Statistical Mechanics and Stochastic Convective Parameterisation

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ABSTRACT

Using elementary considerations from statistical mechanics, it is suggested that convective variability in equilibrium can be characterised as a system of convective updraughts, randomly located in space, and with an exponential distribution of mass flux. The predictions for the magnitude and distribution of convective variability that result from this argument are confirmed to a high degree of quantitative accuracy in simulations in a cloud-resolving model. A stochastic cumulus parameterisation is described that imposes this variability. Initial tests in a single column model conform to expectations.

1. Statistics of convective equilibrium

Cumulus parameterisation schemes are usually based on the assumption of an equilibrium, in which the large scale interacts deterministically with the statistical properties of the convection. For equilibrium to be realised, there must be a scale separation, where the large-scale flow is slowly varying over times and distances large enough to average out the effects of the individual convective clouds. If the resolution of the model using the parameterisation is finer than the intrinsic scales of the convection the behaviour of the convection in a given grid box will be stochastic, but with the known equilibrium statistics. It is clearly of interest to identify the relevant scales: the time scale of adjustment to equilibrium, the length scale required to average out convective variability and the magnitude of the fluctuations about equilibrium that occur in a small region of a given size. In this note, we develop such a statistical characterisation of the convective scale behaviour, and use it to develop a stochastic parameterisation scheme. To do this, we need to consider the underlying processes at the "microscopic" level, that is to say, at the level of the individual clouds.

An essential part of this programme is to test the statistical theory and parameterisation against simulations from a cloud-resolving model (CRM) that has been run to equilibrium with a uniform forcing. The setup is summarised below, and details may be found in the references (Cohen and Craig 2004, 2005).

- Resolution: 2 km x 2 km x 50 levels
- Domain: 128 x 128 km (256 x 256 km for some experiments)
- Boundary conditions: doubly periodic, fixed SST of 300 K
- Forcings: fixed tropospheric cooling of 2, 4, 8, 12, 16 K/day, vertical wind shear of 0, 0.2 m/s/km, 2 m/s/km

2. Timescale of Adjustment to Equilibrium

Following Cohen and Craig (2004), consider the response of a convecting atmosphere to a step function perturbation in cooling rate. As shown in Fig. 1, the convective mass flux adjusts in two stages: an initial rapid adjustment (in the first hour or so), followed by a slow evolution over many days. The rapid shift represents the adjustment to a new equilibrium state, while the slow evolution is associated with changes in tropospheric moisture and can be considered as a drift in the large-scale environment of the convection.

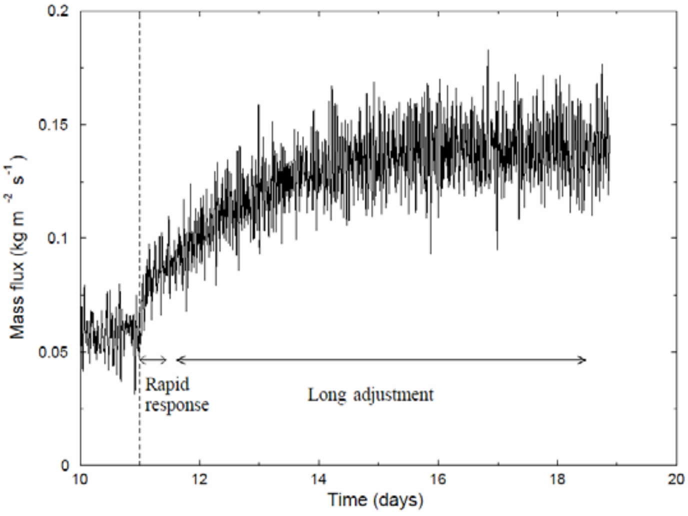

Figure 1: Convective mass flux as a function of time for a simulation in equilibrium with a constant forcing of -8 K/day, changed to -16 K/day at day 11.

An important result is that the time scale is proportional to the spacing of the clouds, being more rapid when the clouds are densely packed. This is shown in Fig. 2 which shows the evolution of the mass flux, when a forcing of -8 K/day is changed to forcings of -4, -12, -16 K/day. It can be seen in Fig. 2(b) that when the mass flux is rescaled by its quasi-equilibrium value (after rapid adjustment) and the time axis by cloud spacing distance, the curves become essentially identical.

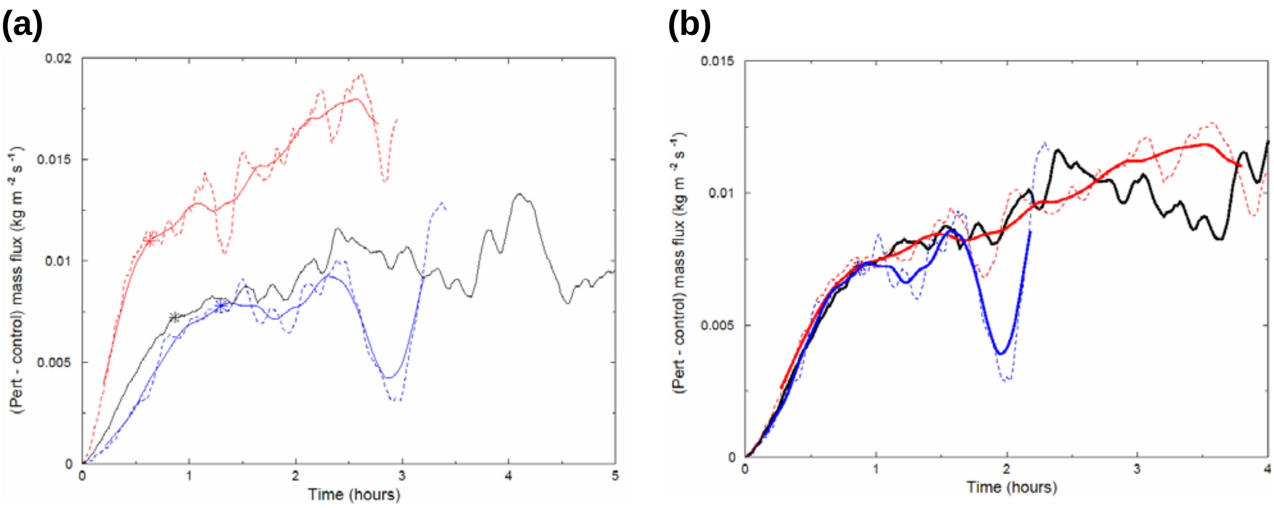

Figure 2: Convective mass flux as a function of time for experiments where the forcing is changed from -8 K/day to -4, -12, or -16 K/day (a) unscaled, and (b) rescaled as described in the text (note that the curve for the experiment where forcing was decreased has been inverted).

That the time scale is proportional to cloud spacing is consistent with the idea that it is determined by the time required for a gravity wave to cross between the clouds, i.e. the time for temperature perturbations to be communicated from convective cores to the rest of the troposphere. To confirm this we would need to consider experiments with different gravity wave speeds, but this is turns out to be difficult to do. It is important to bear in mind that cloud number density depends on the forcing rate, so the adjustment time scale depends on the forcing.

3. Fluctuations in equilibrium

For convection in equilibrium with a given forcing, the mean mass flux should be well-defined. But at a particular time, this mean value would only be measured in an infinite domain. For a region of finite size, we ask what is the magnitude and distribution of the variability, and what scale must one average over to reduce it to a desired level?

Craig and Cohen (2005) describe a theory for convective statistics based on the Gibbs canonical ensemble. The key assumptions are:

1.	Large-scale constraints	mean mass flux within a region: $\langle M \rangle$ mean mass flux per cloud: $\langle m \rangle$
2.	Scale separation	environment sufficiently uniform in time and space to average over a large number of clouds
3.	Weak interactions	clouds feel only mean effects of total cloud field (no organisation)
4.	Equal a priori probabilities	all locations and mass fluxes for a cloud are equally probable

A straight-forward calculation shows that the most probable distribution subject to these constraints has the frequency of clouds with a given mass flux following a Boltzman distribution:

$$d\bar{n}(m) = \frac{\langle N \rangle}{\langle m \rangle} e^{-m/<m>} dm \tag{1}$$

where $<N> = <M>/<m>$ is the mean number of clouds per unit area. The total mass flux within a region, M, has a distribution given by:

$$p(M) = \left(\frac{\langle N \rangle}{\langle m \rangle} \right)^{1/2} e^{-\langle N \rangle} M^{-1/2} e^{-M/\langle m \rangle} I_1 \left(2 \left(\frac{\langle N \rangle}{\langle m \rangle} M \right)^{1/2} \right) \tag{2}$$

which has variance:

$$\frac{\left\langle (\delta M)^2 \right\rangle}{\langle M \rangle^2} = \frac{2}{\langle N \rangle} \tag{2a}$$

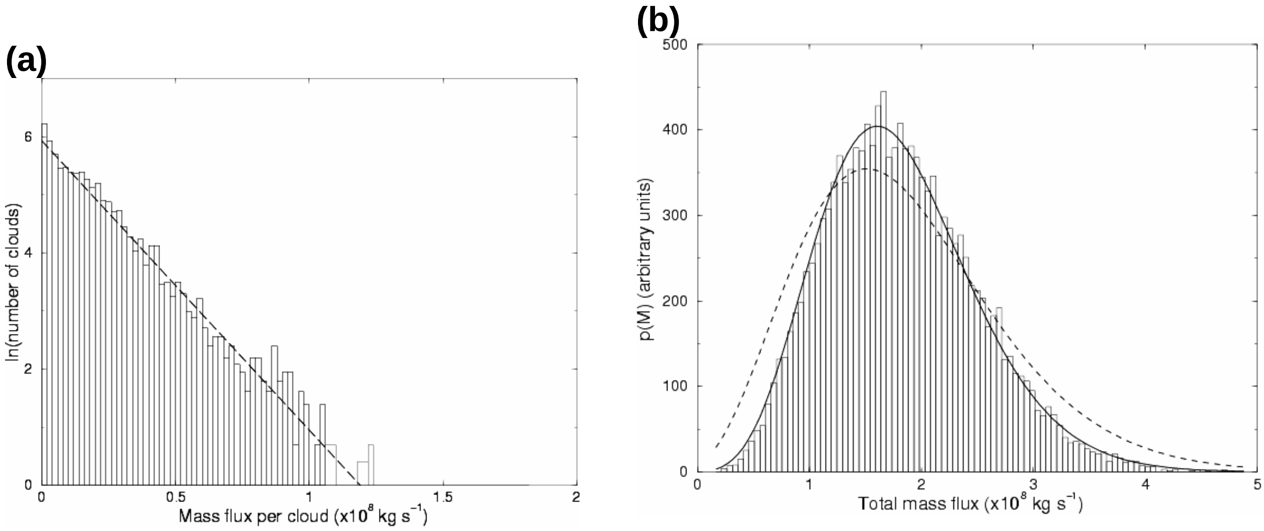

Figure 3: For a simulation with forcing of -2 K/day, (a) histogram of log of mass flux of individual clouds with dashed line indicating a best fit to Eq. (1) ,and (b) histogram of total mass flux in domain with solid line a best fit to Eq. (2), and dashed line the fit obtained with <m> taken from Fig. 3(a) .

The variance is inversely proportional to the cloud number density, as expected for objects randomly distributed in space, but is a factor of two larger because of the variable (exponentially distributed) mass flux of the individual clouds. These distributions are well-reproduced in the CRM simulations, as shown by Fig. 3 (Cohen and Craig 2005).

Fig. 4 shows that Eq. (2a) is valid over a wide range of region sizes, number densities (obtained by changing the forcing), and only weakly influenced by the realistic levels of convective organisation (Fig. 2b) found in the simulations with shear (Fig. 5).

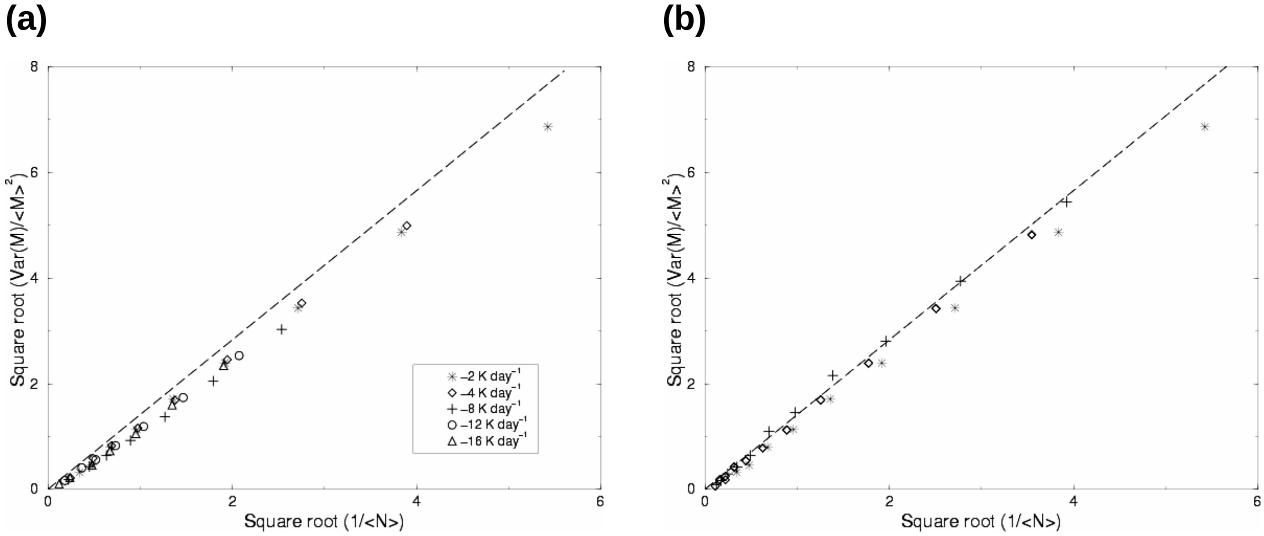

Figure 4: Normalised standard deviation of area-integrated convective mass flux versus characteristic cloud spacing, with line showing prediction of Eq. (2a), for (a) various forcings, and (b) various degrees of convective organisation: unsheared (*), weak shear (o), strong shear (+).

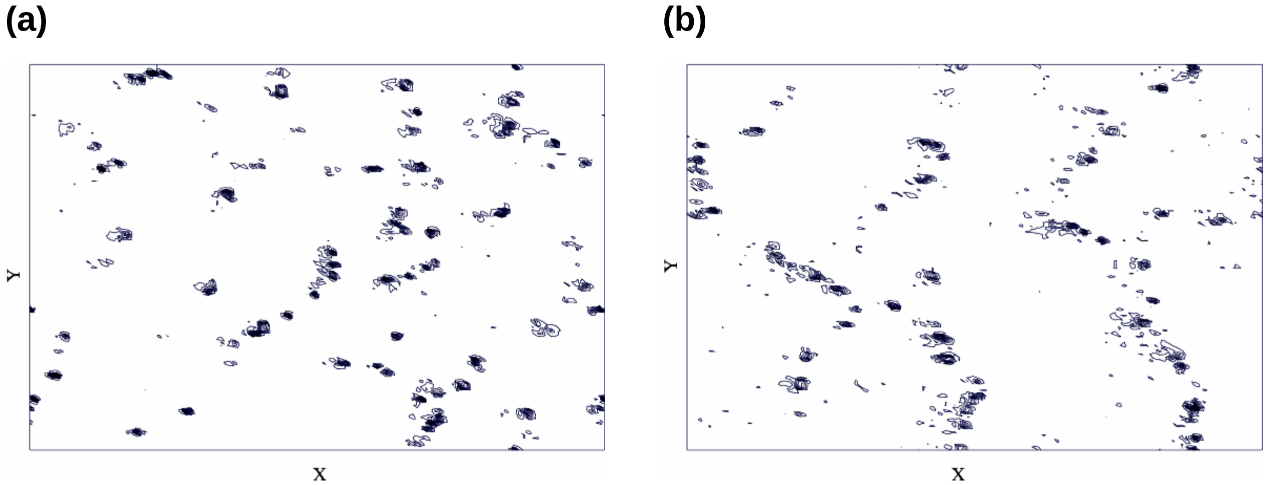

Figure 5: Vertical velocity at a height of 2.8 km for simulations with -8 K/day forcing, (a) weak shear, and (b) strong shear.

4. Outline of a Stochastic Cumulus Parameterization Scheme

As described by Plant and Craig (2005), we are in the process of building a stochastic convection scheme based on the statistical theory described above. The scheme follows the mass flux formalism (based on Kain and Fritsch 1993 and Kain 2004), and has the following main ingredients:

- no trigger function – the presence/absence of convection in equilibrium is due to subgrid variability, represented implicitly by the assumption of a random spatial distribution
- cloud model – an ensemble of plumes with an exponential distribution of cloud base mass flux *m*; each plume acts as representative cloud of given *m*
- CAPE closure - CAPE determined from mean sounding (space-time averaging over the scales defined in section 3 to remove convective variability); total mass flux scaled to remove CAPE over timescale proportional to forcing (see section 2)

The scheme has been implemented in the single column version of the Met Office Unified Model (SCM). The large averaging time interval used in the CAPE closure in the single column tests is to replace the spatial averaging that is not possible in this framework. The results shown here are based on:

- parameterizations for boundary layer transport, stratiform cloud
- forced as in CRM experiment (fixed tropospheric cooling)
- 5 min timestep
- CAPE closure based on sounding averaged over 100 timesteps

A first impression of the behaviour of the parameterisation can be seen in the mass flux time series in Fig. 6. If the column is set to represent a large horizontal area, the fluctuations about the mean value are small (Fig. 6a), while for a smaller area the amplitude increases as expected (Fig. 6b). It is instructive to contrast this behaviour with that of the original Kain-Fritsch scheme (Fig. 6c) where the mass flux tends to oscillate between a value above and a value below the mean, with occasional excursions to very large values or zero. This “deterministic” scheme produces a large amount of random noise, but of the wrong distribution.

In general, single column tests of a parameterisation do not fully test the scheme since interactions between different columns are ignored. This is an even greater limitation for the testing of a stochastic scheme such as the one described here, which is designed to represent spatial variability. There are however three questions that we may hope to answer:

1. Are the mean state temperature and humidity profiles sensible (i.e. not worse than the original Kain-Fritsch scheme)?
2. Are the properties of the individual plumes sensible (*m* constant with height, exponential distribution, as found in the CRM)?
3. Is the desired distribution of *M* obtained for finite-sized grid boxes (can the convective variability be averaged away so that the imposed variability does not feed back on the closure)?

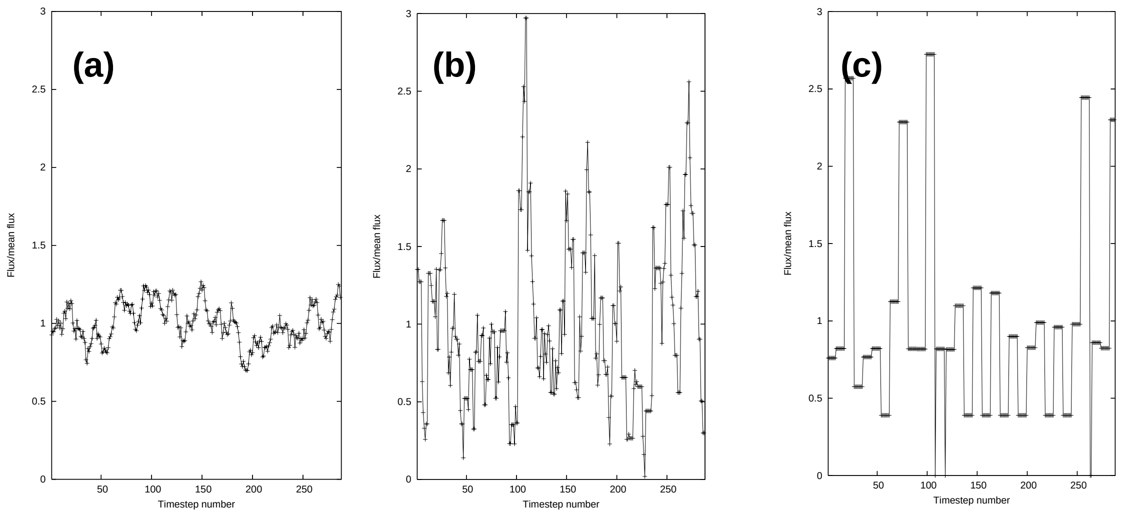

Figure 6: Time series of normalized total convective mass flux for the stochastic parameterisation for a column of (a) area (400 km)^2, and (b) area (64 km)^2, and (c) for the Kain-Fritsch scheme.

Fig. 7 shows the deviations of the mean potential temperature and moisture profiles in the single column runs from profiles obtained in the SCM. The deviations are comparable in magnitude and consistent in structure between the stochastic and Kain-Fritsch schemes, suggesting that they are dominated by the limitations of the entraining plume cloud model that is common to both parameterisations. Although the Kain-Fritsch scheme only has a single cloud type at any given moment, over many time steps it produces a spectrum of cloud comparable to that in the stochastic scheme and the resulting mean profiles are thus similar.

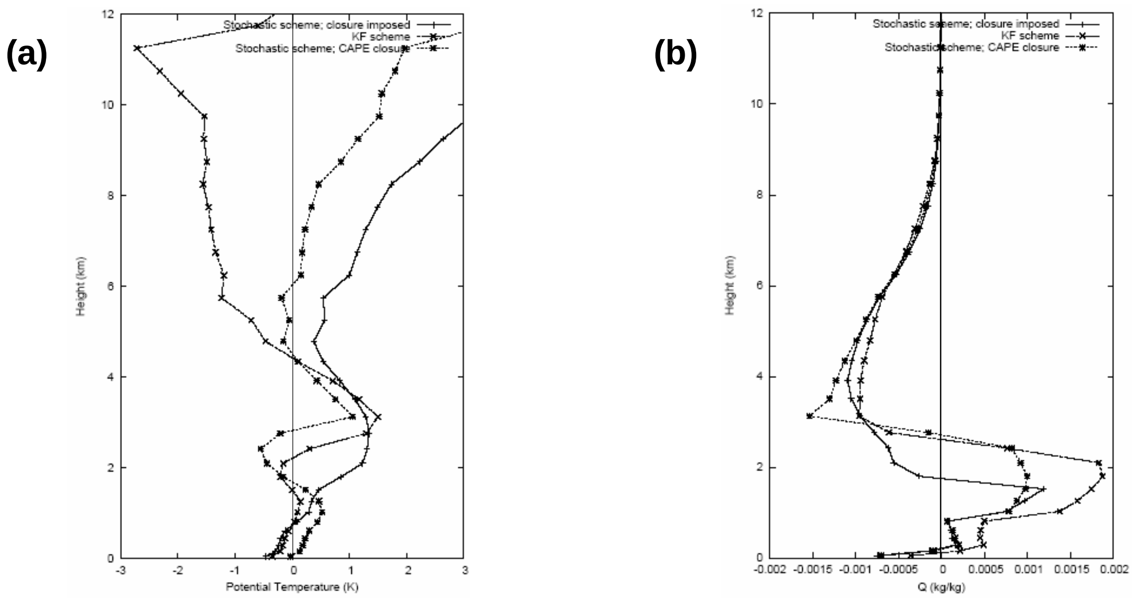

Figure 7: Vertical profiles of deviation between SCM and CRM mean (a) potential temperature, and (b) moisture content.

Although the exponential distribution of cloud mass flux is imposed at cloud base level, there is no guarantee that the entraining plume model in the parameterisation will reproduce this at higher levels, or that the mean mass flux per cloud will remain constant as seen in the CRM. In fact the exponential distribution is maintained through the convecting layer (Fig. 8a shows a typical example for a level in the mid-troposphere), and the mean mass flux per cloud is maintained, but only up to about 8 km. The shift to fewer, larger clouds than seen in the CRM may be due to unrealistic entrainment and detrainment profiles near cloud top, or due to the lack of downdraughts at this level in the parameterisation, making it impossible to reproduce the overturning in convective anvils that occurs in the CRM.

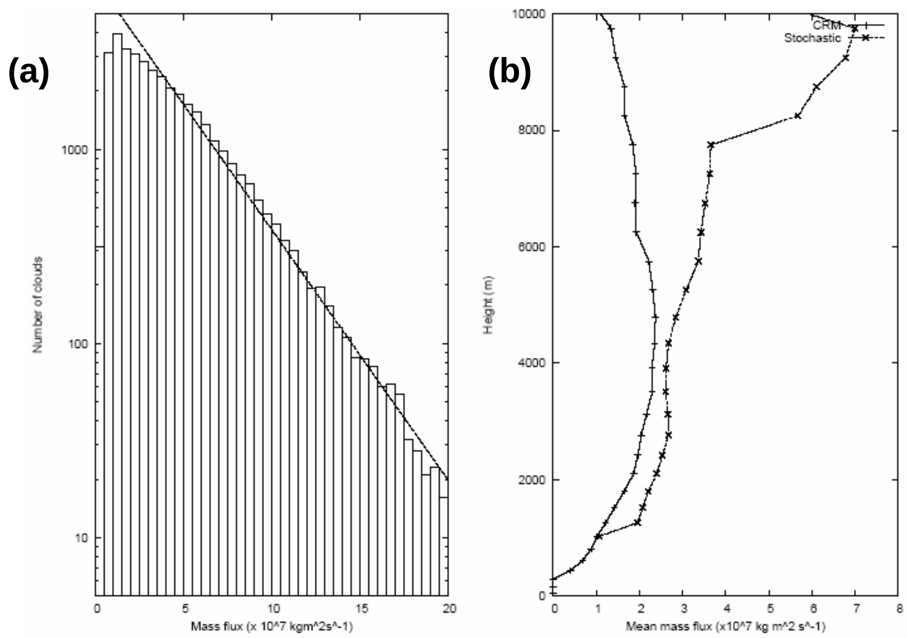

Figure 8: (a) as Fig. 3a, but for SCM at a height of 5.75 km; (b) mean cloud mass flux as function of height for CRM and SCM.

Fig. 9 shows histograms of convective mass flux from the SCM tests for columns of various areas. The agreement with the expected distributions from Eq. (2) is good.

5. Outlook

The next step in development of the scheme will be implementation in the full Unified Model. This will require the input fields to the parameterisation to be averaged in space, and is thus non-trivial, both from an implementation standpoint, and in terms of the convective dynamics since there will additional possibilities for the imposed stochastic variability to feed back on the resolved dynamics. The radiative-convective equilibrium experiments will then be repeated with different horizontal resolutions. The expectation is that the convective behaviour will be resolution independent, in the sense that the results of any given simulation will be statistically identical to that obtained by averaging the results of any higher resolution simulation to the coarser grid.

A second phase of testing is planned where the scheme will be implemented in a regional weather prediction model (the Lokal Modell of the DWD), which is used in the COSMO-LEPS regional ensemble forecasting system. It is difficult to anticipate how a scheme based on statistics of equilibrium convection under uniform forcing will perform for realistically complex weather situations, but correct behaviour in one circumstance should not be a disadvantage in comparison to existing, more *ad hoc* schemes.

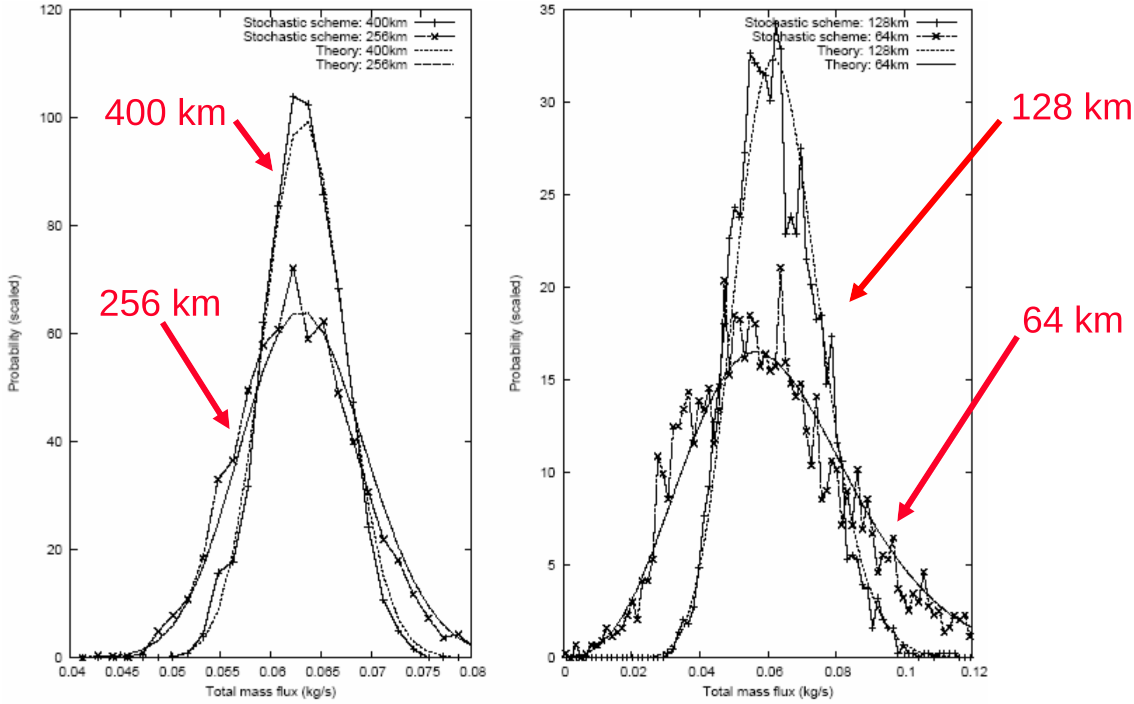

Figure 9: Histogram of total convective mass flux for SCM for columns of different area, with theoretical curves from Eq. (2) for reference.

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