Boundary-Layer Processes for Weather and Climate

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ABSTRACT

An overview of the planetary boundary layer, from the perspective of models for weather and climate, is provided. Emphasis is placed on developing basic concepts, starting from general elementary considerations, and proceeding to the special and complex. In so doing we develop several central themes, such as the role of the boundary-layer depth, the differing characteristics of shear versus buoyancy and the models they most naturally engender. Two complications to these paradigms are considered, as examples of the complexity of real boundary layers. The first being the baroclinic boundary layer as a modification of the shear boundary layer, the second being the stratocumulus-topped boundary layer as an exotic form of the convective boundary layer. It is shown how both have proven difficult to represent with fidelity in large-scale models for weather and climate.

1 The planetary boundary layer (PBL)

In this article I try to develop the readers intuition and appreciation for the planetary boundary layer, and planetary boundary-layer processes by focusing on the big picture in the context of classical boundary-layer theory. In so doing I endeavor to draw out what are enduring and essential themes, including: the role of the boundary layer depth, shear versus buoyancy effects, the role and character of turbulence, and the tension between idealization and realization. This is not a review, as should be evident by my parsimonious approach to citations. Nor is it meant as a substitute for (or even summary of) the many excellent and more comprehensive text-books and compendiums on the subject, for instance Garratt (1992); Holtslag and P.G.Duynkerke (1998); Arya and Holton (2001). Rather it is an attempt at synthesizing what I perceive to be some of the important issues and streams of thought in ongoing endeavors to represent planetary boundary layers in models for weather and climate.

What better way to start, than with a question? What is the planetary boundary layer? Previous lecturers in this series have answered this question in different ways. Wippermann (1976) defines it foremost as "an abstraction." Stull describes it simply as the "bottom 100 - 3000 m of the troposphere," while Beljaars (1994) imbues the definition with physical content by describing the planetary boundary layer as "the lower part of the troposphere where the direct influence from the surface is felt through turbulent exchange." My answer to this question, which is clearly influenced by the above lines of thought, is that:

> the planetary boundary layer is a turbulent layer that emerges due to the destabilizing influence of the surface on the atmosphere, and which is characterized by the length-scale h, such that
>
> $$z_* \ll h \ll \mathscr{H}, \mathscr{L},$$
>
> where z_* measures the scale of surface variations (for instance a roughness hight, or a canopy scale), $\mathscr{H}$ is the depth of the troposphere and $\mathscr{L}$ is a characteristic length scale measuring variations in surface properties.

Describing the boundary layer in this sense incorporates Wippermann's idea of the boundary layer as an abstraction, something that emerges in an ideal sense, *i.e.,* , in the limit as h/H and z_0/h simultaneously approach zero. Because H is on the order of a scale height, or roughly 8-15 km, and z_0 is typically less than a meter, the range of length scales such as those identified by Stull emerge naturally from the above definition.

Fundamentally the idea of layers in the atmosphere embody asymptotic thinking, i.e., the idea that certain limits select a subset of physical processes which we endeavor to represent even when the limit is not strictly satisfied. This applies to many of the "layers" one encounters, from the surface layer, to the roughness sublayer, to the viscous layer, to the troposphere itself. In reality of course, there is no such thing as the planetary boundary layer; still it proves to be a useful concept when thinking about how to represent processes in the lower troposphere. Its utility stems from its success in identifying a set of *essential* processes whose representation proves necessary to adequately match atmospheric flows to the properties of the underlying surface. The emphasis on *essential* reflects a desire to discriminate between things or processes that we believe are important and general from details that are thought to have no general importance. The idea is a classical one, namely that a rough characterization of surface-atmosphere interactions can be accomplished by an accurate description of the general processes, and that refinements of this description can fill in details as required or desired. Asymptotics can be thought of as a way to winnow the details from the essence of the problem, in the way wind winnows chaff from grain. In so doing asymptotics simplifies the system, rendering it amenable to different lines of attack—some of which will be discussed further below.

The importance, or rather necessity, of representing PBL processes with at least some level of detail has long been recognized. As Winn-Nielsen (1976) points out in reference to the incorporation of Ekman pumping and suction in the equivalent barotropic model of Charney and Eliassen, the "incorporation of the planetary boundary layer in a numerical prediction model was done even before the first 1-day forecast was made." This is because boundary-layer processes, and the secondary circulations they entail, set the timescale for the spin-up or spin-down of large-scale atmospheric circulations and thus are essential to the basic structure of the atmospheric circulation.

In subsequent years we have developed a much richer understanding of the ways in which the PBL may help set or determine the characteristics of larger-scale circulations. For example boundary layer convergence has long offered the intriguing, but ever controversial, possibility of amplifying incipient large-scale circulations (in particular low-pressure disturbances, *e.g.,* Charney and Eliassen, 1964; Ooyama, 1964; Moskowitz and Bretherton, 2000). The character of the diurnal cycle over land, in particular the nighttime minimum temperatures, the phasing and character of convective precipitation (Atkins et al., 1998), is also thought to be intricately connected to the development and character of the planetary boundary layer (see also Khairoutdinov and Randall, 2006). boundary-layer research has also been motivated by an attempt to better understand the nature of local circulations, with particular applications related to the dispersion of hazard material. From a larger-scale perspective boundary-layer processes, whether it be those that mediate the exchange of sea-salt between the ocean and atmosphere from ocean breakers, or simply the subtle and seemingly sensitive distribution of haze and clouds, are increasingly thought of as being critical to the behavior of the climate system.

The outline of the remainder of the document is as follows. In section 2 I review some of the basic tools used in the analysis of boundary layers, section 3 draws out essential results from the theory of classical laminar boundary layers. These ideas are developed in the context of idealized planetary boundary layers in section 4. Section 5 presents a flavor of some of the boundary layer regimes that nature presents us with, and in lieu of a conclusion section 6 presents some closing comments and discussion.

2 Tools — The Alpha and Omega

2.1 Asymptotics

Asymptotics provides a rational way of simplifying complex systems. A relevant and expository example of the asymptotic method is the development of Prandtl's boundary-layer equations. We begin with the Navier-Stokes equations for constant density flow, in an inertial-frame, assuming slab symmetry in the y (or cross steam) direction:

$$\partial_t u + (u\partial_x + w\partial_z)u = -\alpha\partial_x p + \nu(\partial_{xx} + \partial_{zz})u \tag{1}$$
$$\partial_t w + (u\partial_x + w\partial_z)w = -\alpha\partial_z p + \nu(\partial_{xx} + \partial_{zz})w \tag{2}$$
$$\partial_x u + \partial_z w = 0. \tag{3}$$

Here symbols have their usual definitions with α being the specific volume, or the inverse density, and ν (not be confused with v, which we shall have recourse to later) the kinematic viscosity. The question Prandtl asked was how, for a mean flow with velocity scale U and spatial variations of scale L, one could reconcile the asymptotics of this system of equations in the limit of vanishing viscosity with the boundary condition of no-slip at a lower surface (say $z = 0$). The trivial limit whereby the viscous term is allowed to vanish as $\nu \to 0$ is singular, as it also removes one of the boundary conditions needed to specify the state of the actual system.

His answer was that a distinct vertical scale, h must emerge such that $h \propto (\nu/UL)^{1/2}$. Only in such a case can the crucial term $\nu\partial_{zz}u$, which represents the effects of the momentum exchange to the surface, be retained in the limit of $\nu \to 0$. Applying this scaling to the above system of equations yields the Prandtl boundary-layer equations:

$$\partial_t u + (u\partial_x + w\partial_z)u = -\alpha\partial_x p + \nu\partial_{zz}u \tag{4}$$
$$\partial_x u + \partial_z w = 0. \tag{5}$$

This scaling, *i.e.,* the asymptotics of Prandtl, introduces several key concepts. First that of a boundary layer itself, whose distinguishing characteristic is its depth, which scales with $\nu^{1/2}$, but also the concept of a secondary circulation. In this case the secondary circulation is represented by w whose magnitude is given, through continuity, by $w \propto uh/L$; because $h \ll L$, $w \ll u$. It is this latter fact that renders the momentum balance in the vertical entirely negligible with regard to the first order evolution of the system.

2.2 Similarity

If *asymptotics* is the alpha of boundary-layer alphabet, then *similarity* is its omega. What is similarity and why is it so important? The book form of the answer to this question is given by the excellent text by Barenblatt (1996). My phrase form of the answer is that similarity is non-dimensional equivalence. Just as two triangles are similar if their non-dimensional measures (angles) are equivalent, we say two systems are similar if they too are non-dimensionally equivalent. *Similarity* forces one to identify the non-dimensional structure of the system—alternatively the constraints imposed by the dimensionality of the parameters of the system. As a result of such an analysis one can often identify rather simple structure in otherwise seemingly complex systems.

Similarity proves to be a useful way to think about systems when a direct assault on the solution of a problem proves unsuccessful. As an example imagine being tasked with solving for the period of a pendulum, which we shall denote $\mathscr{T}$, without knowing Newtons mechanics. One could simply set about measuring pendulum periods and classifying them as a function of the parameters of a pendulum, hoping to identify regularity in the data. However if one stepped back and thought about the problem, they might come to the conclusion that the essential parameters of a pendulum system are the gravitational acceleration, g and the length of the pendulum, ℓ. Dimensionally they allow only one way to form a unit of time, namely $\mathscr{T} \propto (\ell/g)^{1/2}$. Given this one could hypothesize that the non-dimensional period of a pendulum must be universal, by which we mean that all ideal pendulums conform to the law that their non-dimensional period, Π_τ defined such that

$$\Pi_\tau = \mathscr{T}\left(\frac{g}{\ell}\right)^{1/2},$$

is constant. Then one must only make a few measurements to arrive at ones answer, one to establish Π_τ and a couple of others to check that it really is universal.

So doing might reveal the success of the approach. Although, if sufficiently large starting angles (initial pendulum displacements) for the trial pendulum systems are employed, matters would quickly go awry. From which, the investigator would quickly learn that the initial analysis failed to consider the effect of another non-dimensional parameter of the system, namely the starting angle. In other words, the analysis outlined above only applies in the limit of small starting angles, or if you will, linear pendulums. The point of this being that asymptotics can help identify simple systems, which then lend themselves to an enlightened form of empiricism, which we call similarity. Similarity, like asymptotics proves indispensable to the analysis of fluid mechanical phenomena in general, and boundary-layer processes in particular—in large part because the problem of turbulence has proven otherwise so intractable.

3 Laminar Boundary-Layer Theory

In this section we review some results from the theory of laminar boundary layers. The classic reference on the subject remains the book by Schlichting (1979).

3.1 Blasius

Figure 1: A sketch of a flat plate boundary layer

Prandtl's boundary equations can be written as a single equation in terms of the stream function

$$\psi(x,z) \quad \text{where} \quad u = \partial_z \psi, \quad \text{and} \quad w = -\partial_x \psi. \tag{6}$$

Prandtl's student Blasius solved this equation for the special case of steady flow with no pressure gradients downstream of an incident plane or splitter plate along which the stream-wise velocity must vanish, as illustrated in Fig. 1. In terms of the stream function Blasius equation is:

$$\partial_z \psi \, \partial^2_{xz} \psi - \partial_x \psi \, \partial^2_{zz} \psi = \nu \partial^3_{zzz} \psi. \tag{7}$$

In addition to x (the distance from the plates leading edge) and z (the height above the plate) both the free stream (outer) velocity U and ν might be expected to control the nature of the solution. That is we might expect to look for solutions of the form:

$$\psi = g(x,z,\nu,U). \tag{8}$$

Based on the dimensions of the parameters of the problem, the independent variables, and quite likely insight from Prandtl's boundary-layer analysis, Blasius hypothesized a self-similar development of the boundary layer. In his theory the behavior of the system non-dimensionalized by the length-scale $(\nu x/U)^{1/2}$ should behave independently of its free-stream Reynolds number[1] $(Ux/\nu)^{1/2}$ and in so doing depend only on the non-dimensional height $\eta = z\left(\frac{U}{\nu x}\right)^{1/2}$. This implied that x and z are no longer independent, in which case the partial differential equation could be written non-dimensionally as the ordinary differential equation:

$$f(\eta)\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} + 2\frac{\mathrm{d}^3 f}{\mathrm{d}\eta^3} = 0, \tag{9}$$

[1]Barrenblatt calls this incomplete similarity as the Reynolds number similarity (non-dimensional equivalence independent of Reynolds number) emerges only under a particular coordinate transformation.

subject to the boundary conditions

$$f(0) = \left.\frac{\mathrm{d}f}{\mathrm{d}\eta}\right|_{\eta=0} = 0 \quad \text{and} \quad \left.\frac{\mathrm{d}f}{\mathrm{d}\eta}\right|_{\eta=\infty} = 1. \tag{10}$$

Solutions to this system were found to agree well with experiments at sufficiently, but not too, large Re. At very large Re the boundary layer becomes turbulent.

Blasius's solution to this differential equation can also be integrated to yield, interesting properties of the flow. For instance the thickness of the boundary layer can be defined objectively in terms of integrals over the flow. One such measure is the displacement height, defined:

$$h \equiv \frac{1}{U}\int_0^\infty [U - u(z)]\,\mathrm{d}z \approx 1.721\left(\frac{\nu x}{U}\right)^{1/2}, \tag{11}$$

which conforms to the expected scaling of Prandtl. Similarly, the net drag on the flow through some distance L of flow over a plate of some width b can be solved for directly as

$$D = b\int_0^L \nu\left(\frac{\partial u}{\partial z}\right)_{z=0} \mathrm{d},x = f''(0)\nu U\left(\frac{U}{\nu x}\right)^{1/2} \propto \left(\frac{\nu U^3}{x}\right)^{1/2} \tag{12}$$

A remarkable feature of this solution is that the drag scales with $U^{3/2}$ rather than U as would be expected in the absence of a boundary layer. This ability of boundary layers to qualitatively change the drag regime of a flow is one of their most important effects.

Overall Blasius's problem illustrates how a boundary layer, associated with a viscous length scale, develops in the flow. It also shows that this boundary layer develops self-similarly, which means that at each point downstream of the plate's leading edge the flow is non-dimensionally equivalent to the flow at each and every other point. The interesting twist in this solution is that rather than the solution being scaled by what one might expect in the absence of a boundary layer, i.e., U, the appropriate velocity scale is actually a combination of both the external velocity scale and a diffusive velocity scale, i.e., $(U\nu/x)^{1/2}$. The Blasius problem neatly illustrates that the concept of a boundary layer is intrinsically bound up with the development of an additional length-scale in the flow, in this case one associated with viscous effects. It also provides a powerful illustration of similarity concepts.

3.2 Ekman

Another laminar boundary layer of some interest is the Ekman layer. For an Ekman layer one is concerned with how viscous effects and a no-slip lower boundary manifest themselves in the presence of rotation. The framework we take is for a rotating plane with the outer-flow in geostrophic balance, with (u_g, v_g) denoting the geostrophic velocities.

For this flow we look for solutions $u(z)$ and $v(z)$ satisfying a subset of Prandtl's equations on a rotating plane[2]:

$$f(v_g - v) = \nu\partial_{zz}u \tag{13}$$
$$f(u - u_g) = \nu\partial_{zz}v \tag{14}$$
$$\partial_x u + \partial_y v + \partial_z w = 0. \tag{15}$$

subject to the boundary conditions of vanishing flow at the surface, and matching to the free-stream (geostrophic) flow in the interior (as $z \to \infty$). In addition to the presence of the Coriolis terms, which forces us to abandon slab

[2]This leads to the $(-fv, fu)$ terms which measure the apparent (Coriolis) force associated with our non-inertial reference frame. Note that assuming geostrophic balance of the large-scale (outer) flow, allows us to write the pressure gradients as fv_g and fu_g for the x and y momentum equations respectively

symmetry, the above equations differ from the original Prandtl equations in that time-derivatives and advective terms are neglected.

In the case where the second derivatives of u_g and v_g vanish, solutions for u and v are well known and adopt the form

$$u = u_g\left[1 - e^{-\kappa z}\cos(\kappa z)\right] - v_g e^{-\kappa z}\sin(\kappa z) \tag{16}$$
$$v = v_g\left[1 - e^{-\kappa z}\cos(\kappa z)\right] + u_g e^{-\kappa z}\sin(\kappa z), \tag{17}$$

where $\kappa = \sqrt{f/2\nu}$ in the northern hemisphere $(f > 0)$ is an inverse length-scale.

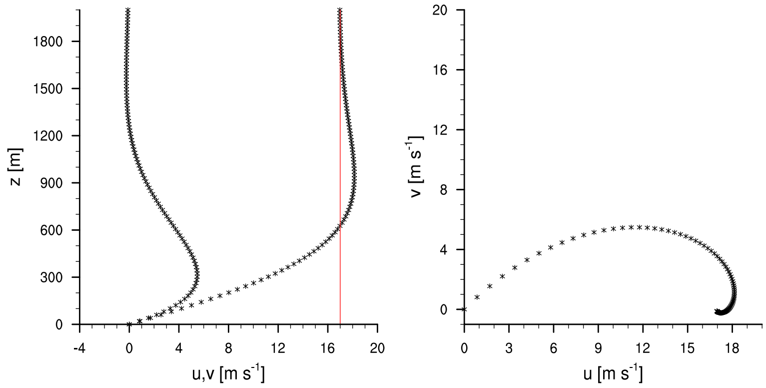

Figure 2: Ekman Boundary-Layer Solutions for $u_g = 17$ m s^{-1}. In the left panel we plot the u and v winds. In the right panel we plot the hodograph [the parameterized curve $u(z), v(z)$ in the $u - v$ plane], this illustrates the famous Ekman spiral, where the wind turns clockwise (veers) with height.

Solutions for (16) and (17) for $v_g = 0$ are plotted in Fig. 2. A characteristic feature of the solution is the turning of the winds with height. Here (in the right panel of Fig. 2 they are seen to turn clockwise, or veer, as one moves away from the surface.

As was the case for the Blasius problem the solutions show that the action of viscosity leads to the development of a new length-scale in the flow. This length-scale defines the Ekman boundary layer. As we did for the Blasius problem we can define the boundary-layer depth in terms of the displacement thickness. For the case where our coordinate system is aligned with the geostrophic wind (so that v_g vanishes)

$$h \equiv \frac{1}{u_g}\int_0^\infty [u_g - u]\,\mathrm{d}z = \int_0^\infty e^{-\kappa z}\cos(\kappa z)\,\mathrm{d}z = \left(\frac{\nu}{2f}\right)^{1/2} = \frac{1}{2\kappa} \tag{18}$$

Alternatively some define the boundary-layer thickness h to be the level where (u, v) is aligned with (u_g, v_g) in which case $h = \pi\sqrt{2\nu/f} = \pi/\kappa$. In either case we find that the depth of the boundary layer increases with viscosity and decreases with rotation.

Other things to notice about the Ekman solution is that the wind at the surface is rotated $\pi/4$ radians relative to the geostrophic wind so that in a coordinate system aligned with the geostrophic wind the surface wind has an equal magnitude in both the x and y directions. Unlike the Blasius solutions the drag at the surface of the Ekman boundary layer

$$\nu\left(\frac{\mathrm{d}u}{\mathrm{d}z} + \frac{\mathrm{d}v}{\mathrm{d}z}\right) = \left[(u_g^2 + v_g^2)\frac{8\nu^3}{f}\right]^{1/2} \tag{19}$$

scales with the magnitude of the geostrophic wind, and the three-halves power of viscosity. Because the Ekman boundary layer allows steady solutions the drag at the surface must be balanced by pressure gradients which accelerate the flow within the boundary layer. These pressure gradients act on the unbalanced (sub/super -geostrophic) component of the flow, i.e., the velocity defect. Physically we can think of surface drag decelerating the flow to the point where the velocity defects become large enough such that the unbalanced part of the pressure gradient balances the drag. As rotation weakens, the acceleration is increasingly less effective and thus must occur over an increasingly deep layer. In the limit as $f \to 0$ $h \to \infty$ which is another way of saying that steady solutions are no longer possible.

3.3 Secondary Circulations

Both the Ekman and the Blasius boundary layers have residual vertical circulations. For the Blasius boundary layer the convergence of the stream-wise wind drives a large-scale circulation away from the plate:

$$w = \frac{1}{2}\left(\frac{\nu u}{x}\right)^{1/2}\left[\eta f' - f\right]. \tag{20}$$

In contrast the Ekman boundary layer only has an implied vertical circulation in the case when the forcing varies spatially. By continuity, and the requirement that the undisturbed (outer) flow is divergence free,

$$w(z) = \int_0^z (\partial_x v_g - \partial_y u_g)\, e^{-\kappa z} \sin(\kappa z)\, \mathrm{d}z, \tag{21}$$

where we note that the term in the brackets is just the geostrophic vorticity, ω_g. In the case where ω_g is constant with height it can be taken outside of the integral so that

$$w(z) = \omega_g \int_0^z e^{-\kappa z} \sin(\kappa z)\, \mathrm{d}z = h\omega_g. \tag{22}$$

This indicates that the vertical motion is positive for cyclonic $(f\omega_g > 0)$ motion and negative for anti-cyclones. Physically we can see the effect of the boundary layer is to turn the flow in the direction of the pressure gradient. This leads to cross isobaric flow which is divergent in high pressure regions and convergent in low pressure regions.

These residual circulations are important because they are thought to interact with the large scale flow. For instance in the Ekman boundary layer it is the residual circulation which organizes the tea leafs into the center of the cup upon stirring. It is also this residual circulation which is responsible for stretching or contracting vortex tubes, thereby either spinning up or spinning down a large-scale circulation on a timescale much faster than the viscous timescale L/ν^2. The effects of residual circulations in atmospheric analogs to Ekman flows are precisely those effects that Charney and Eliassen recognized as critical to their representation of large-scale circulations.

3.4 Rayleigh-Bénard

The Ekman and Blasius theory form the conceptual paradigm for shear driven boundary layers. However in the atmosphere the development of thermal boundary layers also plays a critical role. A guiding paradigm for the thermal, or convective, boundary layer is the framework developed by Rayleigh to understand Bénard's experiments. The experimental framework interpreted by Rayleigh is illustrated schematically in Fig. 3, and while it ended up being different in important respects[3] from Bénard's experiments it has proven enduring. An important feature to note about this paradigm of convective turbulence is that the depth of the convecting layer is imposed as an independent parameter.

The non-dimensional analysis of the Bousinessq equations conforming to the sketch of Fig. 3 identifies two non-dimensional parameters:

$$Ra = -\frac{g(\Theta_0 - \Theta_1)h^3}{\nu\kappa\theta_0} \quad \text{and} \quad Pr = \frac{\nu}{\kappa} \tag{23}$$

[3] In the middle part of the last century it was shown that most of the motions observed by Bénard actually arose from variations in the surface tension with temperature, not thermal instability.

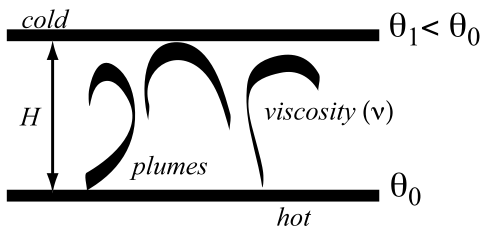

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Figure 3: Basic configuration for the Rayleigh-Bénard convective instability problem.

called the Rayleigh and Prandtl numbers respectively, where now κ denotes the thermal diffusivity. *Ra* and *Pr* encapsulate important aspects of convective flows. The first parameter measures how hard the system is being driven, the second depends on the working fluid. Given a working fluid sets the Prandtl number, leaving only *Ra* free. Conceptually this is a delightful state of affairs as any two flows of a given fluid should be similar in so far as their Rayleigh numbers are similar. Rather than having to study how convection responds to independent variations in the temperature difference between the plates, versus variations in the depth of the convecting fluid, or the strength of the viscosity, all that is necessary is to study the behavior of the flow as a function of the Rayleigh number.

Insight into the Rayleigh number can be derived, for instance, by considering a parcel rising through the fluid with the temperature, θ_0 of the lower boundary. Such a parcel will accelerate through the cooler ambient fluid and will have a convective available potential energy, here measured in terms of a convective velocity scale w_*,

$$w_*^2 = \frac{g(\Theta_0 - \Theta_1)h}{2\Theta_0}. \tag{24}$$

This shows that the Rayleigh number is analogous to square of a Reynolds number defined in terms of a convective velocity scale. It also shows that the depth of the layer, h, selects the characteristics of the flow, rather than the other way around.

The Rayleigh-Bénard problem is often phrased in terms of the transition to turbulence, the character and properties of the ensuing turbulent fluid and its dependence on the particulars of the boundary conditions. Other effects, such as a mean wind, rotational effects, weakly non-linear effects and the planform structure of the convection can all be explored (*e.g.,* Emanuel, 1994). Scaling laws for the fully developed regime can also be investigated, of particular interest is the question of *Ra* number similarity. Apart from its analytic tractability the flow configuration also lends itself well to laboratory experiments, as such it forms a paradigm for convectively driven flows, such as might define the character of atmospheric boundary layers.

4 Turbulent (Planetary) Boundary Layers

A defining characteristic of the planetary boundary layer, as we have defined it, is that it is turbulent. While the laminar boundary-layer theory helps set concepts, it is not directly applicable to the turbulent case. Turbulence arises in the atmosphere because of the range of scales and the small viscosity of the working fluid, which non-dimensionally is expressed in the greatness of the Reynolds, or effective Rayleigh number, of atmospheric flows.

4.1 Turbulence (Reynolds Number Similarity)

By Reynolds number similarity we typically mean that an appropriately defined Reynolds (or Rayleigh) number is so large, that molecular effects can be neglected, *i.e.,* , the flow behaves independently of the particular value of its molecular properties, such as measured by the diffusivity or viscosity of the fluid. Mathematically one way of dealing with turbulent flows is to decomposed any field of interest into an expected value and a fluctuation:

$$\psi = \overline{\psi} + \psi', \tag{25}$$

where an over-line denotes an expected value, and ψ is an arbitrary field. At sufficiently high Reynolds or Rayleigh number we expect (hope) that the divergence of the turbulent fluxes dominates that of the diffusive fluxes:

$$\left|\partial_z \overline{u'w'}\right| >> \left|\nu \partial_{zz}\overline{u}\right|, \quad \text{more generally} \quad \left|\partial_z \overline{\psi'w'}\right| >> \left|\nu \partial_{zz}\overline{\psi}\right|. \tag{26}$$

One class of models of turbulence often assume that the fluxes are related to the structure of the flow at the same point. The simplest of such "local" models assumes a linear relationship between local fluxes and local gradients:

$$\overline{\psi' w\prime} = -K\partial_z \overline{\psi} \tag{27}$$

where the coefficient of proportionality, K has the units of diffusivity, and is often interpreted as an exchange coefficient or an eddy diffusivity/viscosity. It may depend on the quantity being mixed, *i.e.,* differ for scalars versus components of the momentum. Such eddy-diffusivity models of turbulence can be rigorously justified in the limit when the effective length-scale, ℓ, of the mixing eddy is much smaller than the scale of variation of the mean flow, a limit which generally is not well satisfied in atmospheric boundary-layer flows. The most famous example of an eddy diffusivity model of turbulence is Prandtl's mixing-length model, wherein turbulence is assumed to arise due to the mean shear, so that the turbulent diffusivity takes the form:

$$K = \ell^2 \left|\partial_z \overline{u}\right|.$$

This introduces the mixing-length ℓ, which is assumed to be related to the scale of the turbulent eddies. While this discussion focuses attention on the question of how to model the turbulent fluxes, turbulent boundary layers raise additional questions, in particular how to match the mean flow and the turbulence model to the surface, or the flow interior. Each is an issue in the construction of models of the planetary boundary layer.

4.1.1 Turbulent momentum (shear) boundary layers

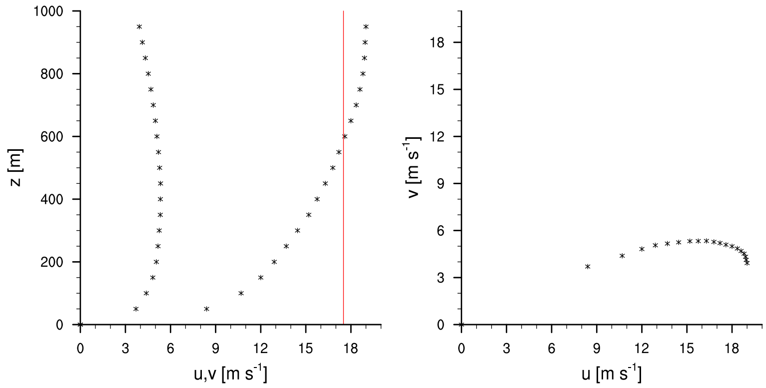

Figure 4: Measured Leipzig wind profiles adapted from data of Lettau. Here $u_g = 17$ m s^{-1}.

Given an eddy-diffusivity representation of turbulent fluxes, Prandtl's or Ekman's equations re-emerge as a useful paradigm for neutrally-stratified shear-driven layers, but with the vertical molecular fluxes being replaced by turbulent fluxes. Consider the ensemble average of Eq. 4, with actual fields replaced by expected values and fluctuations:

$$\partial_t \overline{u} + (\overline{u}\partial_x + \overline{w}\partial_z)\overline{u} = -\alpha\partial_x\overline{p} + \nu(\partial_{xx} + \partial_{zz})\overline{u} - \partial_x\overline{u'u'} - \partial_z\overline{u'w'} \tag{28}$$

$$\partial_t \overline{w} + (\overline{u}\partial_x + \overline{w}\partial_z)\overline{w} = -\alpha\partial_z\overline{p} + \nu(\partial_{xx} + \partial_{zz})\overline{w} - \partial_x\overline{u'w'} - \partial_z\overline{w'w'} \tag{29}$$

$$\partial_x\overline{u} + \partial_z\overline{w} = 0. \tag{30}$$

So that neglecting the diffusive relative to the turbulent fluxes, and assuming the latter are horizontally homogeneous, yields the following analog to Prandtl's horizontal momentum equation, but now in terms of expected values:

$$\partial_t \overline{u} + (\overline{u}\partial_x + \overline{w}\partial_z)\overline{u} = -\alpha\partial_x\overline{p} + \partial_z\left(K\partial_z\overline{u}\right), \tag{31}$$

where in general K may be expected to vary with height.

Overall many of the concepts and results from laminar boundary-layer theory, in particular the Ekman solutions, can be more readily applied to the expected state of the planetary boundary layer. As an example consider the famous Leipzig wind profiles, shown in Fig. 4, which Lettau (1950) used to derive the Eddy Diffusivity (Viscosity) profile of a nearly neutral shear layer. The main features of the Ekman solution are apparent, and differences principally reflect the spatial (height) variations in K. However, because K is not known *a priori*, but rather depends on the evolution of the flow itself, the application of such approaches to models of the planetary boundary layer is often not as straightforward as this discussion may make it seem.

4.1.2 Turbulent thermal (convective) boundary layers

The simplest paradigm for a turbulent thermal (shear-free) boundary layer is that of a uniformly stratified fluid (such that $\partial_z\overline{\theta} = \Gamma$) heated from below. In this case the thermodynamic equation takes the form:

$$\partial_t\overline{\theta} = -\partial_z\overline{w'\theta'} \tag{32}$$

Formulating the problem in this manner (remember the pendulum) leads to a sufficiently simple state of affairs that dimensional analysis helps pose meaningful constraints. Indeed, one can argue on dimensional grounds[4], that the evolution of a thermal boundary layer should proceed such that

$$h = \alpha\left(\frac{Qt}{\Gamma}\right)^{1/2}, \tag{33}$$

where $\alpha > 1$ is the non-dimensional boundary-layer depth, and $Q \equiv \overline{w'\theta'}\big|_0$ is the surface heat flux.

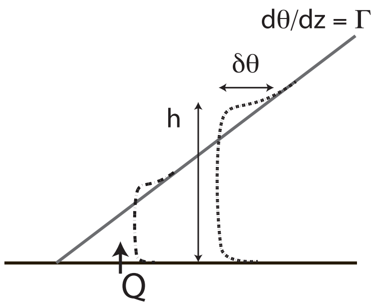

Figure 5: Sketch of the evolution in time of an idealized convective boundary layer, which at $t = 0$ consists of a uniformly stratified layer, and for $t > 0$ a constant surface heat flux is imposed. The dash-dot line shows the expected structure of the layer at an early time, and the dotted line shows the structure of the layer somewhat later.

An example of the temporal evolution of such a boundary layer is shown in Fig. 5. Here the turbulence has little to do with the local structure of the flow, and everything to do with the boundary forcings, as is the case for the Rayleigh-Bénard problem. Hence for convective boundary layers, one needs to know something about the global structure of the problem, in particular the depth of the layer, to say something about the properties of the turbulence. Even in this very simple problem, the depth of the layer reflects the history of the forcing, which thus must be accounted for in any model of the turbulence.

[4]These arguments can be quite subtle, see Stevens (2007); Anotonelli and Rotunno (2007) for a further discussion.

4.2 Surface Matching

In addition to the question of how to represent the effects of turbulence on the mean flow within the boundary layer, one must also decide how best to match the mean flow within the boundary layer to the surface below, and the free (essentially non-turbulent and stratified) flow aloft. The question of surface matching is well addressed in the literature, but bears brief review as it helps frame some of the issues to be discussed subsequently.

The central points of the theory of surface matching again involves a combination of asymptotics and similarity. Asymptotics argues for the existence of a "surface layer" which has the characteristic of being much shallower than the depth of the boundary layer as a whole yet much deeper than the height of variations in surface properties (e.g., what we previously denoted by z_*). In such a case, a neutrally stratified fluid can be expected to develop velocity gradients that depend only on the velocity scale u_* defined by the surface flux (*i.e.,* $u_*^2 \equiv \left|\overline{u'w'}\right|$), and the height, z above the surface, so that

$$\frac{z}{u_*}\partial_z\overline{u} = \frac{1}{k}. \tag{34}$$

Here k is the von Kármán constant, so that $1/k$ is the non-dimensional velocity gradient and is presumed to be universal. Integrating (34) requires the additional specification of a constant, z_0 (or roughness height), which is the height at which the velocity profile would vanish if extended toward the surface (which of course it should not be, as the arguments leading up to Eq. 34 presumed $z \gg z_0$—hence it defines a virtual surface

When thermal effects are included an additional length-scale (measuring effectively the relative role of shear and buoyancy) emerges. This is called the Obukhov length-scale and is defined as follows:

$$L = -\frac{u_*^3}{k\mathscr{B}}. \tag{35}$$

Here $\mathscr{B}$ is the buoyancy flux and has units of $\mathrm{m}^2\ \mathrm{s}^{-3}$. The factor of k is arbitrary, but customary. L is negative in regions of positive buoyancy fluxes, *i.e.,* where convection destabilizes the flow. Hence incorporating $\mathscr{B}$ means that the non-dimensional velocity gradient is no longer expected to be constant, but rather to depend on the non-dimensional height, such that

$$\frac{z}{u_*}\partial_z\overline{u} = \frac{1}{k}\Phi(z/L). \tag{36}$$

Here Φ, the non-dimensional velocity gradient, is determined empirically (Businger, 1973). Its argument, the non-dimensional height, z/L defines an effective Richardson number. This approach, called Monin-Obukhov similarity theory, can (and is) extended to describe profiles of thermodynamic properties, or passive scalars, and overall is one of the success stories in our endeavor to parameterize turbulent atmospheric processes. It should be familiar to most readers of this document.

Thus from the perspective of planetary boundary layers as a whole, the emergence of surface layer theory makes two important contributions. First it introduces a length-scale, L that measures the relative contributions of shear and buoyancy to the destabilization of the flow. Second, it means that models of the planetary boundary layer depend only indirectly (through matching conditions) on the representation of the surface.

5 Planetary Boundary-Layer Regimes

5.1 Basic Issues

Rather simple boundary layers have been presented in terms of two classes. One in which $L \approx 0$ and shear-instability is relatively unaffected by stability effects, and another wherein $L \ll 0$ and the presence of a mean flow has relatively little effect on the turbulence. For the neutrally stratified shear layers, turbulence is thought to exhibit a predominantly local character in that its properties reflect the local structure of the flow. Mixing-length models work well for such flows, although how best to represent the mixing-length itself remains a topic of research. If one adopts von Kármán's initial suggestion as taking $\ell \propto \partial_z \overline{u}/\partial_{zz}\overline{u}$ then the logarithmic velocity profile near the surface, as would be expected by integration of Eq. (34), implies that the distance from the lower boundary (the wall) is the only length-scale in the system, which is consistent with the assumption that lead to (34).

On the other hand we have endeavored to argue that for convective layers, plumes convert potential to kinetic energy over the depth, h, of the convective layer. In such a situation the boundary-layer depth can be expected to play a larger role, and the character of the turbulence depends more strongly on the structure of the layer as a whole. Moreover, how to match the vigorously-mixed convective layer, to the outer flow emerges as perhaps a more critical issue. This is evident, for instance, in a comparison of Figs. 4 and 5. For the case of the Leipzig wind profile the flow properties transition gradually to those of the outer layer, but for the cartoon of the convective boundary layer there is a distinct transition layer (the so-called entrainment interfacial layer) separating the convective layer from the outer layer.

What about $L \gg 0$? In this case we expect shear and stability effects to work against one another, as buoyancy effects will resist the tendency of shear-flow instability to destabilize the flow. Such stability effects can be measured by the Brunt-Väisällä frequency, $N^2 = \frac{g}{\Theta}\partial_z\overline{\theta}$ thereby introducing another timescale into the flow. So for instance, for stable Ekman layers one could look for solutions as a function of N/f. The effect of stability introduces further complications as it supports waves and intermittency (bursts, perhaps associated with breaking waves) in the turbulent activity, which greatly complicates the treatment of such layers.

In summary, even for very basic planetary boundary layers the dimensionality of the solution space is non-trivial. In addition to L which characterizes the surface driving, the boundary-layer depth, h, the Coriolis parameter f, and (at least for stably stratified layers) N^2, all emerge as parameters. Among these, both h and L are ambiguous as they depend on the character of the flow itself. But if one can take them as given it says that the structure of the planetary boundary layer, in even the simplest circumstances, should depend on at least two non-dimensional numbers, h/L and f/N.

5.2 Complications

Of course nature is much more complex than this. Indeed, one might argue that the planetary boundary layer is best defined by the complexity of the regimes it encompasses. In this section we just try to develop a taste for some of the complicating factors, and their relevancy to outstanding issues in our endeavor to represent the effects of boundary-layer processes in weather and climate.

5.3 Baroclinicity

One important complication, is also an original one, as it was a focus of the introductory remarks of Winn-Nielsen (1976). And that is the effect of baroclinicity, *i.e.,* vertical shear in the geostrophic wind arising from large-horizontal temperature gradients. Even such a simple change to the experimental configuration (Ekman's problem) effectively introduces two additional timescales into the problem. Winn-Nielsen (1976) among others showed that these baroclinic effects can significantly alter the character of the Ekman spiral, and that for some conditions the winds can be expected to back (turn counter-clockwise) rather than veer with height. An example is shown in the left panel of Fig.6. These different wind profiles in turn are characterized by distinctly different secondary circulations, with concomitant effects on the mean flow.

The difficulty state-of-the-art boundary-layer representations have in capturing such effects, is illustrated in a recent study by Brown et al. (2005) from which the right panel in Fig. 6 is excerpted. The figure shows that the wind turning in the boundary layer is less pronounced in the model than it is in the data. Brown shows that reducing the mixing in stable boundary layers tends to mitigate this issue, but exacerbate other problems, particularly over land where enhanced mixing of momentum in situations where one would not expect much turbulent exchange improves the forecast skill of the model (see Beljaars, 1994). Notwithstanding the extensive

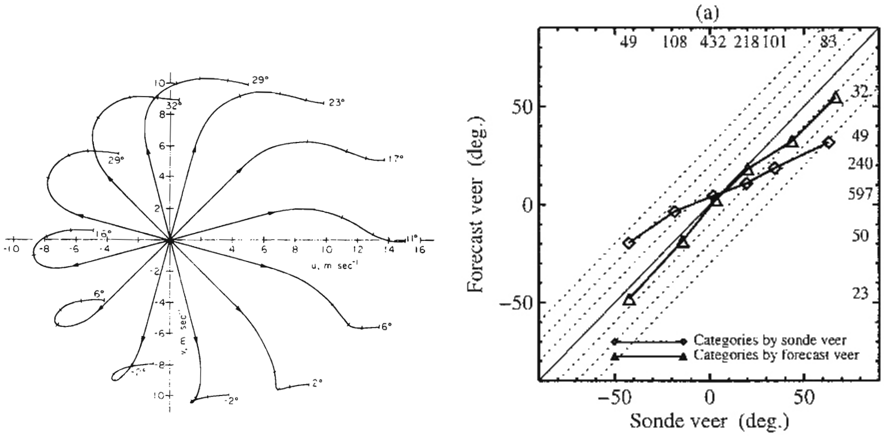

Figure 6: Left Panel: Wind Hodographs, taken from Fig. 5 of Winn-Nielsen (1976). Here the parameters are $u_g = 10$ m s^{-1}, f = 0.729 10^{-4} s^{-1}, $K = 3.6$ m^2 s^{-1}. The thermal wind gradient is specified as having a magnitude of 4 m s^{-1} km^{-1} with a direction relative to the geostrophic wind as indicated on the Figure. Right Panel: Composites of forecast veer versus measured (radiosonde) veer for 24 hour forecasts for four ocean weather-ship sites in the Northern Hemisphere. Taken from Brown et al. (2005).

theoretical work, a clear understanding of this issue remains elusive.

5.4 Cloudy Boundary Layers

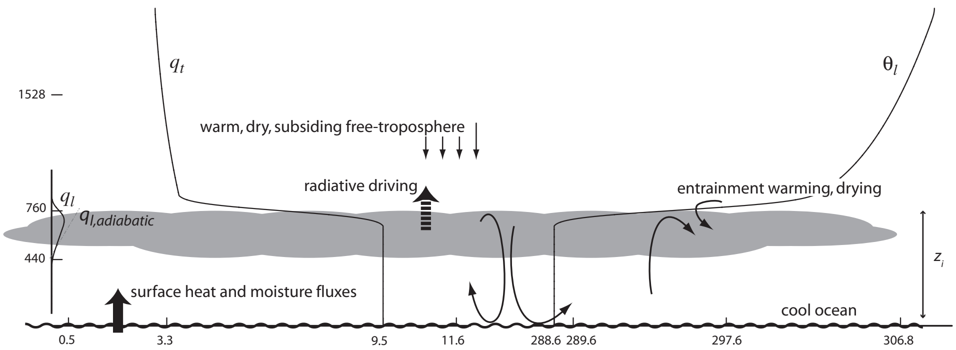

Figure 7: Mean Structure of the stratocumulus-topped boundary layer as observed in June 2001 in the North-East Pacific as analyzed by (Stevens et al., 2007). Here q_t and q_l are measured in g kg^{-1} and θ_l (the liquid water potential temperature) is measured in K.

While issues such as stability and baroclinicity complicate the treatment of shear-driven planetary boundary layers, clouds complicate the treatment of convective boundary layers. The composite structure of a stratocumulus-topped boundary layer is shown in Fig. 7. A characteristic feature of this boundary layer is that the main source of turbulence is not at the surface, but rather at the top of the boundary layer where radiative cooling generates convective plumes that mix the layer. The radiative cooling depends on the presence of cloud, which in turn depends on strong surface coupling as this is the source of moisture for the cloud, which is why we think of this as a boundary-layer regime. Another characteristic feature of such layers is the sharpness of the entrainment interfacial layer, that thin layer at the top of the boundary layer where the gradients in thermodynamic properties are very large. So while physically such boundary layers principally introduce only one more process (that of radiative driving of turbulence at the top of the boundary layer) physically the delicacy of the coupling between moisture, radiation and boundary-layer structure proves very challenging to model.

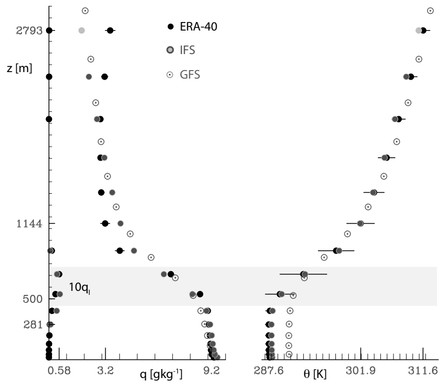

Figure 8: Mean Structure of the stratocumulus-topped boundary layer as represented in the ERA40, the IFS and the GFS for the period and time corresponding to the observations presented in Fig. 7, taken from (Stevens et al., 2007).

This point is illustrated in Fig. 8 where the model representation of the boundary layer observed in Fig. 7 is presented. Here we see that the models "resolve" the transition between the boundary layer and the free-troposphere over the characteristic 5-6 grid points. Because the vertical discretization is so coarse, this results in a much diffuser transition which distorts the mixing processes between the troposphere and the boundary layer, and hence the properties of the boundary-layer state (see for instance the thermal structure of the global forecast system (GFS) model). As a result the amount of cloud in the modeled boundary layer tends to be too little (here underestimated by at least a factor of ten), which results in insufficient radiative cooling to drive turbulence and mixing within the boundary layer.

5.5 And So On

The complications, of course, do not end here. As our demands of global models increase, myriad other questions come into play. A by no means non-exhaustive list includes: the coupling of the planetary boundary layer to clouds that evacuate mass from the boundary layer (for instance the cumulus topped boundary layers discussed by Siebesma, 1998), but also the coupling of boundary layers to canopies, water waves, complex underlying orography and so on.

6 Discussion & Outlook

In this presentation I have endeavored to develop the big picture of boundary-layer processes, and the recurring themes that emerge from such a picture. The first is that the boundary layer indeed embodies the concept of a distinct (sometimes very distinct) layer of the atmosphere, one in which vertical transport (through turbulence) and surface coupling are very active. The depth of this layer plays an important role in the character of the turbulence, particularly for convective layers. The driving forces of turbulence are either shear in the horizontal winds, or convection. Shear is usually manifest through the drag at the surface, but also modified by stability and varying pressure gradients (for instance in baroclinic boundary layers). Convection emerges from surface heating, for conventional boundary layers over land, but also can arise due to radiative processes acting at the top of the boundary layer (such as one finds in stratocumulus layer).

Approaches to modeling boundary layers are varied; but invariably are faced with similar questions. What is the boundary-layer depth? What type of mixing rule is required within the boundary layer? How is the boundary layer coupled to the free-troposphere above, versus the surface below? Depending on one's emphasis, very different types of models emerge to address these questions. Interest in convective layers favors the use of global rules, in that the mixing depends on the structure of the layer as a whole; while interest in shear and momentum boundary layers favor local approaches. More fundamentally, models differ in whether one views boundary layers in terms of idealized regimes (through which similarity approaches might be useful to lend insight into their workings) or the outcome of specific processes.

In the former approach (regime based methods) models are first tasked with identifying the regime and then applying appropriate mixing rules. In the latter processes are allowed to compete in the hope that they will find the correct regime. An example of a regime based model is that of the UK Met Office (Lock, 2004). Regime based methods suffer from the need to identify a small set or regimes (to be practical) and the question of how to deal with intermediate regimes. Process based methods have the potential of being more general, but depend on a mature understanding and capacity for modeling the underlying processes, and resolving their vertical structure—something which most large-scale models still fail to do.

Throughout most of this discussion we have identified boundary layers as ideas, abstractions. This may be somewhat old fashioned. Increasingly interest is demanding representations of planetary boundary layers that break the classifications we wish, for reasons of theory and tractability, to confine them too. For instance as models represent ever more processes at ever finer resolution, interest in layers where h may be smaller than $\mathscr{L}$ is encouraged, as is interest in layers where turbulence is not continually coupling the flow to the surface, or in representations where statistical theories become less applicable. In all challenges continue to mount; but then so do the opportunities for new and novel contributions.

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