknowns, namely 5 variances and 5 covariances, and just as in the previous section there are 10 equations. Hence the problem is solvable.

The collocation data set is obtained with the same procedure as the operational data set from the previous Section, except that the ENVISAT superobservations are averaged over 11 individual observations. Only results for wave height larger than 1 *m* are presented. This is done to avoid having the ERS-2 altimeter wave height data been unnecessarily penalized because of the low wave height problems. When correlated errors are taken into account we find that for the whole period the relative errors in wave height are 6%, 7%, 9%, and 5% for respectively ENVISAT, ERS-2, buoys and wave analysis. The errors for ERS-2, buoys and analysis are in fair agreement with results from the previous section. The results for the correlations are in descending order: ENVISAT & ERS-2 = 77%, first-guess & analysis = 73 %, analysis & ERS-2 = 48 %, analysis & ENVISAT = 24%, and first-guess & ERS-2 = 11%. Note the high correlation between ENVISAT and ERS-2 Altimeter wave height errors. A likely reason for the high correlation is that both instruments share the same measurement principle, although the ENVISAT Altimeter has an improved treatment of the wave form and in the algorithms deviations from the Gaussian sea state are incorporated. However, a thorough study is required to understand better the systematic part of the Altimeter wave height error.

Last, in Fig.7 we show monthly time series of ENVISAT, first-guess, analysis, ERS-2 and buoy relative wave height error. It confirms that errors for ERS-2, first-guess and analysis are consistent with the findings of the previous section. It also shows the high quality of the ENVISAT altimeter wave height results. In Fig.8 the monthly time series for the slopes of ENVISAT, ERS-2, first-guess and analysis are shown. In agreement with the operational results, slopes for ERS-2, first-guess and analysis are less than 1 when compared with the buoys, hence, according to this standard, are underestimating wave height. In contrast, the ENVISAT altimeter wave heights are higher by about 3% on average.

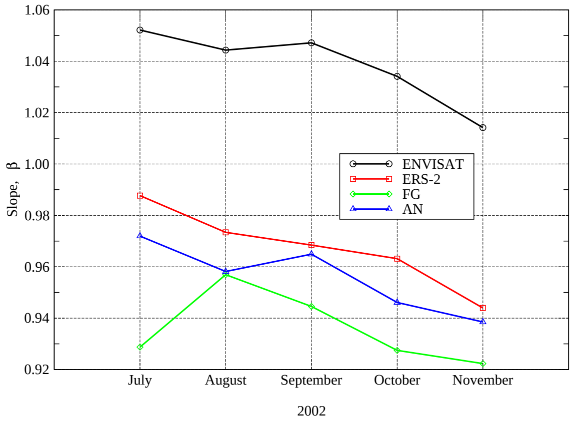

Figure 8: Monthly slopes β for ENVISAT and ERS-2 Altimeter, First-guess and Analyzed wave height over the period July until November 2002. Reference is the buoy wave height.

To conclude, we have given a preliminary validation of ENVISAT Altimeter wave height. The first impression is that the quality of these data is high. In addition, it has been shown that there is a significant correlation between the errors of the ENVISAT and ERS-2 Altimeters, presumably because they share the same measurement principle. This requires a study of the systematic error of these Altimeters, but it should be emphasized that the error level is small and therefore the systematic error is only a minor problem.

5 Conclusions.

We have used a triple collocation method to estimate the rms error of collocated wave height data sets, and we have combined this with a neutral regression approach to obtain a relative calibration of the data sets. The basic assumption of this method is that the three data sets have uncorrelated errors.

We have applied this approach to the estimation of wave height error in the ECMWF operational analyzed and first-guess wave fields, in buoy and ERS-2 Altimeter wave height observations, while we also applied this approach to results from the ENVISAT-ERS-2 tandem mission.

At ECMWF the wave analysis uses ERS-2 Altimeter data and therefore an additional data set is required, which is independent of the observations. This was provided by a hindcast with the wave model over a two year period, rather than by the first-guess fields because a correlation with the Altimeter data was suspected. Results from the application of the triple collocation method show that there is indeed a correlation between first-guess error and ERS-2 Altimeter wave height error of at most 20%. Time series of the monthly errors show that the relative error is typically between 5% and 10% which may be regarded as small. We also studied the dependence of the statistical results on the number N of individual Altimeter observations involved in the Altimeter superobservation. This resulted in an alternative method to estimate the first-guess error and Altimeter error. Results are consistent with the triple collocation method provided one assumes a considerable correlation between the individual Altimeter observations. This correlation is plausible when it is realized that ERS-2 Altimeter tracker results are smoothed over 10 consecutive observations. For JASON wave height results are smoothed over 5 observations and as a consequence the correlation length scale for JASON Altimeter wave height is half of the one from ERS-2 (Jean-Michel Lefèvre, private communication, 2002).

Preliminary results from the ENVISAT-ERS-2 tandem mission do suggest that the ENVISAT Altimeter wave heights are of high quality. It is also found that there is a high correlation between the ENVISAT and ERS-2 Altimeter wave height errors.

Finally, although the triple collocation method is a powerful tool it should be realized that one cannot apply this approach blindly. It is important to point out that this method may only be applied when data sets may be regarded as independent. Otherwise, inconsistencies may result as is evident when the method was applied to the ENVISAT-ERS-2 tandem mission. But the restriction to independent data sets is not a weakness of the triple collocation method, rather it is a strong point: because of the inconsistencies in results it was realized that there was a correlation between ENVISAT and ERS-2 Altimeter wave height error.

Acknowledgment

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