The Sensitivity of Analysis Errors to the Specification of Background Error Covariances

M Fisher

European Centre for Medium-range Weather Forecasts
Shinfield Park, Reading, UK
m.fisher@ecmwf.int

1. Introduction

Background error covariances in the ECMWF analysis system are specified from two sources. The correlation structure is determined from the statistics of an ensemble of 4dVar assimilations and is static, whereas standard deviations of background error are determined using a cycling algorithm (Fisher, 1995). Recent attempts to change the background error covariances, either by re-calibrating the statistics using a more recent ensemble, or by changing the method used to estimate background error standard deviations, have shown a surprising lack of sensitivity to changes in the covariance specification.

In this paper, we address this lack of sensitivity by considering simple one- and two-dimensional analogues of the assimilation process.

2. Analysis Sensitivity to Changes in the Standard Deviation of Background Error

The ECMWF analysis system has for many years used a cycling algorithm (Fisher, 1995) to determine the standard deviation of background errors that are used to define the background cost function, J_b. The algorithm consists of an initial estimate of analysis error standard deviation, based on the leading eigenvectors of the Hessian of the analysis cost function, followed by a simple inflation to account for the growth of error between one cycle and the next. The error growth model is due to Savijärvi (1995), and expresses the rate of change of forecast error standard deviation as:

$$\frac{d\sigma}{dt} = (a\sigma + b)\left(1 - \frac{\sigma}{\sigma_\infty}\right)$$

Here, a and b are constants, and σ_∞ is specified from climatology. For very small errors, the model produces linear growth at a rate determined by b. For somewhat larger errors, the growth is exponential and determined by a. As σ approaches σ_∞, the growth rate decreases and errors saturate. The rate of error growth given by this model is independent of the flow. This is unrealistic.

A small degree of flow dependence is introduced by the use of the 4dVar Hessian to estimate analysis error. However, the leading eigenvectors of the 4dVar Hessian largely identify areas of high data density. They are not markedly different from those of the 3dVar Hessian.

A more significant source of flow-dependence in background error standard deviation is produced through the use of linearised versions of nonlinear balance equations in the analysis change of variable (Fisher, 2003). The standard deviations of the balanced components of divergence, temperature and surface pressure are not directly specified. They are determined implicitly through the action of the balance operators on the covariance matrix for vorticity. By using balance operators that are linearised about the background state, a significant degree of flow-dependence is achieved (see Isaksen's paper in these proceedings).

Despite these sources of flow-dependence in the current operational method for estimating background error standard deviation, it is believed that significantly improved estimates, that reflect the dynamical characteristics of the underlying flow, could be generated from the spread of an ensemble of analyses. An initial investigation of this possibility was conducted by Kucukkaraca and Fisher (2006), who investigated two case studies. More recently, longer assimilation experiments have been run.

In the experiments presented here, each member of the analysis ensemble consisted of an independent run of the analysis-forecast system for a period of around 30 days. Ten members were used. For each member, random perturbations with the statistical characteristics of observation error were added to the observations. Spatially-correlated perturbations were used in the case of atmospheric motion vectors, using Bormann *et al.*'s (2003) estimate of spatial correlation. Perturbations for all other observations were spatially uncorrelated. In addition, sea-surface temperatures were perturbed to account for uncertainty in the prescribed field, and forecasts were perturbed (to account for model error) using a version of the stochastic backscatter scheme of Schutts (2004). Despite the attempt to add perturbations that account for the main sources of error, the analysis ensemble seriously underestimates the standard deviation of background error. For this reason, the spread of the ensemble must be inflated in order to produce a realistic estimate of the standard deviation of background error. A simple linear scaling by a factor of two has been used for the experiments discussed here.

Figure 1 shows two estimates of the standard deviation of background error for zonal wind. The panel on the left shows the estimate produced using the cycling algorithm, as used in the current ECMWF operational analysis system. On the right is an estimate corresponding to twice the spread of an ensemble of analyses. The corresponding plots for the standard deviation of temperature are shown in Figure 2.

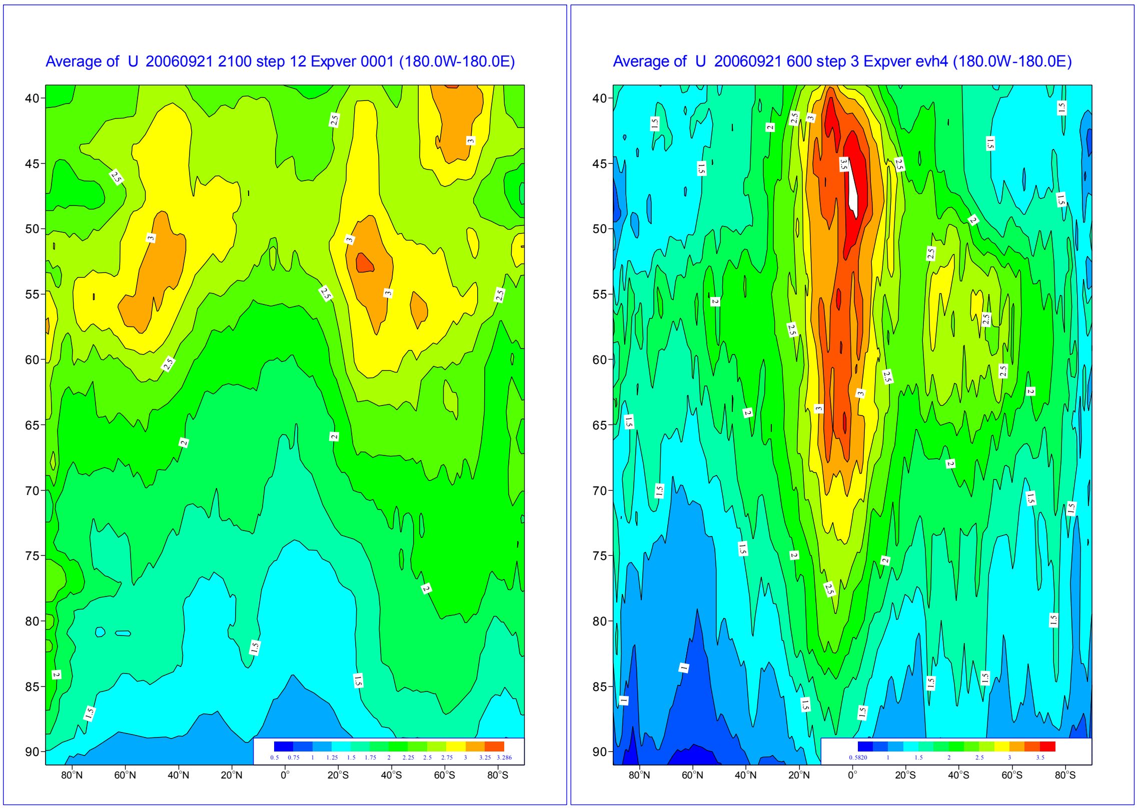

Figure 1: Zonal mean standard deviation of background error for zonal wind estimated using a cycling algorithm (left panel) and as twice the spread of an ensemble of analyses (right panel). The vertical axis corresponds to model level in the 91-level configuration of the ECMWF model. Pressure levels 50hPa, 100hPa, 200hPa, 500hP and 850hPa correspond approximately to model levels 30, 39, 49, 64 and 77.

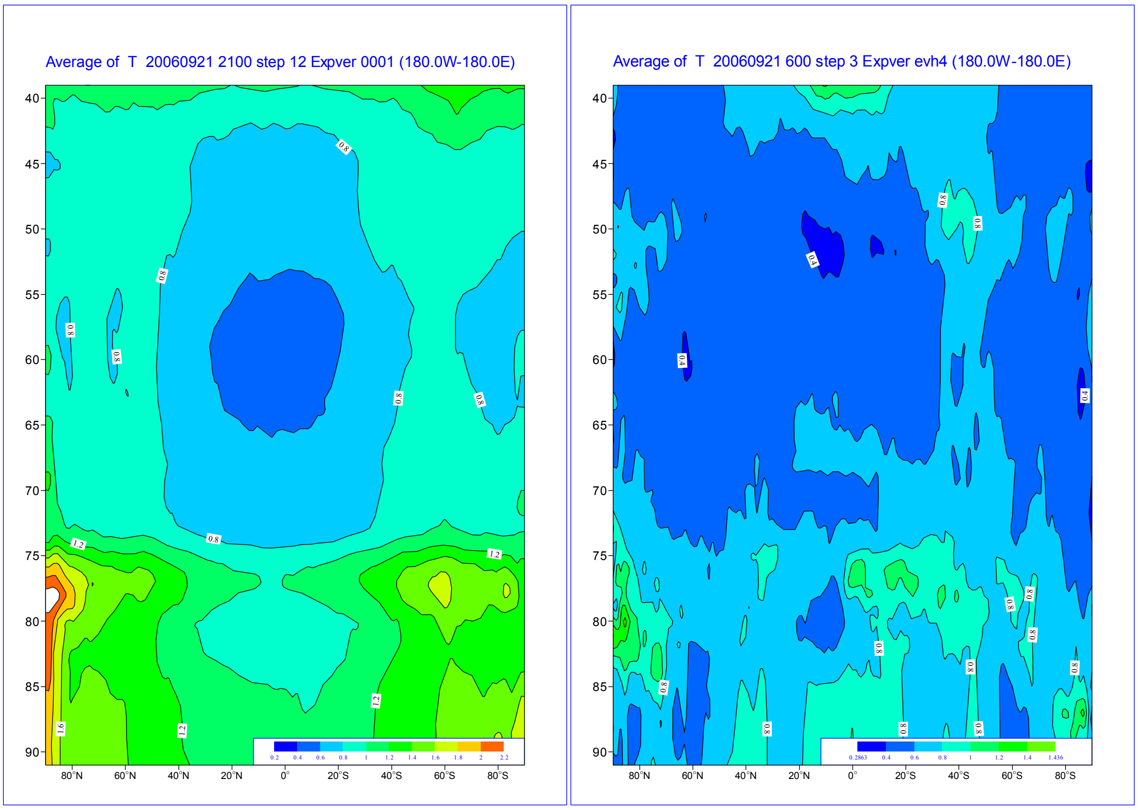

Figure 2: As Figure 1, but for temperature.

Clearly, the two methods for estimating the standard deviation of background error produce very different results. However, when analyses were run using the two estimates there was no discernable impact on the quality of the analyses, as measured by forecast scores. Figure 3 shows scores for forecasts run from the two sets of analyses, verified using the ECMWF operational analysis and averaged over 33 cases. There is a similarly small impact for the southern hemisphere (not shown), and a slightly larger (but still small) impact on forecast skill in the tropics.

A second set of experiments also showed a surprising lack of sensitivity to the specification of background error standard deviations. In these experiments, the standard deviations of background error for all variables and at all model levels were multiplied by a scaling factor. As in the case above, the analysis system was cycled for more than a month. Figure 4 shows scores for forecasts run from analyses with scaling factors between 0.4 and 1.4. There is little difference in forecast skill between any of the forecasts, despite variation in background error standard deviations by a factor of 3.5. Figure 5 shows the fit to observations for analyses with two different scaling factor. As expected, analyses with larger standard deviations of background error draw more closely to the observations. However, there is little difference in the fit of the background fields to observations, despite the fact that they are produced from short forecasts initialized from analyses with very different fits to observations.

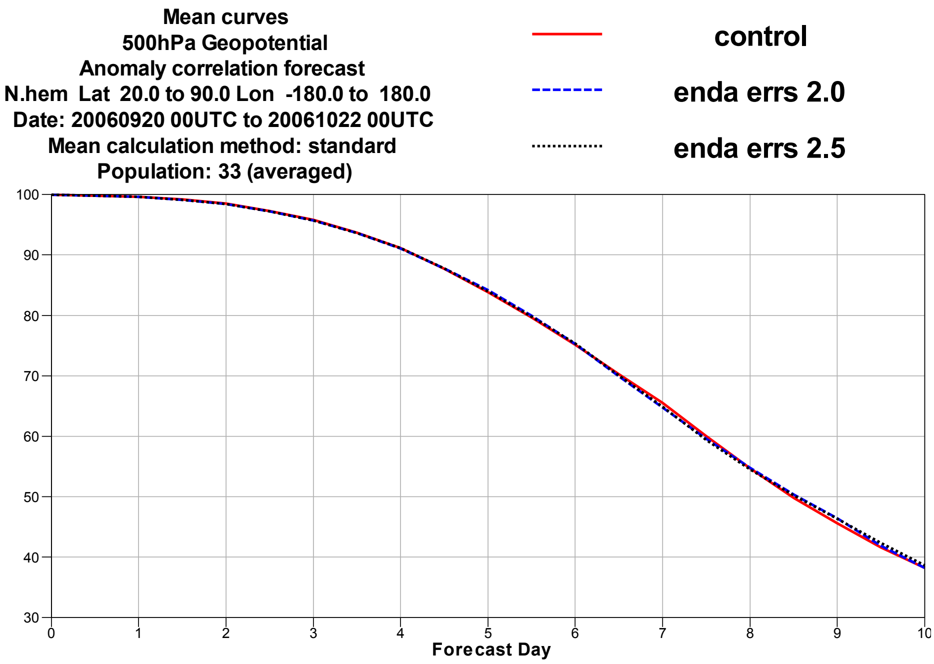

Figure 3: Forecast scores (anomaly correlation for 500hPa geopotential over the northern hemisphere) for forecasts run from analyses that used background error standard deviations estimated using a cycling algorithm (red curve), or estimated as twice (dashed blue curve) or 2.5 times (dotted black curve) the spread of an ensemble of analyses.

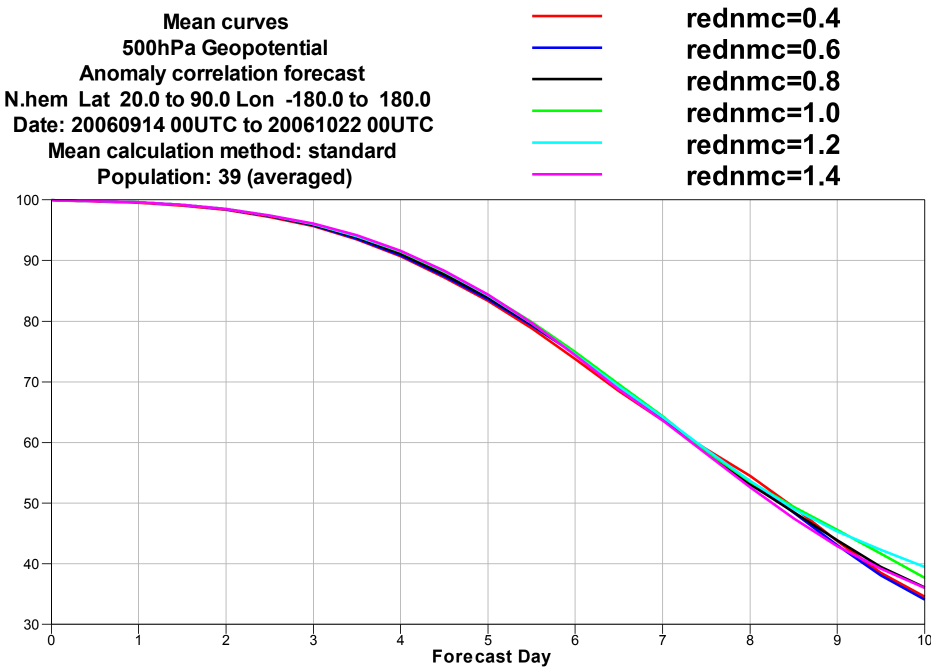

Figure 4: Forecast scores (anomaly correlation for500hPa geopotentia over the northern hemisphere) for forecasts launched from analyses in which the specified standard deviations of background error for all variables were multiplied by a factor "rednmc" ranging from 0.4 to 1.4.

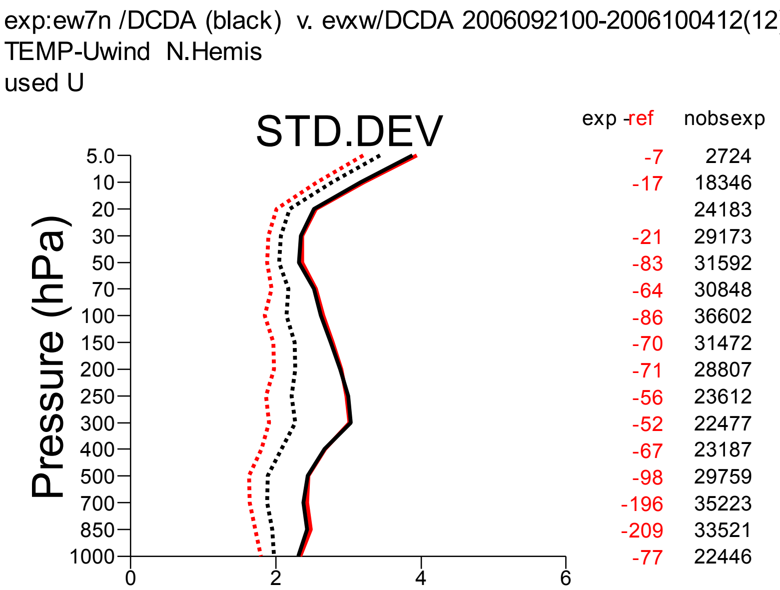

Figure 5: Root-mean-square fit to radiosonde observations of zonal wind in the northern hemisphere for two sets of analyses. Solid curves show the fit of the background to the observations. Dotted curves show the fit of the analyses. Red curves correspond to analyses for which background error standard deviations were scaled by a factor 1.2. Black curves correspond to analyses for which background error standard deviations were scaled by a factor 0.6. The right hand column of numbers show the total numbers of observations in the sample that generated the black curves. The column of red numbers shows the difference between the numbers of observations used to generate the red and black curves.

3. Sensitivity to Misspecified Background Error in a Simple Analogue

To understand the results presented above, consider a simple zero-dimensional analogue. That is, we consider the estimation of a single scalar quantity, x, given an observation, y, and an a priori estimate x_b.

The optimal analysis may be written as $x_a = \left(x_b\sigma_b^{-2} + y\sigma_o^{-2}\right)\Big/\left(\sigma_b^{-2} + \sigma_o^{-2}\right)$, where σ_b and σ_o are respectively the standard deviations of background and observation error. The standard deviation of analysis error in this case is $\sigma_a^* = \left(\sigma_b^{-2} + \sigma_o^{-2}\right)^{-1/2}$.

Let us consider the case in which the standard deviation of background error is misspecified as $\tilde{\sigma}_b$. It is easy to show (see e.g. Daley 1991, section 4.9) that the standard deviation of analysis error is given by:

$$\sigma_a = \sqrt{\frac{\tilde{\sigma}_b^4\sigma_o^2 + \sigma_o^4\sigma_b^2}{\sigma_o^2 + \tilde{\sigma}_b^2}}$$

Figure 6 shows the ratio of the standard deviation of analysis error to its optimal value, as a function of $\tilde{\sigma}_b/\sigma_b$ and σ_o/σ_b. The largest degradations in the quality of the analysis occur when accurate information is not drawn to. The top left corner of the figure corresponds to the case of accurate observations and too-small background error standard deviation. The bottom left of the figure corresponds to the case of an accurate background and too-large background error standard deviation. Note, in particular, that if the background is more accurate than the observations (i.e. $\sigma_o/\sigma_b>1$), it is better to underestimate rather than overestimate σ_b.

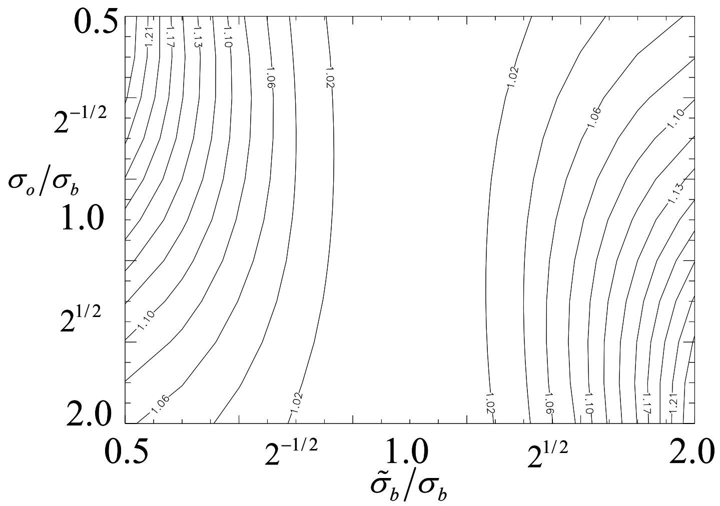

Figure 6: Ratio of actual to optimal standard deviation of analysis error for a zero-dimensional analysis in which the background error standard deviation is misspecified. The horizontal axis represents the specified background error standard deviation, divided by its true value. The vertical axis represents the observation error standard deviation divided by the true standard deviation of background error.

For a range of values of $\tilde{\sigma}_b/\sigma_b$ between about $1/\sqrt{2}$ and $\sqrt{2}$, the analysis is degraded by less than 5%. It is interesting to translate this degradation into a corresponding loss of forecast skill. Let us suppose that forecast error increases exponentially with a doubling time T_D. Consider two forecasts, A and B, started from analyses with standard deviations of analysis error a and b. Suppose that $b<a$. After a time t, forecast A will reach a level of forecast error $\sigma_f = a\,2^{t/T_D}$. Forecast B, which was launched from a more accurate analysis,

will reach the same level of forecast skill somewhat later, at time $t+\Delta t$. That is, $b\,2^{(t+\Delta t)/T_D} = a\,2^{t/T_D}$. Solving for Δt gives: $\Delta t = T_D \ln(a/b)/\ln(2)$. Table 1 lists values of Δt for a range of ratios of analysis error standard deviation, a/b. Comparing Figure 1 and Table 1, we see that, for this simple model, a factor-of-two range of standard deviations around the true value (i.e. $\tilde{\sigma}_b/\sigma_b$ between $1/\sqrt{2}$ and $\sqrt{2}$) results in a loss of forecast skill of only around 2½ hours.

a/b	**Δ*t* (hours)**
1.02	1.03
1.05	2.53
1.10	4.95
1.20	9.47

Table 1: Loss of forecast skill (hours) as a function of degradation in analysis quality, assuming a forecast-error doubling time of 36 hours.

4. The Effect of Misspecified Statistics in a One-Dimensional Analogue

Seaman (1983) considered the effect of misspecified analysis statistics in an idealised two-dimensional analysis system consisting of 42 grid-points arranged in a 7×6 rectangular array with a grid spacing of 200km, and for an observing network corresponding to a subset of the Australian surface synoptic network. Two parameters were varied around their true values, and the corresponding degradation in analysis quality calculated. The parameters were the length scale of background error correlation, for which the true value was taken to be L=500km, and the ratio of observation to background error standard deviation, for which the true value was assumed to be $\sigma_o/\sigma_b = 0.1$. Daley (1991, figure 4.14) reproduces Seaman's (*op. cit.*) figure 2a, showing the area-averaged degradation in analysis quality.

The values of L and σ_o/σ_b assumed by Seaman are unrealistic for modern data assimilation systems. To investigate the effect of varying these parameters in a modern system, a simple one-dimensional analysis system was constructed. The system consists of a cyclic one-dimensional domain of 10,000km, with observations randomly distributed over the domain. Background errors are assumed to be spatially correlated with a homogeneous Gaussian correlation function with length scale L=500km. Two configurations were considered: an "ancient" system, for which $\sigma_o/\sigma_b = 0.1$, and the mean distance between observations was 500km; and a "modern" system, for which $\sigma_o/\sigma_b = 1$ and the mean distance between observations was 100km.

Figure 7 shows the mean analysis error standard deviation, normalised by the background error standard error deviation, for the "ancient" system. The figure should be compared with Seaman's figure 2a (Daley's figure 4.14), which it resembles closely. This suggests that the choice of a one-dimensional, rather than two-dimensional, domain does not greatly modify the results. The large black dot shows the position of the minimum of the analysis error standard deviation, and the tips of the arrows correspond to factors of $\sqrt{2}$ increases and decreases in the parameters. Changes in the standard deviation of observation error over a factor-of-two range result in about a 1.2% degradation in the analysis, whereas a corresponding range of values of length scale results in a much larger change in analysis quality. (Note that Seaman considered σ_b to be correct, with σ_o misspecified. However, since a linear analysis depends only on the ratio of σ_o to σ_b, we may equally consider σ_o to be fixed and σ_b to be varied.)

The corresponding figure for the "modern" system is shown in Figure 8. The analysis is less sensitive to changes in correlation length scale, but more sensitive to changes in σ_o/σ_b. The change of around 6% in the standard deviation of analysis error for a factor-of-two range of σ_o/σ_b corresponds well with the results for the zero-dimensional analogue shown in Figure 6.

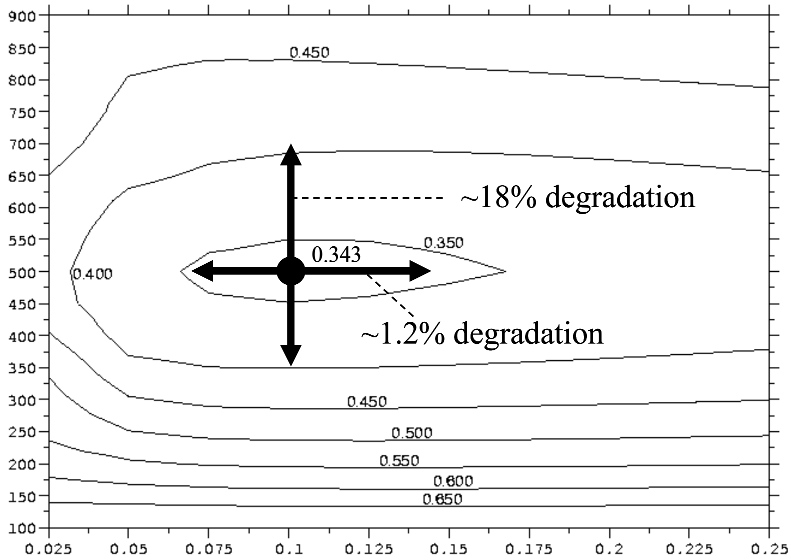

Figure 7: Ratio of mean analysis error to background error for the "ancient" system, with the parameters used by Seaman (1983).

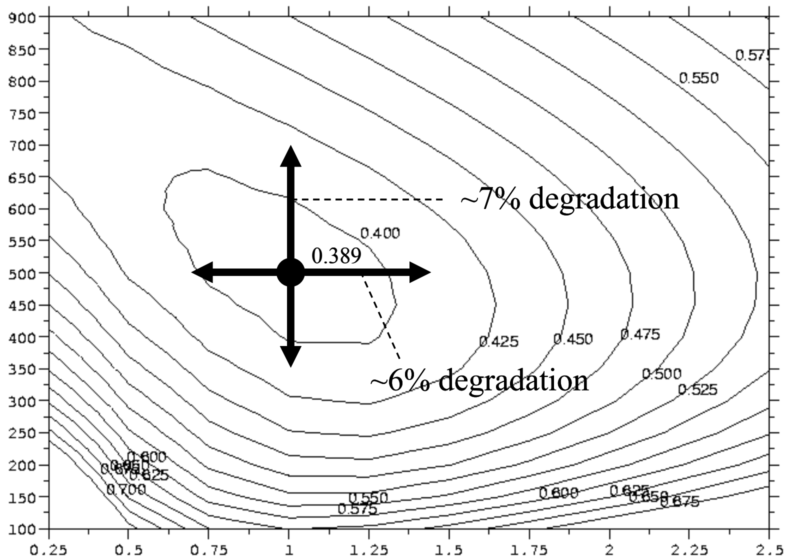

Figure 8: As Figure 7, but for the "modern" system.

Two mixed configurations of the system were also evaluated, corresponding to "modern" observation separation combined with "ancient" σ_o/σ_b and *vice versa*. The results are summarised in Table 2. Increased accuracy of the background (i.e. decreased σ_o/σ_b) results in increased sensitivity to σ_o/σ_b. However, the sensitivity remains less than 6%. Increased density of observations results in increased sensitivity to errors in both parameters. (Note that the extreme sensitivity of 62% for the case of an accurate background and sparse observations corresponds to an unrealistic system. In this case, for correctly specified parameters, the normalised analysis error is around 0.06. That is, errors in the background are around 17 times larger than errors in the analysis.)

	Degradation due to misspecified σ_b	Degradation due to misspecified L
Poor background, sparse observations	1.2%	18%
Poor background, dense observations	2.5%	62%
Accurate background, sparse observations	4.8%	2.4%
Accurate background, dense observations	6%	7%

Table 2: The effect of misspecified statistics on four configurations of the one-dimensional analysis system. Configurations denoted "poor background" and "accurate background" correspond to true values of σ_o/σ_b of 0.1 and 1, respectively. Configurations denoted "sparse observations" and "dense observations" correspond to mean distances between observations of 500km and 100km, respectively. Degadations are given as the larger of the two increases in σ_a/σ_b resulting from an increase and a decrease by a factor of $\sqrt{2}$ in the value of the parameter.

Next, suppose that the true length scale of background error correlation is in the "modern" configuration is 100km, rather than 500km. The effect (not shown) of this reduction in length scale is to slightly reduce the sensitivity to changes in σ_o/σ_b from 6% to 4.6%, as measured by the maximum degradation in analysis error for a $\sqrt{2}$ increase or decrease. The sensitivity to a corresponding relative change in specified length scale is reduced from 7% to 1.9%. Finally, suppose that the true value of σ_o/σ_b is 2. That is, the background is more accurate than the observations. In this case, the sensitivity of the "modern" configuration (assuming a true length scale of background error correlation of 500km) is 7% for changes in σ_o/σ_b, and 3% for changes in *L*.

5. Conclusions

Radical changes to the standard deviations of background error used to construct the background covariance model in the ECMWF analysis system result in surprisingly little effect on the quality of the analyses, and of forecasts launched from them. Investigations with very simple zero- and one-dimensional analogues confirm this result, and suggest that for a factor-of-two range of background error standard deviation around the true value, the expected analysis error remains within a few percent of its optimal value, resulting in a reduction of forecast skill of only a few hours.

6. References

Bormann, N., Saarinen, S., Kelly, G., Thépaut, J. 2003: The spatial structure of observation errors in atmospheric motion vectors from geostationary satellite data. *Monthly Weather Review,* **131**, 706-718.

Daley R., 1991: *Atmospheric Data Analysis*. Cambridge University Press.

Kucukkaraca E. and M. Fisher, 2006: Use of analysis ensembles in estimating flow-dependent background error variances. *ECMWF Technical Memorandum*, No. 492.
(Available from http://www.ecmwf.int/publications/)

Fisher M. and P. Courtier, 1995: Estimating the covariance matrices of analysis and forecast error in variational data assimilation. *ECMWF Technical Memorandum* No.220.
(Available from http://www.ecmwf.int/publications/)

Fisher M. 2003: Background error covariance modelling. *ECMWF Seminar on Recent developments in data assimilation for atmosphere and ocean*, 8-12 September 2003.
(Available from http://www.ecmwf.int/publications/)

Savijärvi, H., 1995: Error growth in a large numerical forecast system. *Monthly Weather Review*, **123**, 212-221.

Schutts G., 2004: A stochastic kinetic energy backscatter algorithm for use in ensemble prediction systems. *ECMWF Technical Memorandum* No.449. (Available from http://www.ecmwf.int/publications/)

Seaman R.S., 1983: Objective analysis accuracies of statistical interpolation and successive correction schemes. *Australian Met. Mag*., **31**, 225-240.