Local Diagnostics to Measure the Efficiency of the Ensemble in Representing the Error Space

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Acknowledgements

The general approach and some of the results were published in:

- Satterfield, E., and I. Szunyogh, 2010: Predictability of the Performance of an Ensemble Forecast System: Predictability of the Space of Uncertainties. *Mon. Wea. Rev.*, **138,** 962-981.
- Satterfield, E. and I. Szunyogh: 2011 Assessing the Performance of an Ensemble Forecast System in Predicting the Magnitude and the Spectrum of Analysis and Forecast Uncertainties. *Mon. Wea. Rev.*, **139,** 1207-1223.

The unpublished (new) results were also obtained in collaboration with E. Satterfield

Outline

- I will argue that the linear space **spanned by the local ensemble perturbations** provides a good representation of the local error space at, not only the analysis, but also the longer forecast times
- Implications for
  - the **representation** of model uncertainties and errors **in numerical models** and
  - **post-processing**

Mapping a Local Vector into the Space of Ensemble Perturbations, $\mathbb{S}_\ell$

- An arbitrary local state vector $\mathbf{x}_\ell$ can be decomposed as

$$\mathbf{x}_\ell = \bar{\mathbf{x}}_\ell + \delta\mathbf{x}_\ell,$$

  where $\delta\mathbf{x}_\ell$ is the difference between $\mathbf{x}_\ell$ and the ensemble mean $\bar{\mathbf{x}}_\ell$.
- The perturbation vector $\delta\mathbf{x}_\ell$ can be decomposed as

$$\delta\mathbf{x}_\ell = \delta\mathbf{x}_\ell^{(\|)} + \delta\mathbf{x}_\ell^{(\perp)},$$

  where $\delta\mathbf{x}^{(\|)}$ is the component that projects into $\mathbb{S}_\ell$
- The vector $\delta\mathbf{x}_\ell^{(\perp)}$ is the component of $\delta\mathbf{x}_\ell$ that does not project into $\mathbb{S}_\ell$.

Formal Definition of $\mathbb{S}_\ell$: Part I

- The K-member **ensemble of local state estimates**:

$$\{\mathbf{x}_\ell^{(k)}, k = 1 \dots K\}$$

- The **ensemble mean**:

$$\bar{\mathbf{x}}_\ell = K^{-1} \sum_{k=1}^{K} \mathbf{x}_\ell^{e(k)}$$

- The **ensemble perturbations**:

$$\{\mathbf{x}_\ell'^{(k)} = \mathbf{x}_\ell^{(k)} - \bar{\mathbf{x}}_\ell,\ k = 1 \dots K\}$$

- The **ensemble-based estimate of the covariance matrix**:

$$\hat{\mathbf{P}}_\ell = (K-1)^{-1} \sum_{k=1}^{k} \mathbf{x}_\ell'^{(k)} \left(\mathbf{x}_\ell'^{(k)}\right)^T,$$

Formal Definition of $\mathbb{S}_\ell$: Part II

- The range of $\hat{\mathbf{P}}_\ell$ (spanned by the K ensemble perturbations) defines a linear space $\mathbb{S}_\ell$ [$dim(\mathbb{S}_\ell) \leq K - 1$]
- The normalized eigenvectors associated with the first $K - 1$ eigenvalues of $\hat{\mathbf{P}}_\ell$,

$$\{\mathbf{u}_k, k = 1, \ldots, K - 1\}$$

  define an orthonormal basis in $\mathbb{S}_\ell$
- The basis vectors represent linearly independent patterns of uncertainty in the ensemble perturbations in the local region at ℓ.

Decomposition of the Error in a Local State Estimate

- The **true local state**, $\mathbf{x}_\ell^t$, can be written as

$$\mathbf{x}_\ell^t = \bar{\mathbf{x}}_\ell + \delta\mathbf{x}_\ell^t = \bar{\mathbf{x}}_\ell + \delta\mathbf{x}_\ell^{t(\|)} + \delta\mathbf{x}_\ell^{t(\perp)}$$

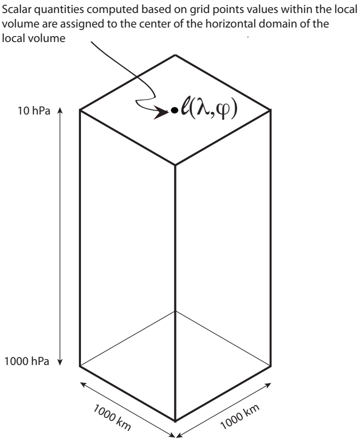
- The **local ensemble spread**, $VS_\ell = trace(\hat{\mathbf{P}}_\ell$ is an estimate of the TV_ℓ expected value of $\|\mathbf{x}_\ell^t\|^2$. Thus the expected value, VS, of VS_ℓ over all locations and verification times, should be equal to the expected value, TV, of TV_ℓ over all locations and verification times.
- The projection of $\delta\mathbf{x}_\ell^t$ into $\mathbb{S}_\ell$ is $\delta\mathbf{x}_\ell^{t(\|)}$. We introduce the notation TVS for the expected value of $\left(\delta\mathbf{x}_\ell^{t(\|)}\right)^2$
- When $\delta\mathbf{x}_\ell^t$ can be expressed as a linear combination of the ensemble perturbations, $\delta\mathbf{x}_\ell^{t(\|)} = \delta\mathbf{x}_\ell^t$ and $TVS = TV$

Analysis-Forecast System

- **Data Assmilation:** Local Ensemble Transform Kalman Filter with 40 ensemble members. (Szunyogh et al. 2008)
- **Model:** 2004 version of NCEP GFS at resolution T62 (about 210 km) and 28-levels
- **Statistics:** Collected for 45 days (January and February 2004), all results shown are for NH extratropics
- **Observations:** (Non-radiance) observations of the atmosphere
- **Variance Inflation:** Was tuned to satisfy $VS \approx TVS$ (larger VS was found to degrade the analyses and ensuing forecasts)

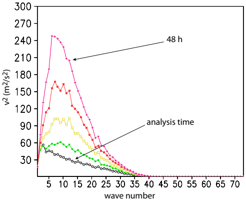
Local State Vector

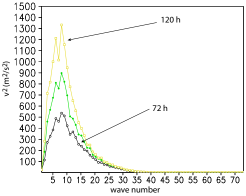
We define a **local state** vector $\mathbf{x}_\ell$ with all N state variables of the model representation of the state within a local volume centered at location (grid point) ℓ



The Motivation for Choosing a 1000 km by 1000 km Local Domain

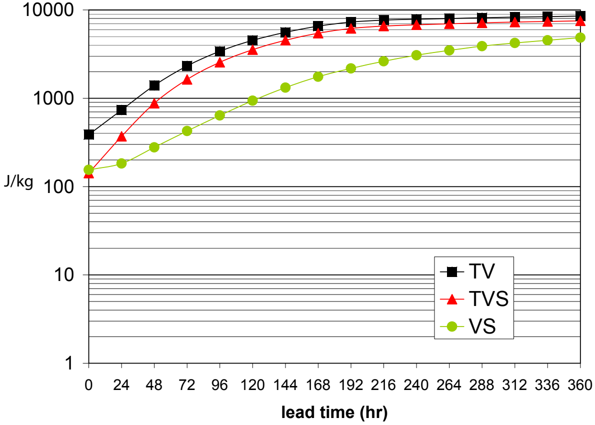
Time Evolution of the Power Spectrum of the Forecast Error (Meridional Wind at 500 hPa): The errors in the longer term forecasts are dominated synoptic scale patterns





The Evolution of *VS*, *TV*, and *TVS* with Forecast Time

For forecast times longer than about 3 days, $\mathbb{S}_\ell$ provides a good representation of the state $\mathbf{x}_\ell^t$, but the ensemble underestimates the magnitude of $\delta\mathbf{x}_\ell^t$



Discussion of the Results on V, TV, TVS

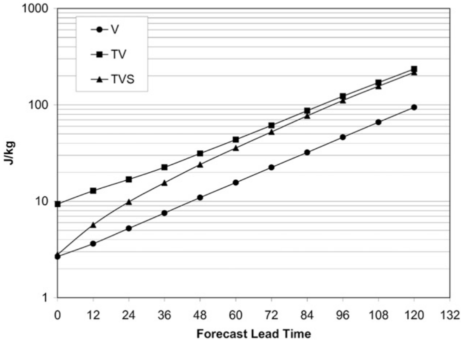
- **The Good News:** Because $\mathbb{S}_\ell$ provides a good representation of the difference between the ensemble mean and the true state (dominant potential error patterns), linear post-processing techniques have great potentials. New approaches for the interpretation of the ensemble based statistics may also exist.
- **The Problem:** Why does the ensemble spread, *VS*, grows slower than the projection of the forecast uncertainty on the ensemble space, *TVS*?

Potential Answers

- Sub-optimality of the data assimilation system, which generates the initial perturbations
- Lack of accounting for the effects of model uncertainties
- Lack of accounting for patterns of uncertainty in the initial conditions, which later pay an important role in the evolution of the forecast uncertainty, due to the use of a small ensemble
- Some combination of the above (e.g., a larger ensemble may have benefits only if the effects of model uncertainties are better represented)

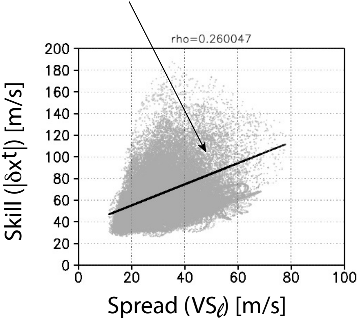
Lack of Accounting for the Effects of Model Uncertainty Is Unlikely to Be the Sole Explanation

The results with randomly (nearly uniformly) distributed simulated observations are surprisingly similar to those with observations of the atmosphere

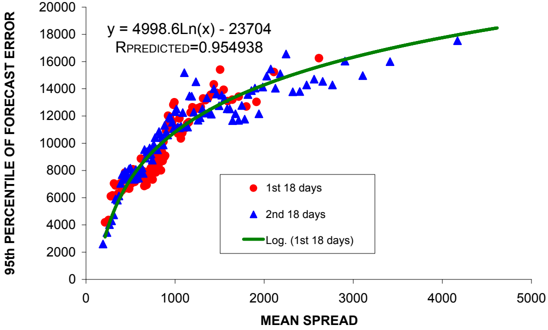


Spread-Skill Relationship Part I: Linear Regression is Not a Good Approach

Because there is a large spread of the 'skill' for large values of the spread, the conditional expectation is not a sharp predictor of the actual value of the skill



Spread-Skill Relationship Part II: 95th Percentile of $\delta^2 \mathbf{x}^t$ can be Well Predicted Based on V_S



Concluding Remarks Part I

- We outlined one possible approach for the validation of the ensemble
- The linear space spanned by the ensemble perturbations provides a good representation of the possible states of the system
- The same ensemble underestimates the magnitude of the forecast uncertainty. (The ensemble is not skillful, either, in distinguishing between the importance of the error pattern–results were not shown.) The lack of accounting for the effect of model uncertainties may not fully explain this result.

Concluding Remarks Part I

- Using the same ensemble for data assimilation and forecasting may not be optimal, which provides support for the approach of mixing ensemble perturbations from the data assimilation with SVs in the ensemble forecast system (e.g., Meteo France, ECMWF)
- Exploring the relative merits and drawbacks of post-processing vs. a representation of model uncertainties within the models promises to be an interesting research topic
- **Cautionary Note:** The proposed approach has not been tested on fields of high spatial variability (e.g., in the tropics, on precipitation, etc.)