Two extra components in the Brier Score Decomposition

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$$BS = \frac{1}{n}\sum_{k=1}^{m}\sum_{j=1}^{n_k}(f_{kj} - o_{kj})^2$$

$$= \bar{o}(1-\bar{o}) + \frac{1}{n}\sum_{k=1}^{m} n_k (f_k - \bar{o}_k)^2 - \frac{1}{n}\sum_{k=1}^{m} n_k (\bar{o}_k - \bar{o})^2$$

= Uncertainty + Reliability - Resolution

Brier score components

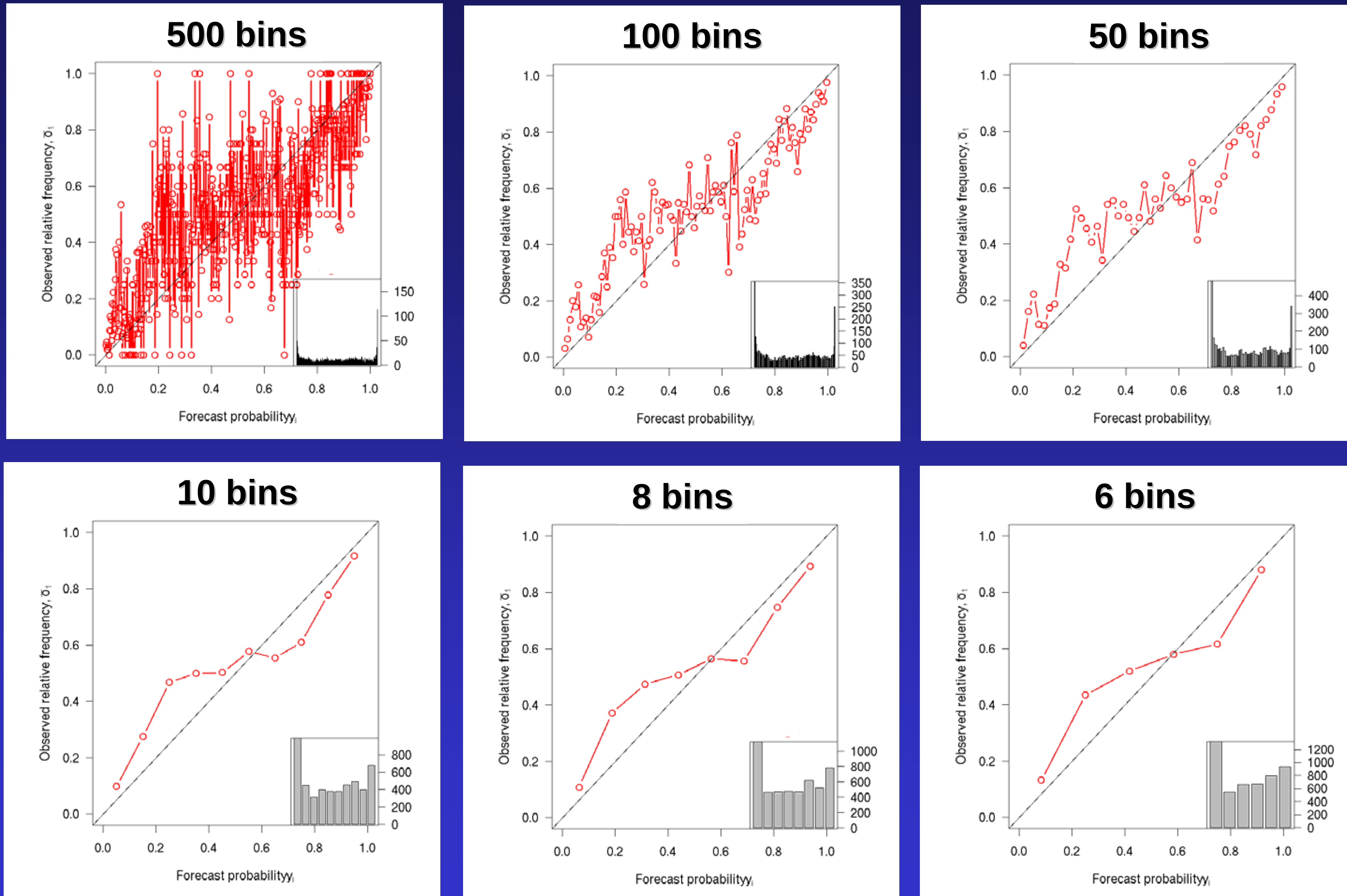
To calculate the components (e.g. E(o|f)):

Stratify on ALL issued probability values {f}

OR

Stratify into m distinct probability bins:

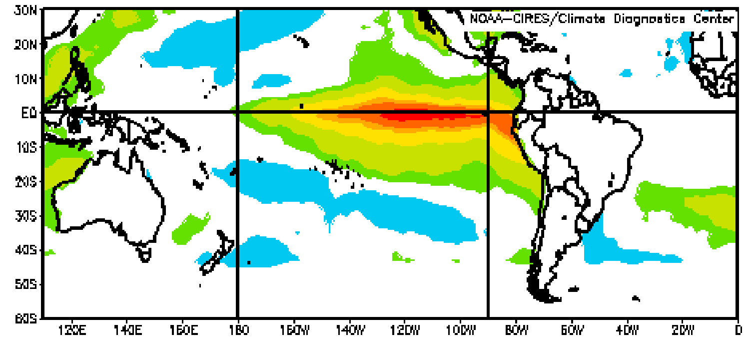
- More reliable estimates (smoothing);
- Can avoid sparseness issues;
- Comparison of different forecasting systems.



→ Forecast system is over-confident

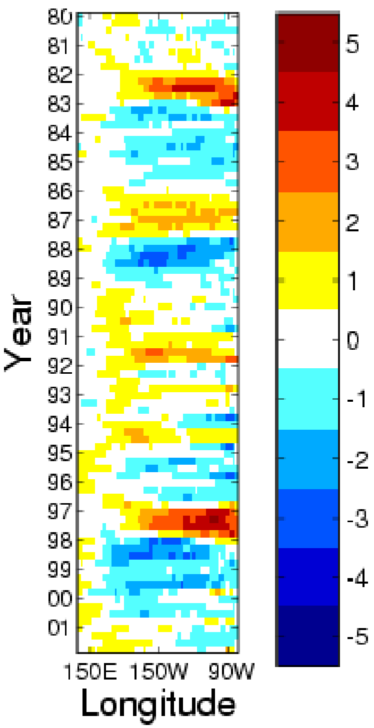
Example:Equatorial Pacific SST

88 seasonal probability forecasts of binary SST anomalies at 56 grid points along the equatorial Pacific. Total of 4928 forecasts.

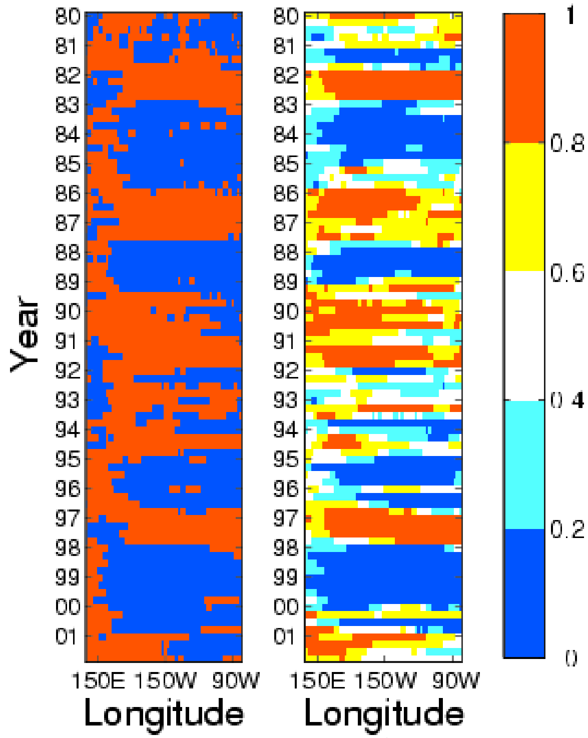


SST $\quad o = (SST > 0) \quad f = \Pr(\hat{o})$

OBS



OBS ENS

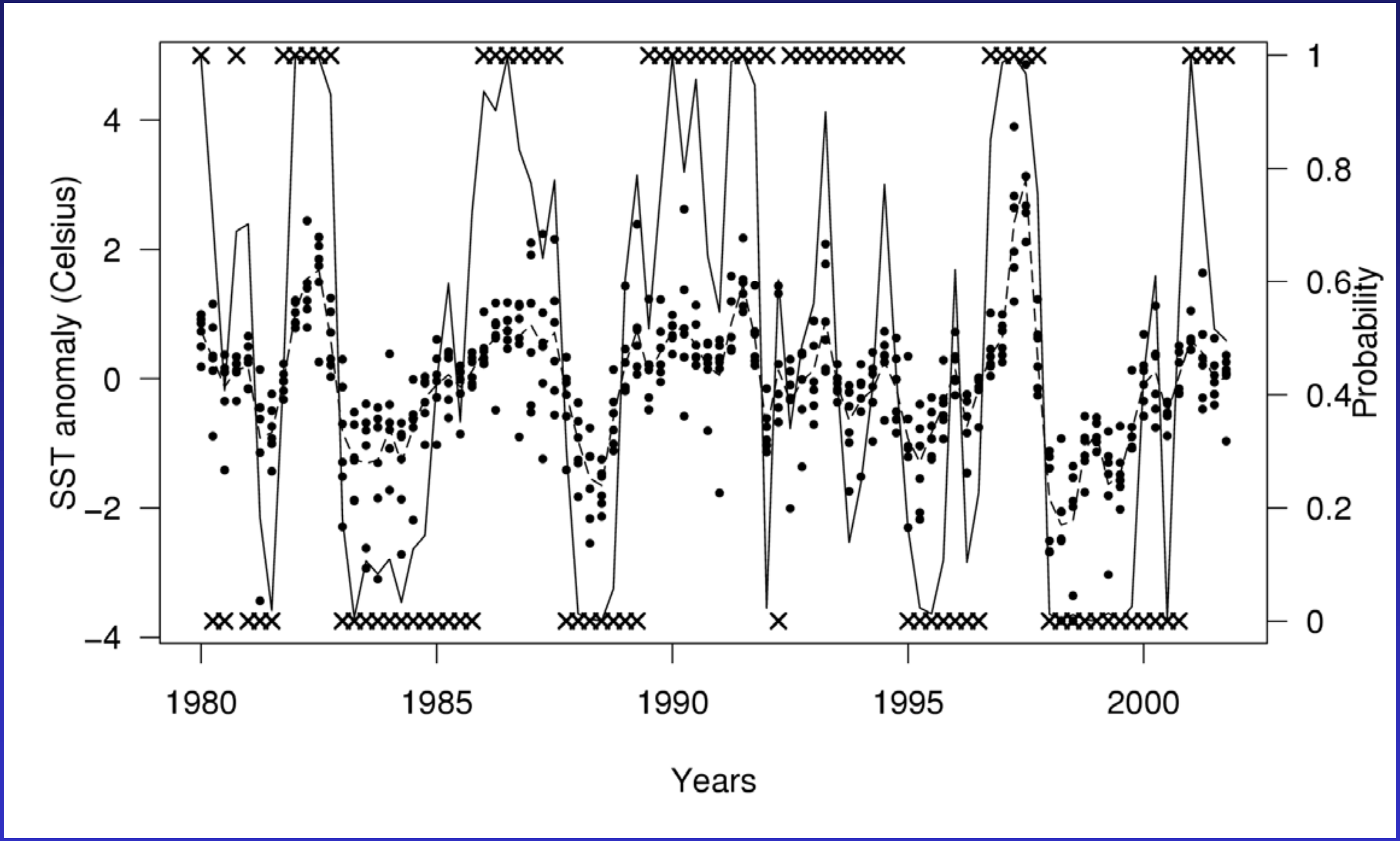


The probability forecasts were constructed by fitting Normal distributions to the ensemble mean forecasts from the 7 DEMETER coupled models, and then calculating the area under the normal density for SST anomalies greater than zero.

SST anomalies (°C)

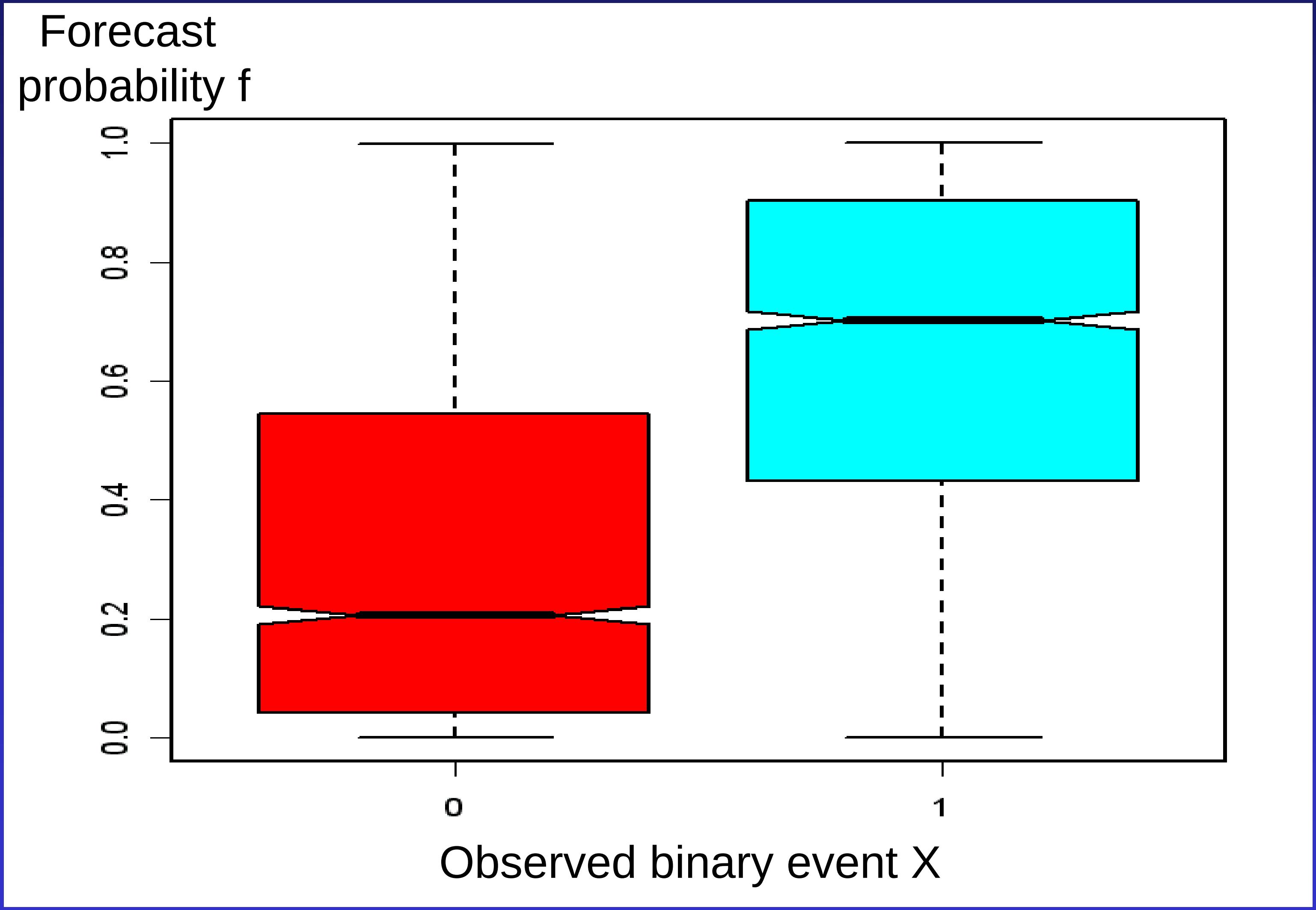
Forecast probabilities: f

Forecasts and observations at 150W



X = observed binary event: =1 for above average SST
Dots = ensemble mean forecasts of SST
Solid line = probability forecast estimated from ensemble means

Prob. forecasts stratified on observations



→ Forecast system has discrimination

Brier score for probabilties in m bins

$$BS = \frac{1}{n}\sum_{k=1}^{m}\sum_{j=1}^{n_k}(f_{kj} - o_{kj})^2$$

$$= \bar{o}(1-\bar{o}) + \frac{1}{n}\sum_{k=1}^{m} n_k (f_k - \bar{o}_k)^2 - \frac{1}{n}\sum_{k=1}^{m} n_k (\bar{o}_k - \bar{o})^2$$

$$+ \frac{1}{n}\sum_{k=1}^{m}\sum_{j=1}^{n_k}(f_{kj} - \bar{f}_k)^2 - \frac{2}{n}\sum_{k=1}^{m}\sum_{j=1}^{n_k}(o_{kj} - \bar{o}_k)(f_{kj} - \bar{f}_k)$$

← NEW!!

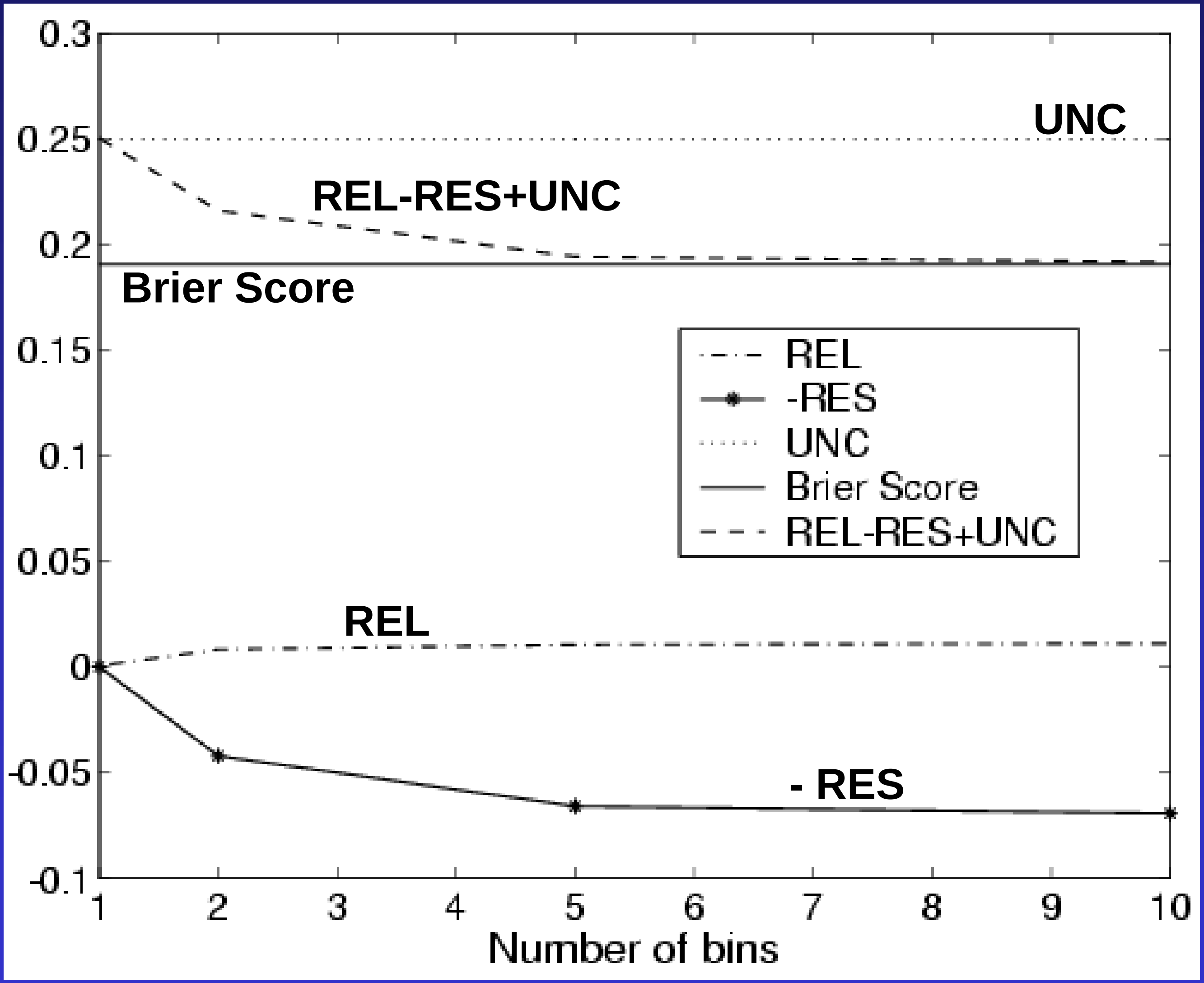
= Uncertainty + Reliability - Resolution

+ Within-Bin Variance - Within-Bin Covariance

For mathematical derivation please refer to:

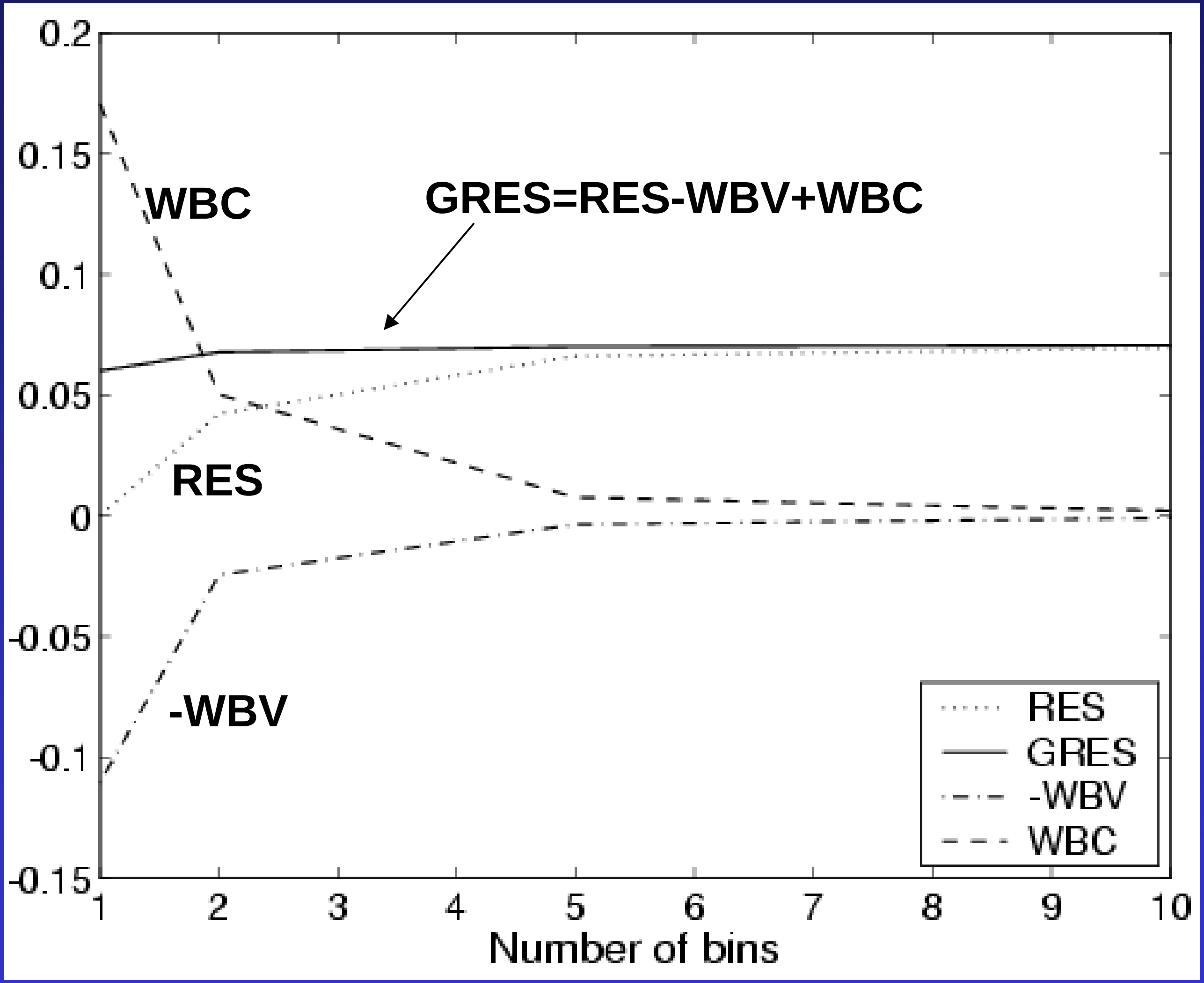
Stephenson, D.B., Coelho, C.A.S., and Jolliffe, I.T., 2007: Two extra components in the Brier Score decomposition, Weather and Forecasting (submitted).

Brier score components vs. num. of bins



→ Brier score is less than REL-RES+UNC!

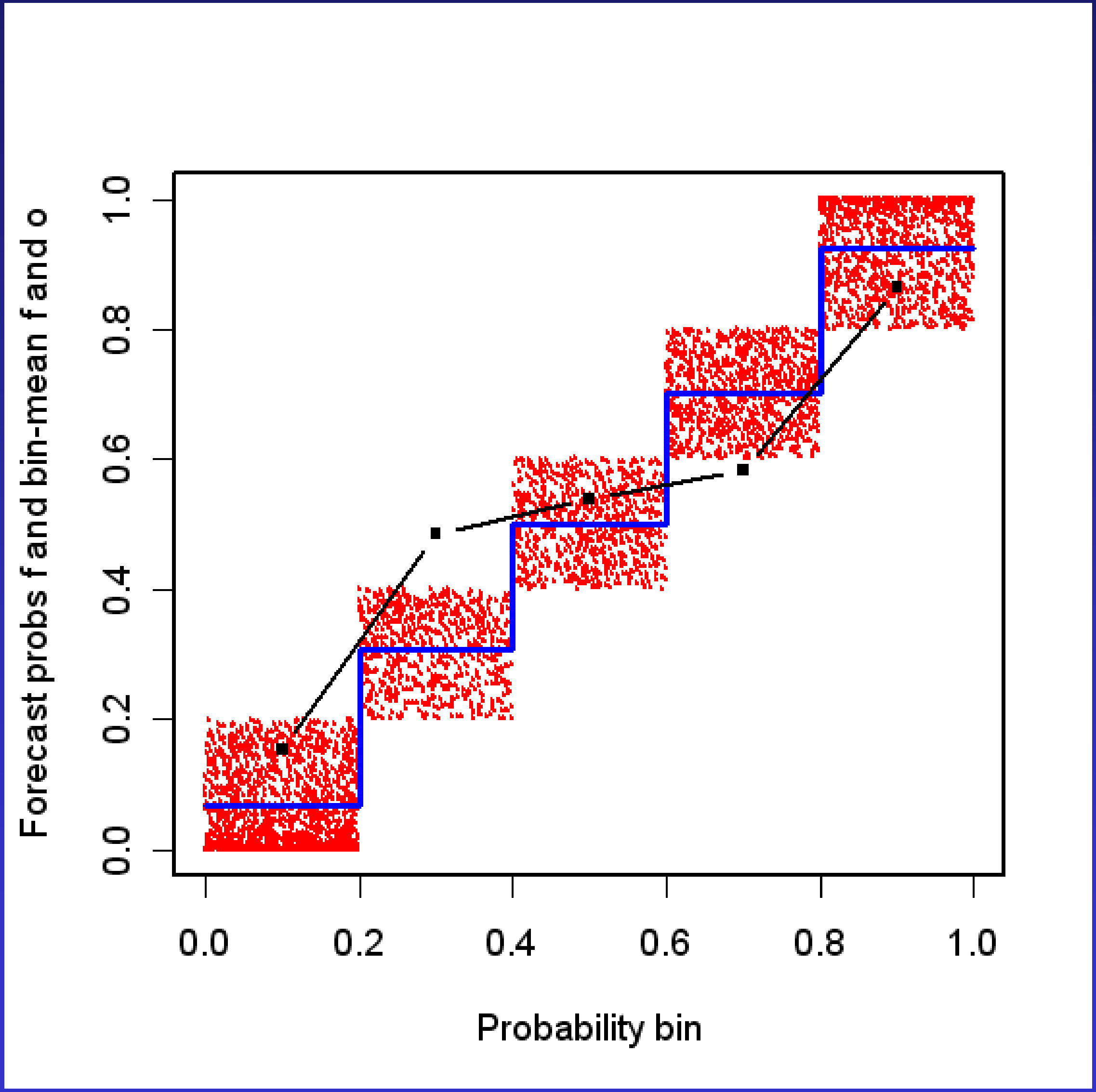
Within-bin terms and Generalised RESolution



→ GRES=RES-WBV+WBC is more constant than RES

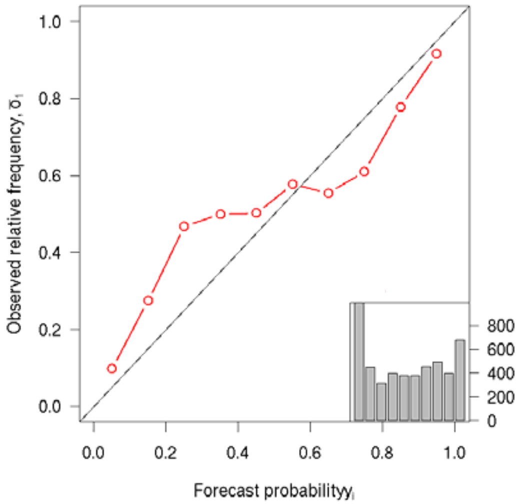
The End

Within-bin Variance of Probabilities

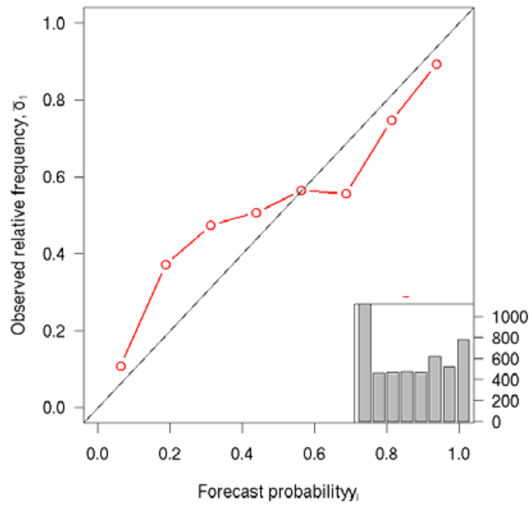


Red dots = probabilities f
Blue line = bin-average f
Black line = bin-average o
(reliability diagram)

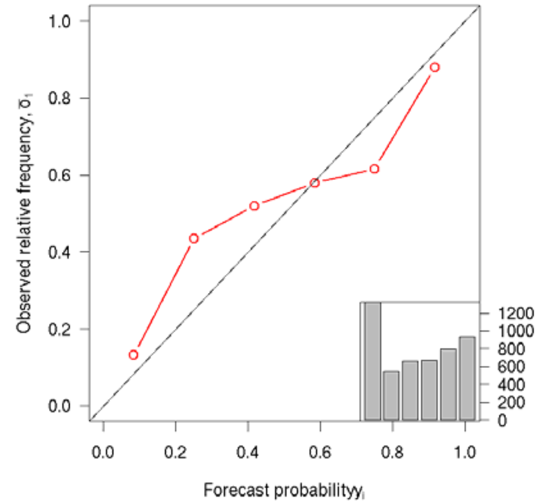
10 bins



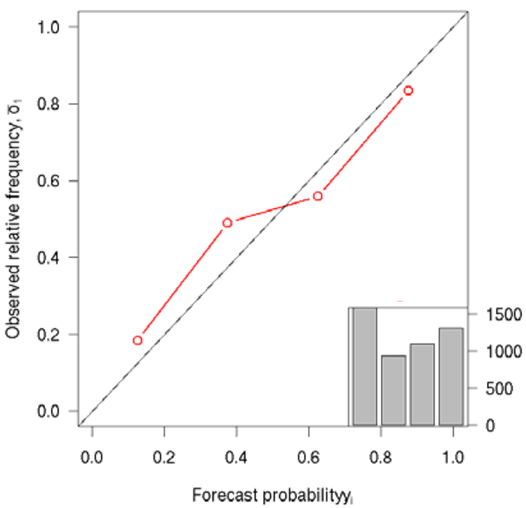
8 bins



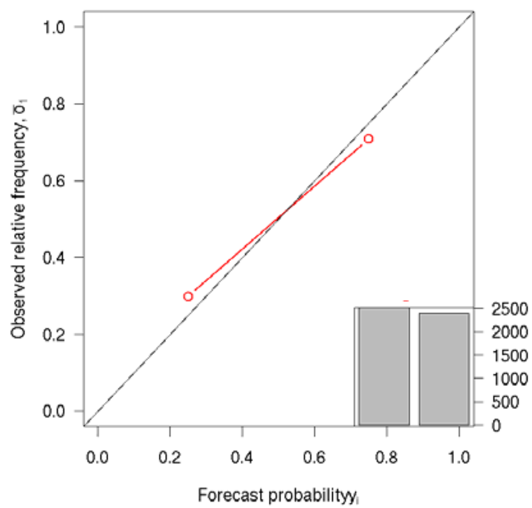
6 bins



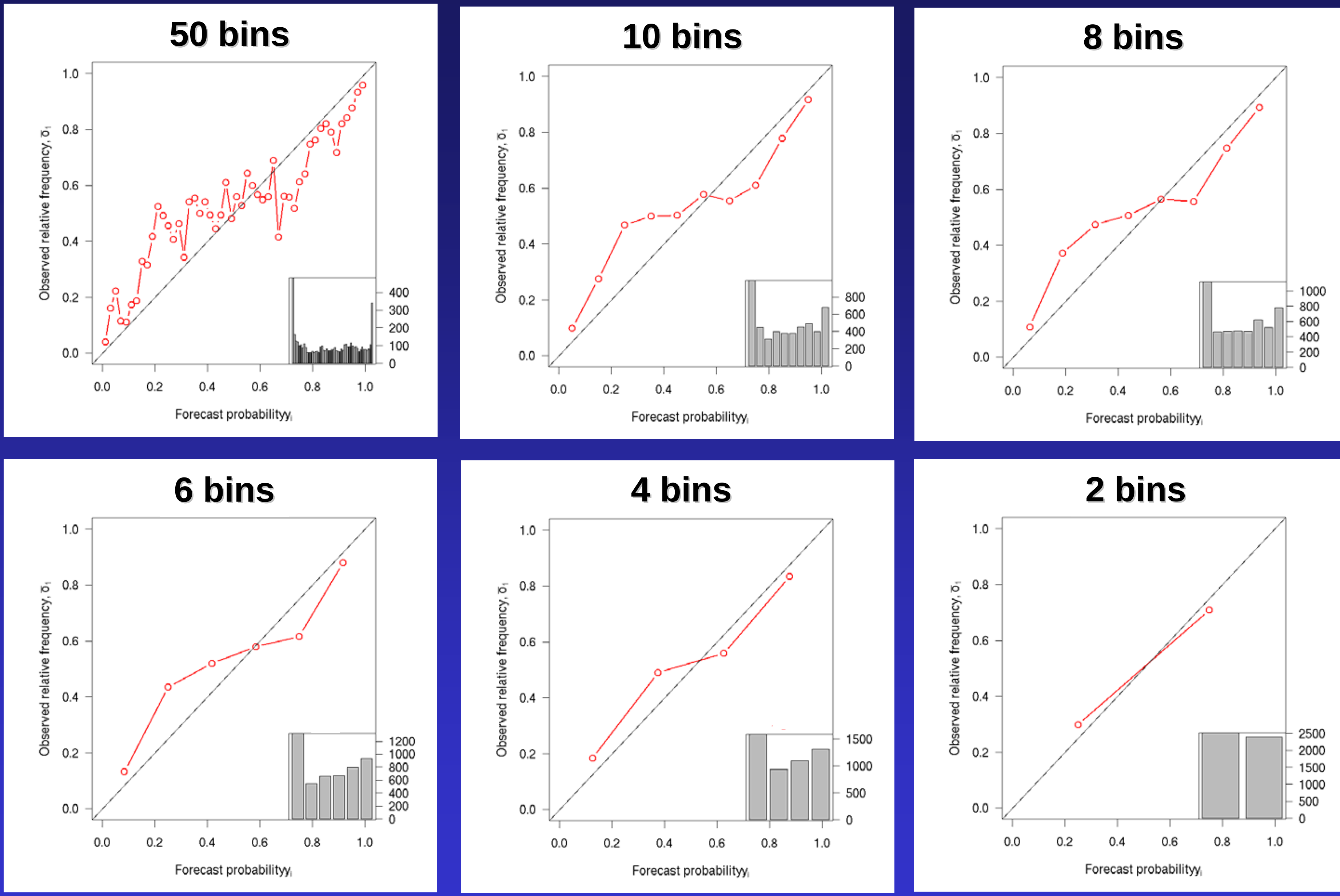
4 bins



2 bins



→ Forecast system is over-confident



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