Issues in Stochastic Parameterization

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1. Introduction

Introducing stochastic perturbations in weather and climate models is one way to account for the second and higher moments of small-scale eddy feedbacks on the resolved scales at each model time step. Traditional parameterizations only deal with the first moment. In an NWP context, it is hoped that the additional stochastic forcing will increase the spread of forecast ensembles and make it more consistent with the forecast error. The forcing could also have a mean effect. At present such stochastic forcing terms are either ignored or treated in an *ad hoc* fashion.

To fix ideas, consider a model's equation in the form

$$\begin{aligned}\dot{X} &= N(X) + N_1 \\ &= N(X) + M(X) + R \\ &\approx N(X) + M(X) + B(X) \bullet F_s\end{aligned} \qquad (1)$$

where X is the resolved model state vector, $N(X)$ is the resolved model tendency, $\dot{X}$ is the true tendency, and N_1 is the tendency error associated with finite model truncation. Deterministic parameterization involves approximating N_1 as $M(X)$. The error of this approximation is R. One may further approximate R as a stochastic noise vector F_s times a matrix B, which may in general depend upon X. Thus $M(X)$ represents the 'deterministic' parameterization and $B(X)\cdot F_S$ the 'stochastic' parameterization. F_s need not be of the same dimension as X. One may also assume here without loss of generality that F_s is white in time. If it is not, then one may always declare it an auxiliary model variable and augment Eq (1) with equations for its evolution.

For a weather or climate model of finite resolution, the unparameterized remainder R must be non-zero. It needs to be accounted for, even if it is 'just' noise. Clearly, it affects the variability of X, but if its amplitude depends on X, it can also affect the expected mean of X as illustrated in Fig 1. The expected mean position of a ball that is kicked around in a symmetric well will be zero if the kicks have the same strength regardless of where the ball is, but if they are stronger on the right hand side, then the expected mean position will shift to the right. In an ensemble forecasting context, this means that neglecting R affects not only the ensemble spread but also the ensemble mean. In a climate modeling context, it affects both a model's climate variability and its mean climate.

Several modeling groups are investigating the possibility of estimating R and incorporating stochastic approximations of it in their models. One way to estimate R is from the difference of the model tendency from that of a much higher resolution model; another is from a budget analysis of high-resolution observations; and a third from observed or theoretically derived turbulence statistics. At present all of these approaches are in their infancy. At issue are also basic uncertainties in the overall impacts of such stochastic forcing terms in weather and climate models, and their sensitivities to (i) the prescribed amplitudes and space and time scales of the stochastic perturbations, (ii) the model variables that are stochastically perturbed, and (iii) numerical implementation.

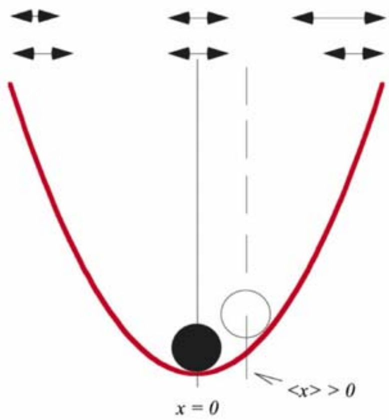

Figure 1

To guide us in these efforts, it would be nice to have some analytical results on the impacts of stochastic perturbations in simple meteorologically relevant settings and to provide checks on stochastic numerical integration schemes. Analytical results however generally exist only for linear systems forced by additive and/or linear multiplicative white noise, of the form

$$\dot{x} = Lx + \sum_j A_j \eta_j x + SF_s + F_{ext} \tag{2}$$

where x and F_{ext} are N-vectors, F_s is an additive white noise K-vector, η is a multiplicative white noise J-vector with components η_j and L, S and A_j are NxN , NxK, and NxN matrices. Note that L, S, A_j and F_{ext} may depend explicitly on time. Denoting the first and second moments of x by the N-vector $<x>$ and the NxN matrix C with elements $C_{nm} = <x_n x_m>$, the equations for these moments may be obtained from the Fokker-Planck equation as

$$<\dot{x}> \; = \; [L + \frac{1}{2}\sum_j A_j^2] \; <x> \; + \; F_{ext} \tag{3a}$$

$$\begin{aligned} \dot{C} \quad &= \; [L + \frac{1}{2}\sum_j A_j^2]\, C \;\; + \;\; C\, [L + \frac{1}{2}\sum_j A_j^2]^T + \sum_j A_j C A_j^T \\ &\quad + \; SS^T \; + \; F_{ext} < x^T > \; + \; < x > F_{ext}^T \end{aligned} \tag{3b}$$

The chief conceptual and practical simplification afforded by the linear Eq. (2) with white stochastic forcing is this closed set of deterministic coupled linear equations (3) for the first two moments of x. Eq. (2) is useful as an approximate error growth equation in data assimilation and forecasting contexts, and as an anomaly evolution equation in climate contexts.

In reality the stochastic forcing is not white but colored. A serious consequence of this, even in the linear system (2), is that the equations for the moments are no longer closed and some closure approximation must be made. If the autocorrelation time scales of the forcing are sufficiently small compared to those of the resolved scales, an attractive alternative is to approximate the colored noise as white and use the theory outlined above. The consistent way to do this, given that white noise has a flat spectrum and infinite variance, is to choose its flat spectral power density to be that of the colored noise at the lowest frequencies. The rationale for this is the so-called 'dynamical central limit theorem' (DCLT) (Penland 2003, and references) which may be intuitively understood by supposing that the noise affects the resolved scales primarily through forcing at the resolved frequencies, at which its own spectrum is approximately flat. Figure

2 provides an illustration. Note that the approximation yields the same white noise for the two red noises shown, but is quantitatively better for the noise with the shorter autocorrelation time scale.

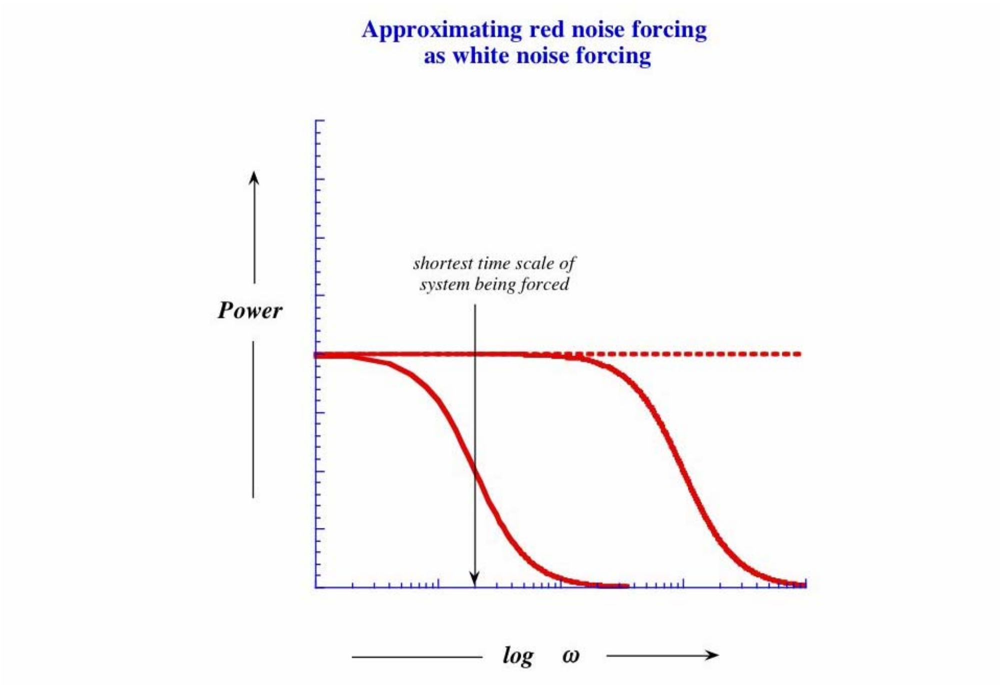

Figure 2

Figure 3 summarizes results from Sardeshmukh et al. (2001, 2003). The top panel shows the steady streamfunction response to a steady circular 30º diameter Rossby wave source imposed over Tibet on a 15 m/s superrotation zonal flow in a barotropic model. The time-mean response obtained when red noise perturbations are introduced in the amplitude of the super-rotation or in the Rayleigh wave damping coefficient are shown in the lower right panels. The mean response is damped relative to the unperturbed case for the noisy ambient flow, whereas it is amplified for the noisy damping rate. The lower left panels show analytical approximations to these numerical solutions obtained by approximating the red noises as white using the DCLT and determining the steady solutions to Eq. (3a). The contrasting damped and amplified responses are well captured, but the effects are exaggerated under these approximations. Sardeshmukh et al. (2003) have developed a closure approximation for red noise perturbations that accurately captures all the details of the numerical response.

Even these simple Rossby wave examples provide sobering reminders that (i) the impact of stochastic perturbations can be very different depending on what quantities (ambient flow or damping coefficient) are perturbed, (ii) the impact can be sensitive to the time scale of the noise (red or white), and (3) pretending the noise to be white can lead to large error.

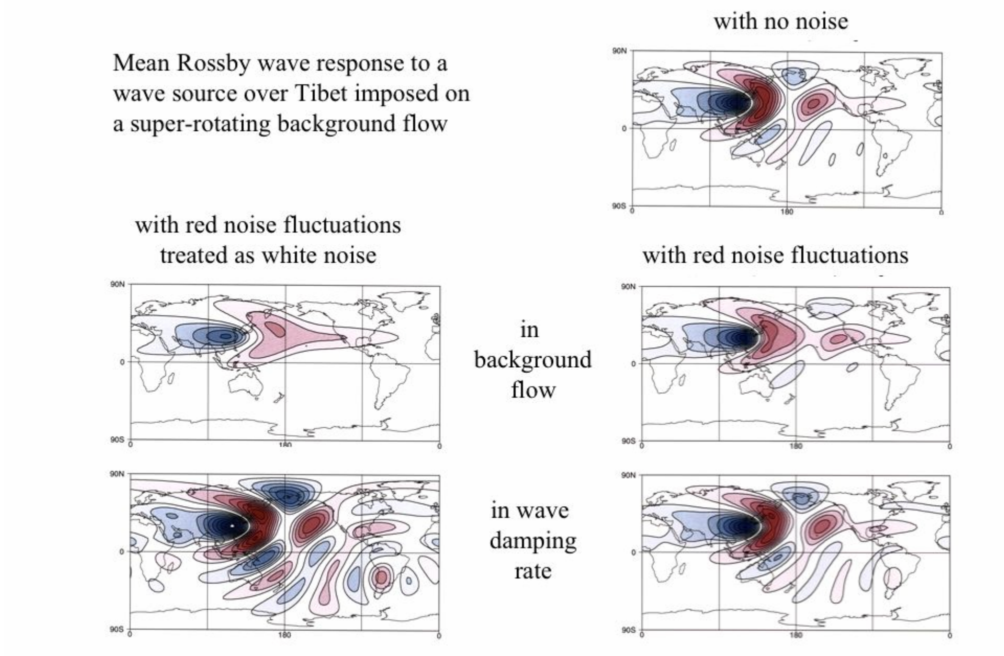

Figure 3

We have also investigated (jointly with H.-P. Huang) the sensitivity of stochastic forcing impacts in a research version of NCEP's T62 medium range forecast (MRF) model. Specifically, two-week ensemble forecast experiments were conducted without and with additional stochastic terms in the model. Our original motivation was simply to confirm some of Buizza et al's (1999) findings in a different forecasting environment and assess their sensitivity to how the stochastic forcing was specified. As in their study, we multiplied the model's diabatic tendency at each time step by a random number, in our case between 0 and 2, as in

$$\dot{X} = \dot{X}_{adiabatic} + \left(1 + r\right)\dot{X}_{diabatic} \tag{4}$$

with r as red noise with a Normal probability density N (0, 0.4^2) clipped to 0 for magnitudes greater than 1. A control set of 20 eleven-member two-week forecast ensembles was generated with $r = 0$ for 20 cases in the northern winter of 1997/98, another set of 20 eleven-member ensembles (with identical initial conditions to those in the control set) with $r \neq 0$, and a third set of 5 eleven-member ensembles with $r \neq 0$ only in the tropics (± 10º).

We generated r as an evolving red noise process on the globe with prescribed spatial and temporal autocorrelation scales. This was accomplished by advancing a red noise process with the prescribed temporal autocorrelation scale for each spectral coefficient, and then spatially smoothing the field at each time step using the spectral smoother of Sardeshmukh and Hoskins (MWR 1984) with parameters appropriate for the desired spatial autocorrelation scale. We implemented a spatial scale of 300 km and temporal scales of 12 and 24 hours.

Figure 4 shows meridional profiles of the zonally averaged rms difference between the spreads of the stochastically forced and unforced ensembles as a function of forecast lead time for 500 mb streamfunction (top panel) and precipitation (bottom panel). The results for the global and tropical stochastic perturbation experiments are shown in black and red, respectively. Comparing the two, it is evident that the dominant impact of the stochastic forcing is on the tropical precipitation, from where it spreads globally on a Rossby wave dispersion time scale of about a week, so that by week 2 it makes a substantial contribution to the extratropical spread.

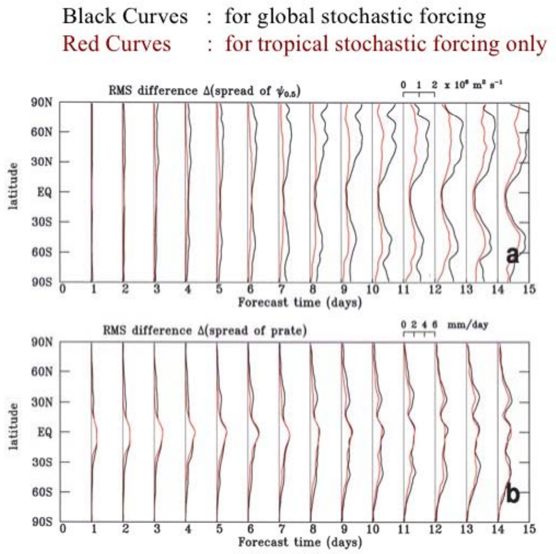

Figure 4

Fig 5 summarizes a remarkable result from these experiments, that although the stochastic forcing leads to an increase of global mean spread, locally it leads to a decrease in many areas, particularly in the medium and extended forecast ranges. Indeed the figure shows that the ratio of the area of decreased spread of global 200 mb streamfunction to that of increased spread increases monotonically as a function of lead time. This result is robust with respect to ensemble size, as shown, Examination of the spread difference maps (not shown) suggests that the stochastic perturbations both amplify and shift the spread patterns, resulting in a decreased spread in some areas even though the global effect is an increase. Some of this effect may be anticipated from the linear error variance evolution Eq. 3b discussed earlier (strictly, with the contribution from 3a subtracted), and the two distinct effects of the multiplicative noise in introducting an additional growth term ACA^T and modifying the error evolution operator through the noise-induced drift term A^2C. In a multivariate system with multiple noises η_j, these two effects are not the same. In particular, as we saw in Fig 3, the noise-induced drift can be stabilizing, which can lead to a variance decrease.

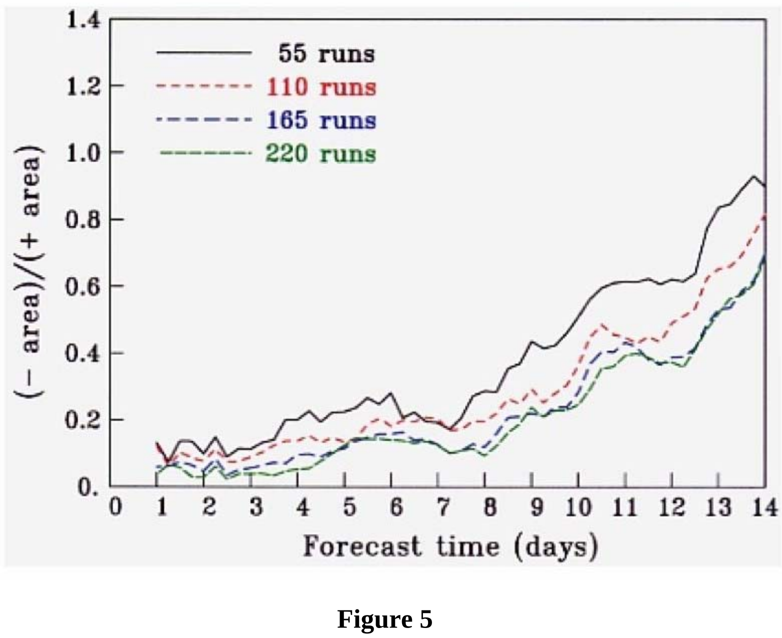

Figure 5

There was also a mean effect in these experiments (Fig 6), a weakening of the mean tropical precipitation and a dynamically consistent weakening of the subtropical jets. similar to that associated with weak La Nina

-type tropical SST forcing. Interestingly, this is also the sense of the mean effect shown in 40-yr ensemble runs made at ECMWF with prescribed SSTs, described by Palmer et al elsewhere in this volume,

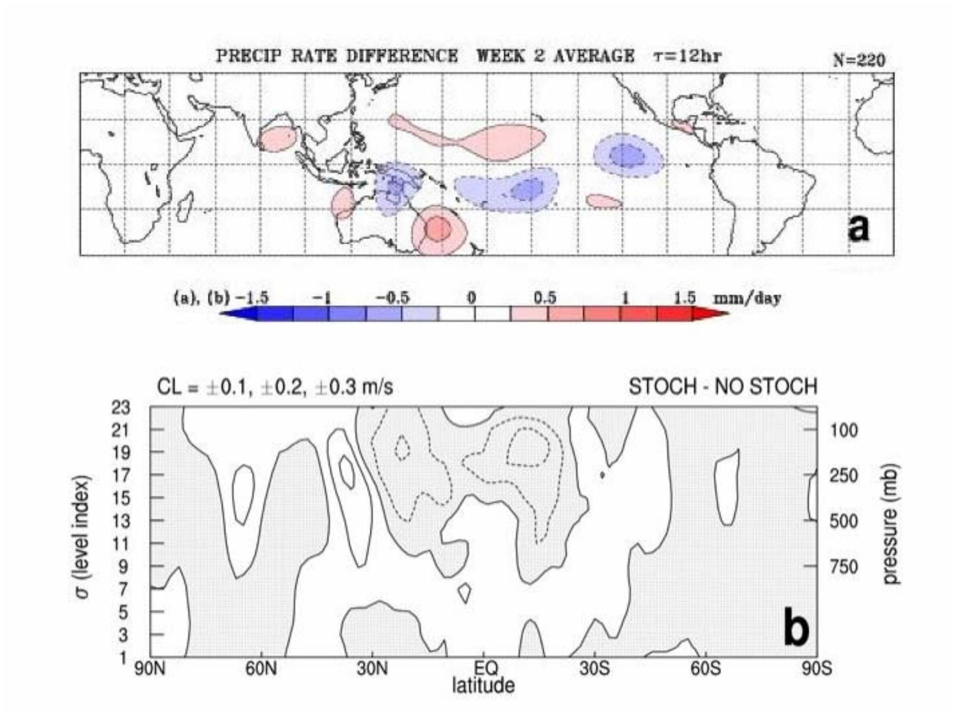

Figure 6

Discussion

The principal task of stochastic parameterization is to estimate and approximate the unparameterized remainder *R* in Eq. (1) as a relatively simple stochastic process. One could of course attempt to develop ever more sophisticated mesoscale and microphysical models of *R*, but this would defeat the purpose of a 'parameterization' without necessarily leading to more accurate probabilistic descriptions of the resolved circulation.

The simplest thing to do is to treat *R* as white in time and space. The next simplest is to treat it as white in time but not space; and the third simplest to treat it as red in time. Linear stochastically forced models of *R* of reasonably small dimension are the logical limit of this approach. Other approaches, such as using cellular automata to generate possible realizations of *R* with desired space-time correlation statistics, are discussed elsewhere in this volume.

To what extent can *R* be approximated as red noise in time and space, as we did in our experiments with the NCEP model? And even if it can, what are the appropriate space and time scales? For tropical noise, a hint is provided by the study of Ricciardulli and Sardeshmukh (2002), who suggested that organized tropical deep convection has a large red noise component with space and time scales of about 130 km and 6 hours, respectively, over the oceans and slightly shorter scales over land (Fig. 7).

Outside the tropics, stochastic approximations of *R* should be consistent with observed wavenumber-frequency spectra (e.g. Nastrom and Gage 1985), a daunting task. For some purposes, linear stochastic models of large-scale turbulence may suffice and allow the stochastic forcing of large scales to be approximated as red noise in space with a K^{-2} spectrum, and also in time invoking the Taylor approximation. A K^{-2} line drawn on Nastrom and Gage's spectrum in Fig 8 is actually not a bad approximation, especially near the much discussed 'kink' between the downscale enstrophy cascading K^{-3} and the upscale energy-cascading $K^{-5/3}$ turbulence regimes.

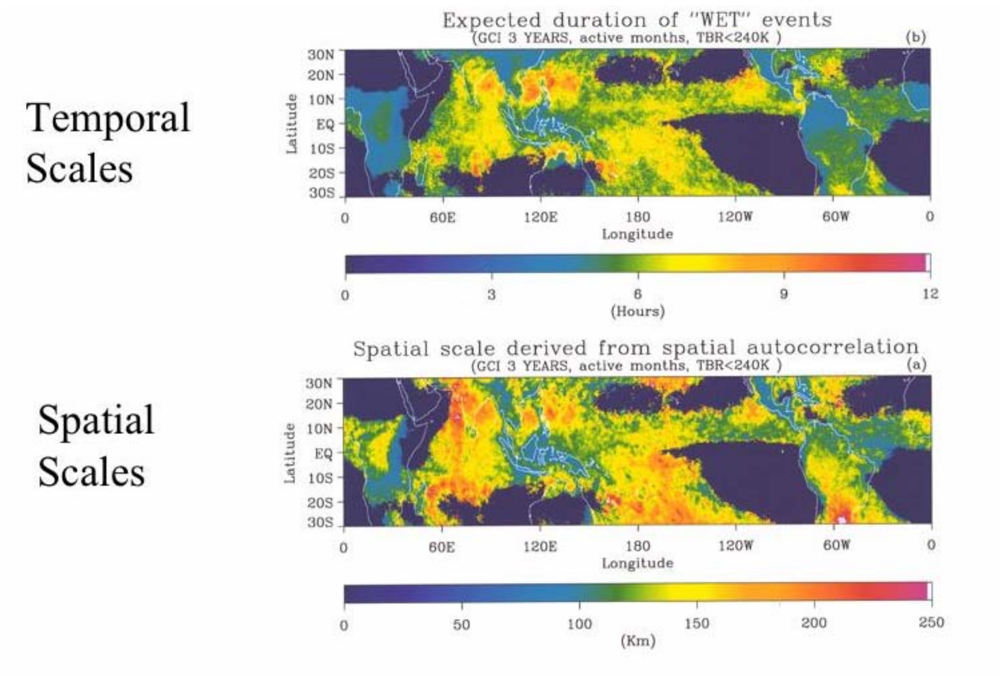

Figure 7

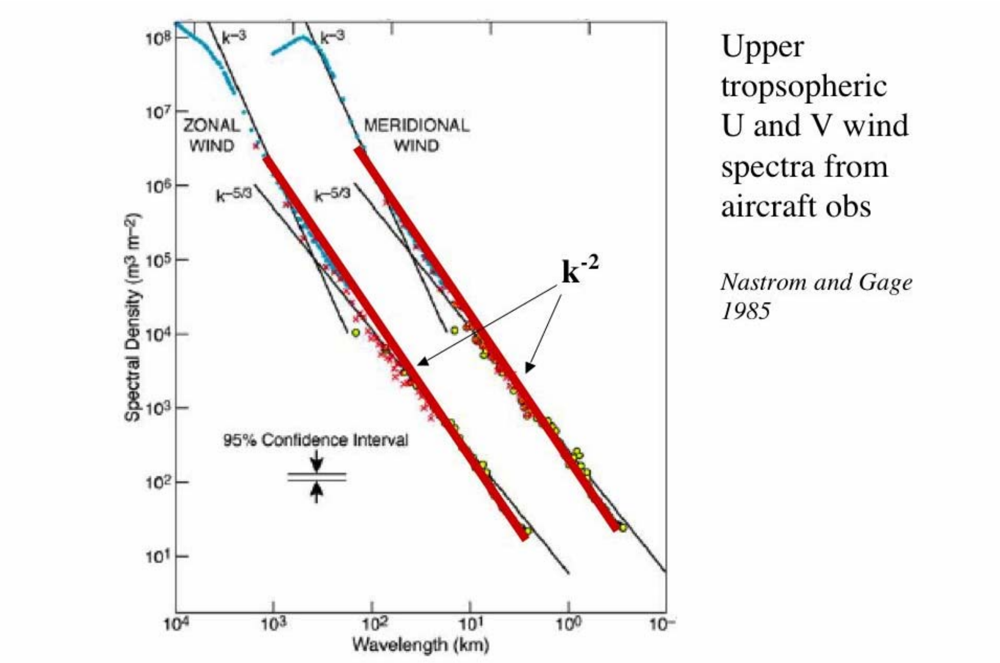

Figure 8

Concluding remarks

The need for some stochastic input in NWP and climate models is clearly physically justified (because *R* is not zero). From a practical viewpoint, however, arriving at an accurate stochastic parameterization *B(X)* in Eq (1) is an enormous challenge. As we have seen from our simple Rossby wave examples, the impact of stochastic perturbations can depend sensitively on the structure of *B(X)*. Our experiments with the NCEP model further highlight the importance of representing *B(X)* accurately in the tropics, for which there is little theoretical guidance at present. Ensuring a proper balance in *B(X)* between the mass and wind stochastic forcing associated with small scale diabatic interactions among convection, boundary layer, and clouds is theoretically intractable given the complexity of those interactions Careful experimentation with cloud resolving models might provide a way forward.

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