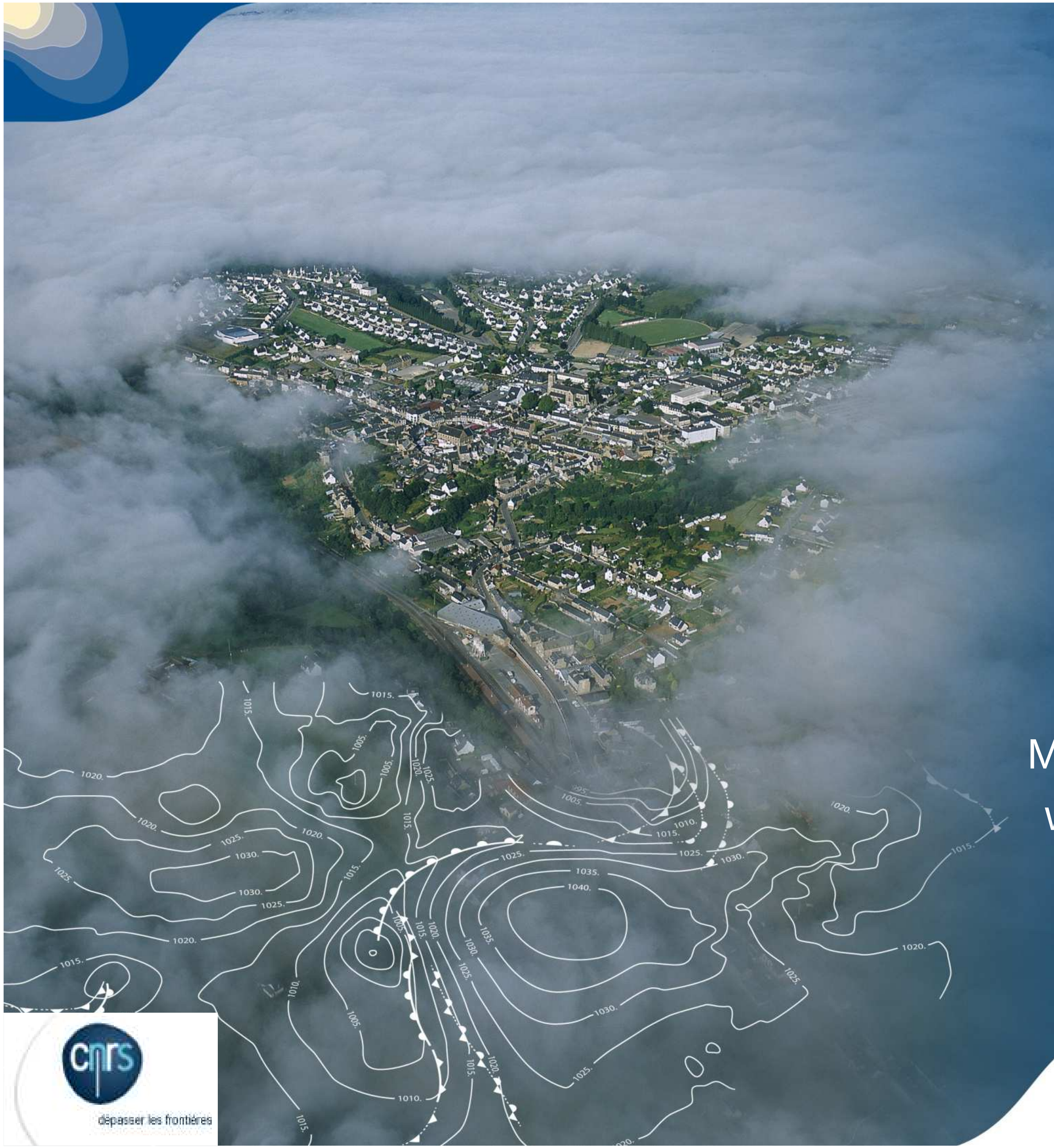

Observation error specification

Gérald Desroziers
Météo-France and CNRS
with many contributions

Outline

1. General framework
2. Methods for estimating observation error statistics
3. Diagnostic of observation error variances
4. Diagnostic of observation error correlations
5. Observation error correlation specification in the assimilation
6. Conclusion

General formalism

- Statistical linear estimation :

$$\mathbf{x}^a = \mathbf{x}^b + \delta\mathbf{x} = \mathbf{x}^b + \mathbf{K}\ \mathbf{d} = \mathbf{x}^b + \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T+\mathbf{R})^{-1}\ \mathbf{d},$$

with $\mathbf{d} = \mathbf{y}^o - \mathcal{H}(\mathbf{x}^b)$, innovation, $\mathbf{K}$, gain matrix,

$\mathbf{B}$ et $\mathbf{R}$, covariances of background and observation errors.

- Solution of the variational problem

$$J(\delta\mathbf{x}) = \mathbf{dx}^T\ \mathbf{B}^{-1}\ \mathbf{dx} + (\mathbf{d}-\mathbf{H}\ \delta\mathbf{x})^T\ \mathbf{R}^{-1}\ (\mathbf{d}-\mathbf{H}\ \delta\mathbf{x}).$$

- Incremental formulation (Courtier et al, 1994):

$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}^b)^T\ \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}^b) + (\mathbf{y}^o-\mathcal{H}(\mathbf{x}))^T\ \mathbf{R}^{-1}\ (\mathbf{y}^o-\mathcal{H}(\mathbf{x})).$$

General formalism

- Even in such a (slightly) non-linear problem,
analysis, background, model and observation errors are linked, at first order, by

$$\varepsilon^a = (\mathbf{I} - \mathbf{KH})\, \varepsilon^b + \mathbf{K}\, \varepsilon^o ,$$

with $\varepsilon^a = \mathbf{x}^a - \mathbf{x}^t$, $\varepsilon^b = \mathbf{x}^b - \mathbf{x}^t$, $\varepsilon^o = \mathbf{y}^o - \mathcal{H}(\mathbf{x}^t)$

$\varepsilon^{b+} = \mathbf{M}\, \varepsilon^a + \varepsilon^m$, with ε^m model error.

- Evolution of estimation error covariance matrices:

$$\mathbf{A}^t = (\mathbf{I} - \mathbf{KH})\, \mathbf{B}^t\, (\mathbf{I} - \mathbf{KH})^T + \mathbf{K}\, \mathbf{R}^t\, \mathbf{K}^T$$

$$\mathbf{B}^{t+} = \mathbf{M}\, \mathbf{A}^t\, \mathbf{M}^T + \mathbf{Q}^t .$$

General formalism

- Observation errors (Daley, 1993):

$$\begin{aligned} \varepsilon^o &= \mathbf{y}^o - \mathcal{H}(\mathbf{x}^t) \\ &= \mathbf{y}^o - \mathbf{y}^t + \mathbf{y}^t - \mathcal{H}(\mathbf{x}^t), \text{ where } \mathbf{y}^t \text{ is the true state equiv. of } \mathbf{y}^o \\ &= \varepsilon^o_i + \varepsilon^o_H . \end{aligned}$$

- ε^o_i is the instrument error.

- ε^o_H is a complex function of the

✓ type of observation (in situ or integrated),
✓ resolution of the state (representativeness error),
✓ precision of the observation operator (satellite observation) ...

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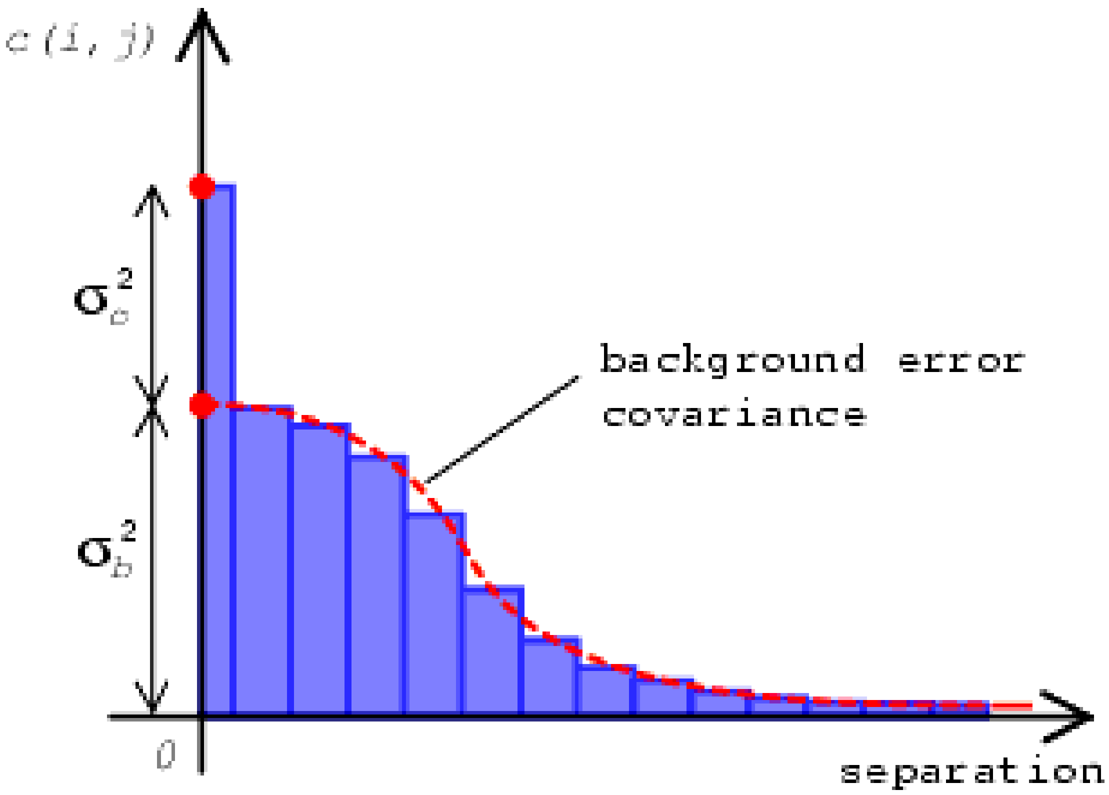
Hollingsworth and Lönnberg method



(From Bouttier and Courtier, ECMWF)

A posteriori « Jmin » diagnostics

- We should have

$$E[J(\mathbf{x}^a)] = p, \text{ with}$$

p = total number of observations.
(Bennett et al, 1993)

- More precisely, for a sub-part of J^o :

$$E[J^o_i(\mathbf{x}^a)] = p_i - Tr(\mathbf{R}_i^{-1/2}\,\mathbf{H}_i\,\mathbf{A}\,\mathbf{H}_i^T\,\mathbf{R}_i^{-1/2}), \text{ with}$$

p_i : number of observations associated with J^o_i,
$\mathbf{R}_i$, $\mathbf{H}_i$: associated error cov. matrix and obs. operator.
(Talagrand, 1999)

A posteriori « Jmin » diagnostics: optimization of **R**

- Normalization of $\mathbf{R}_i$: $s^o_i\ \mathbf{R}_i$

  Coef. s^o_i diagnosed with $s^o_i = E[J^o_i(\mathbf{x}^a)]/(E[J^o_i(\mathbf{x}^a)])^{opt}$
  $= E[J^o_i(\mathbf{x}^a)]/(p_i - Tr(\mathbf{R}_i^{-1/2}\mathbf{H}_i\mathbf{A}\mathbf{H}_i^T\mathbf{R}_i^{-1/2}))$,

  (Desroziers and Ivanov, 2001; Chapnik et al, 2004;
  Desroziers et al 2009)

- Equivalent to a Maximum-likelihood estimation (Dee, 1998)

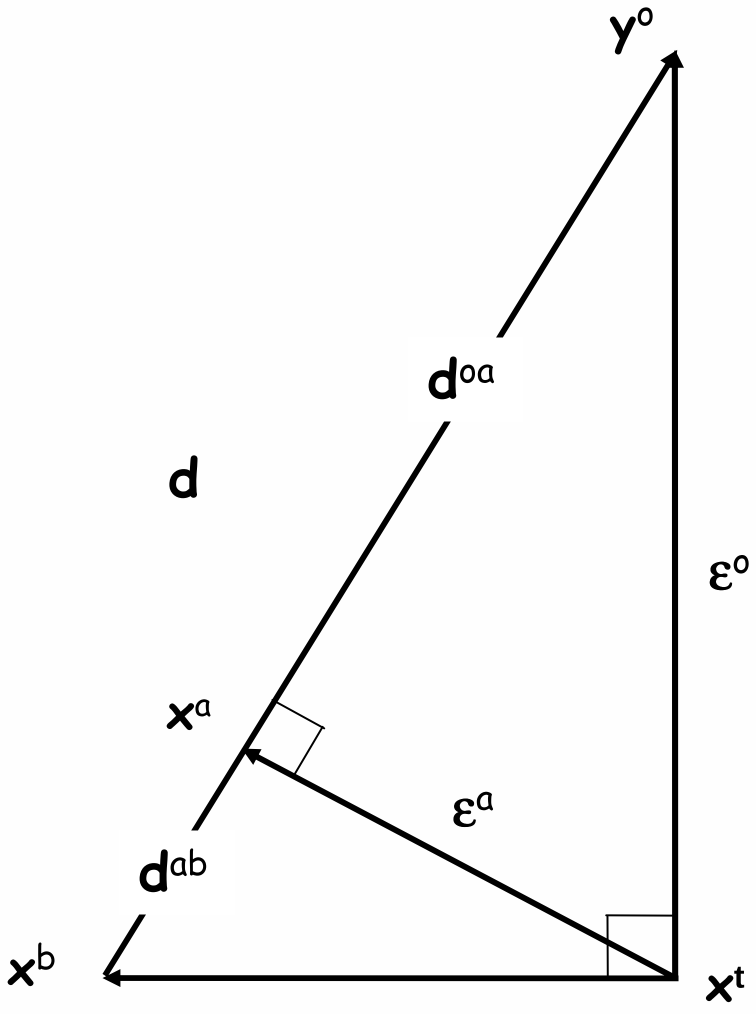
  $f(\mathbf{d}|\mathbf{s}) = 1 / ((2p)^p \det(\mathbf{D}(\mathbf{s}))^{1/2} \exp(-1/2\ \mathbf{d}^T\ \mathbf{D}(\mathbf{s})^{-1}\ \mathbf{d})$,

  where $\mathbf{D}(\mathbf{s})$ is the covariance matrix of parameters $\mathbf{s}$.

  Optimal parameters $\mathbf{s}$ are those that minimize the Log-likelihood

  $L(s) = -\log(\ f(\mathbf{d}|\mathbf{s})\ )$.

Diagnostics in observation space



(Desroziers et al, 2005)

- $\mathbf{d} = \mathbf{y}^o - H(\mathbf{x}^b)$
- $\mathbf{d}^{oa} = \mathbf{y}^o - H(\mathbf{x}^a)$
- $\mathbf{d}^{ab} = H(\mathbf{x}^a) - H(\mathbf{x}^b)$
- $E[\mathbf{d}^{oa}\ \mathbf{d}^T] = \mathbf{R}$
- $E[\mathbf{d}^{ab}\ \mathbf{d}^T] = \mathbf{HBH}^T$
- $E[\mathbf{d}^{ab}\ \mathbf{d}^{oaT}] = \mathbf{HAH}^T$
- $<\varepsilon, \varepsilon'> = E[\varepsilon\ \ \varepsilon'^T]$

Diagnostics in observation space: practical implementation

- For any subset i with p_i observations, simply compute

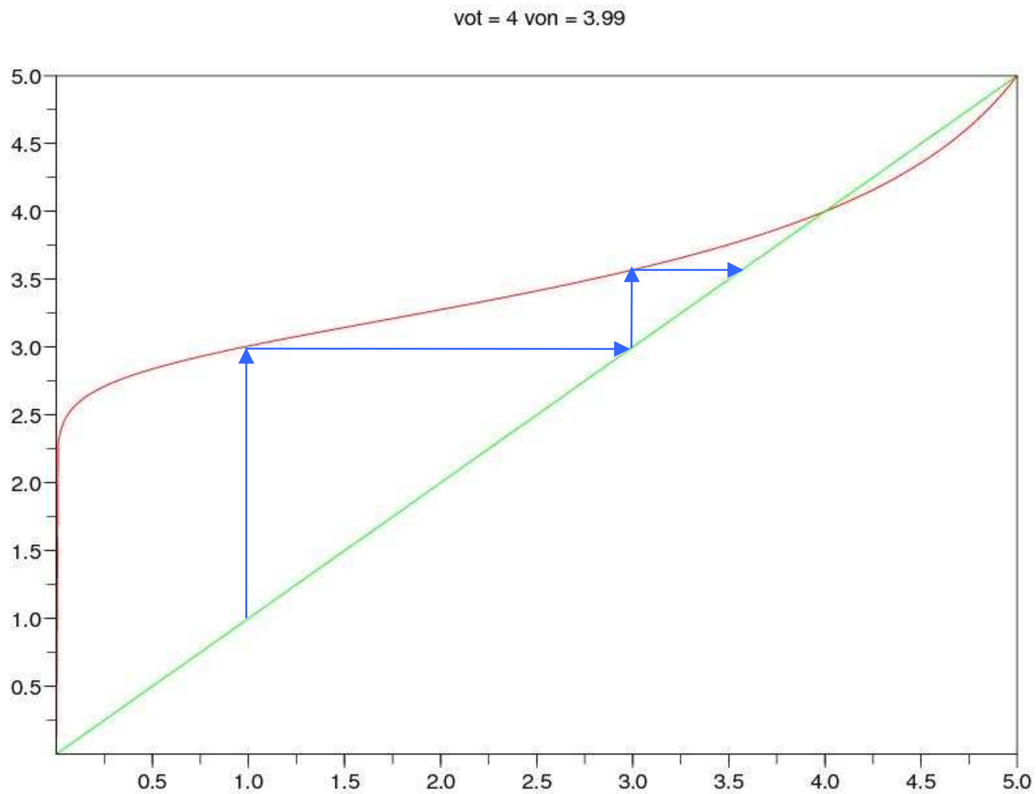
$$(\sigma^{oi})^2 = \Sigma_{k=1,pi} (y^{oi}_k - y^{ai}_k)(y^{oi}_k - y^{bi}_k) / p_i .$$

- Covariances between different observation errors can also be computed:

$$(C^{oi,j})^2 = \Sigma_{k=1,pi,j} (y^{oi}_k - y^{ai}_k)(y^{oj}_k - y^{bj}_k) / p_{i,j}$$

✓inter-channel covariances,
✓spatial covariances ...

Convergence: $v^o_{diag}(v^o)$

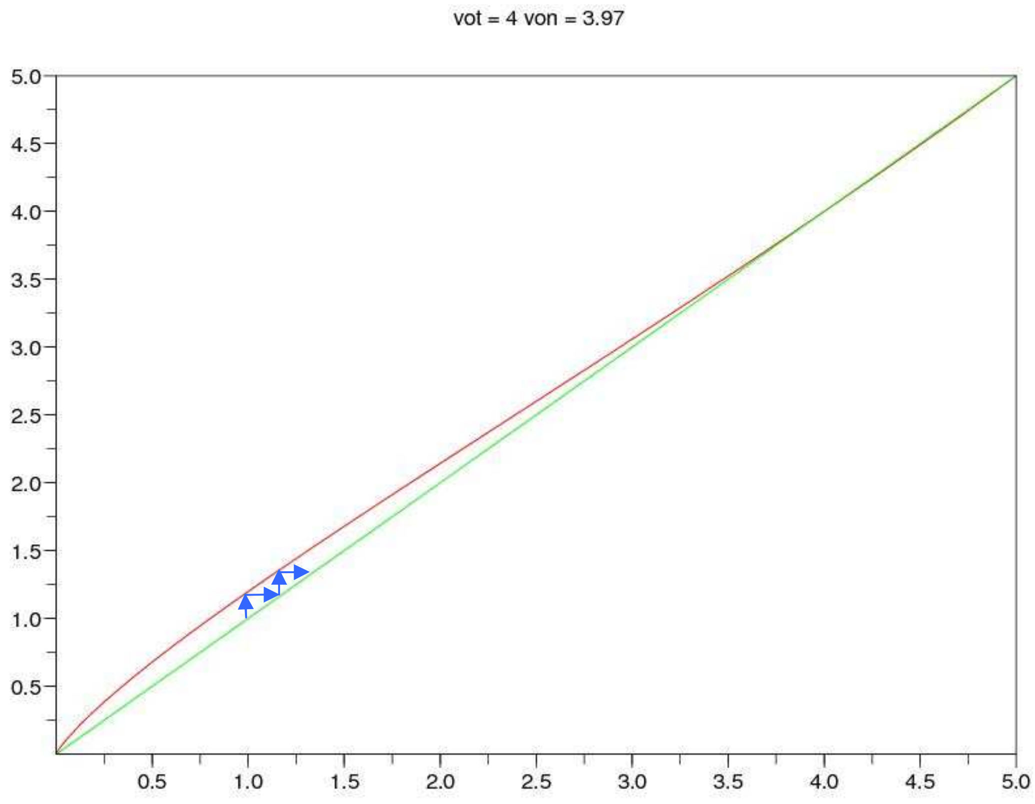


Idealized case: analysis on an equatorial circle (40 000km).

$v^o_{true} = 4.$

$L^b = 300$ km / $L^0 = 0$ km.

Convergence: $v^o_{diag}(v^o)$



Idealized case: analysis on an equatorial circle (40 000km).

$v^o_{true} = 4$.

$L^b = 300$ km / $L^0 = 200$ km.

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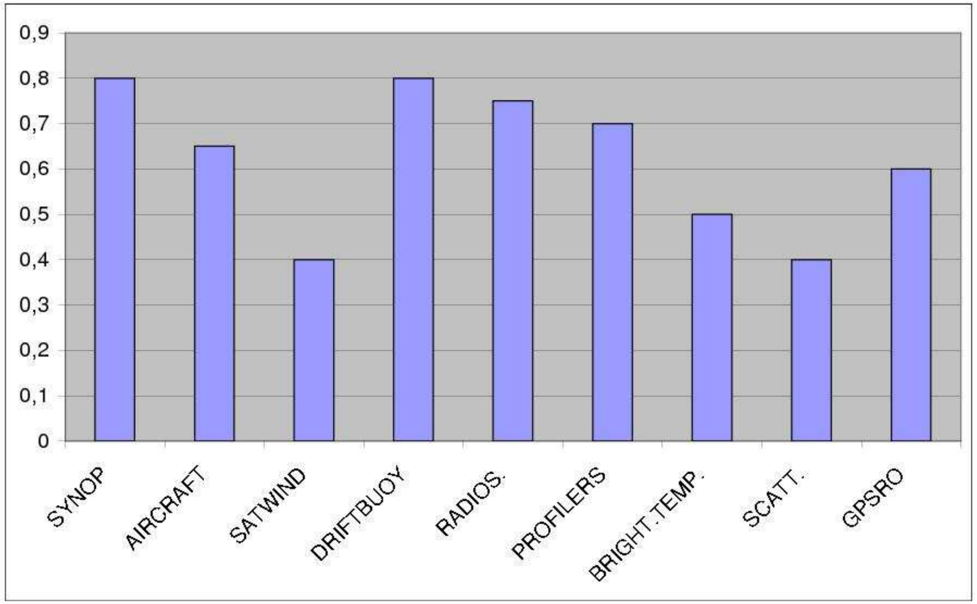
Observation error standard-deviations



Normalization of $\mathbf{R}_i$:

$s^o_i \mathbf{R}_i$

Coef. s^o_i diagnosed with

$s^o_i = E[J^o_i(\mathbf{x}^a)]/(E[J^o_i(\mathbf{x}^a)])^{opt}$.

Normalization coefficients of σ^o_i in the French Arpège 4D-Var

(Chapnik, et al, 2004; Buehner, 2005; Desroziers et al, 2009)

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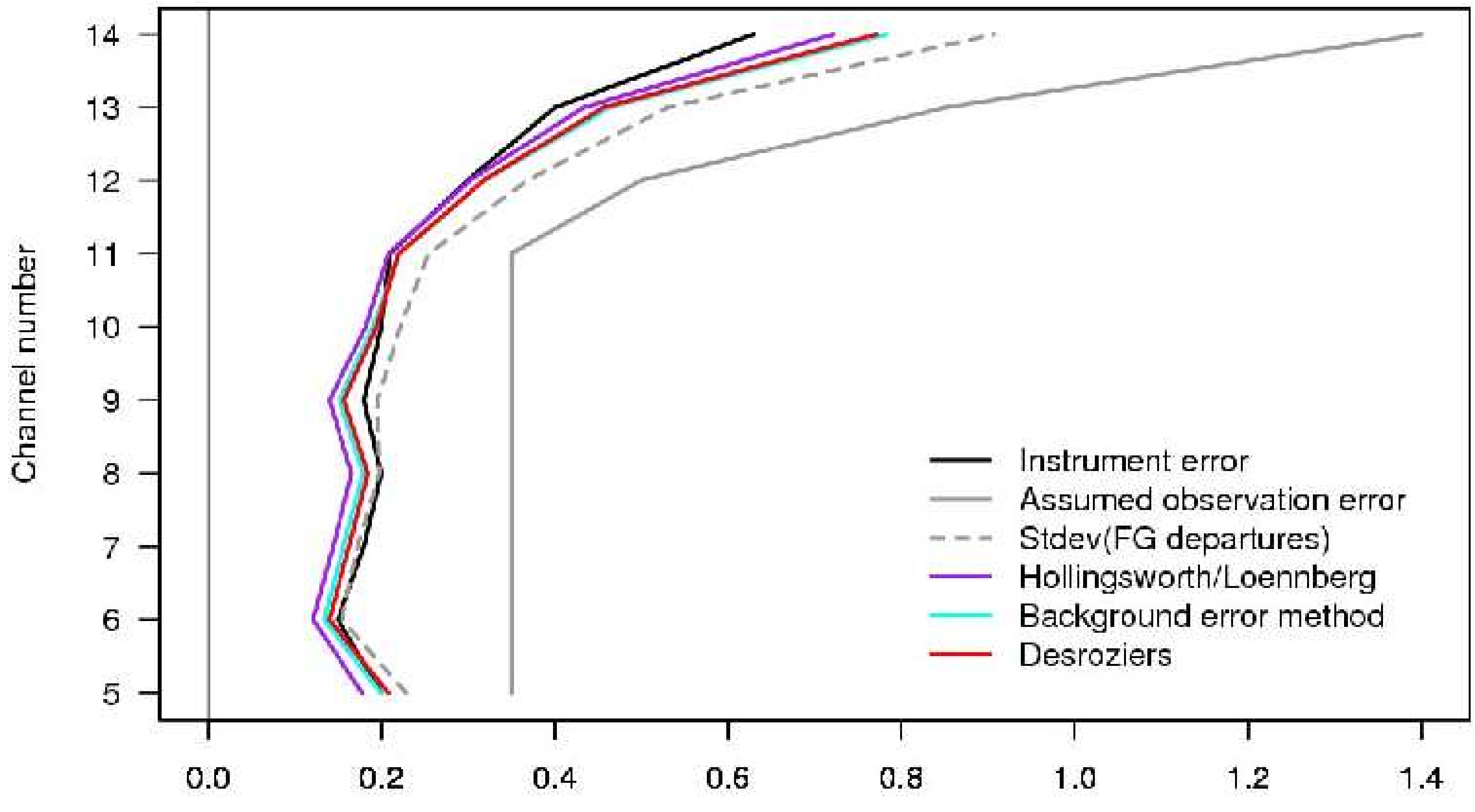
Satellite error standard-deviations

N-18 AMSU-A: Estimated observation errors (σ_O)



(Bormann et al, ECMWF, 2010)

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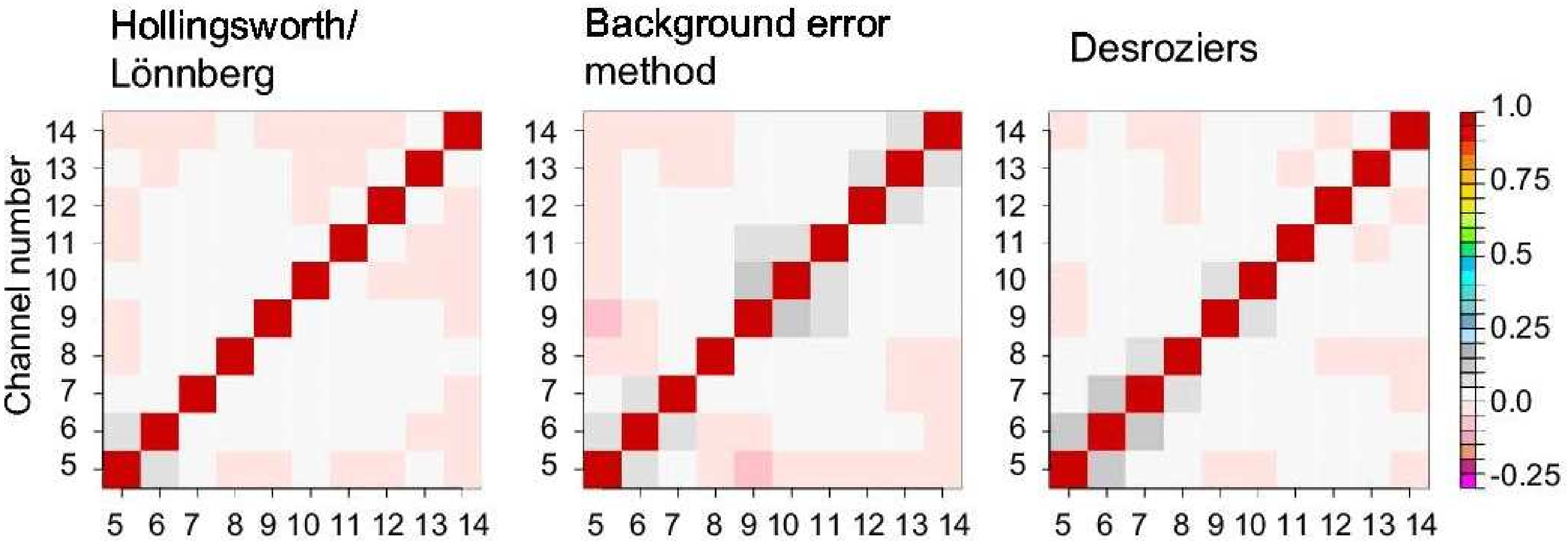
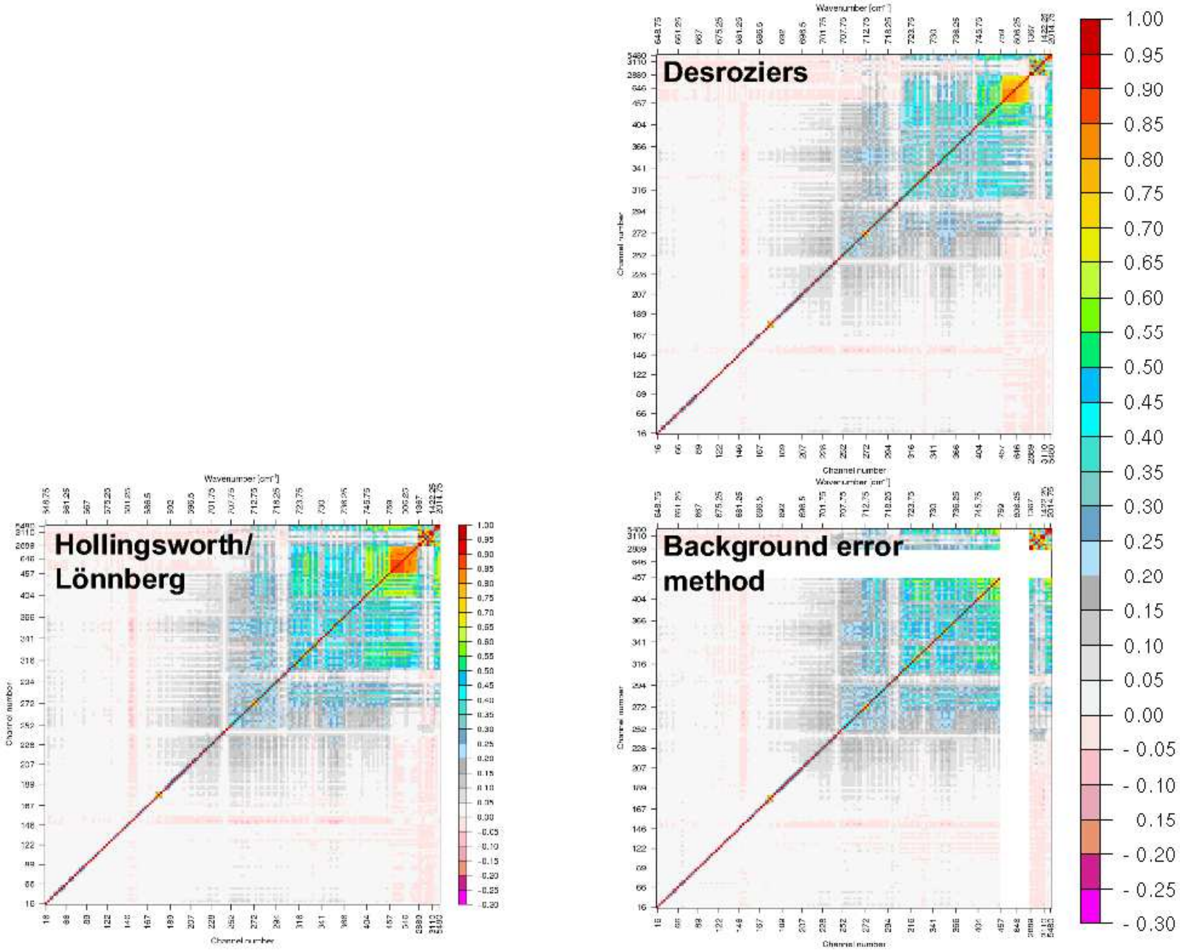
Outline

AMSU-A inter-channel error correlations



(Bormann and Bauer, ECMWF, 2010; Bormann et al, ECMWF, 2011)

IASI inter-channel error correlations



(Bormann et al, ECMWF, 2011)

IASI inter-channel error correlations

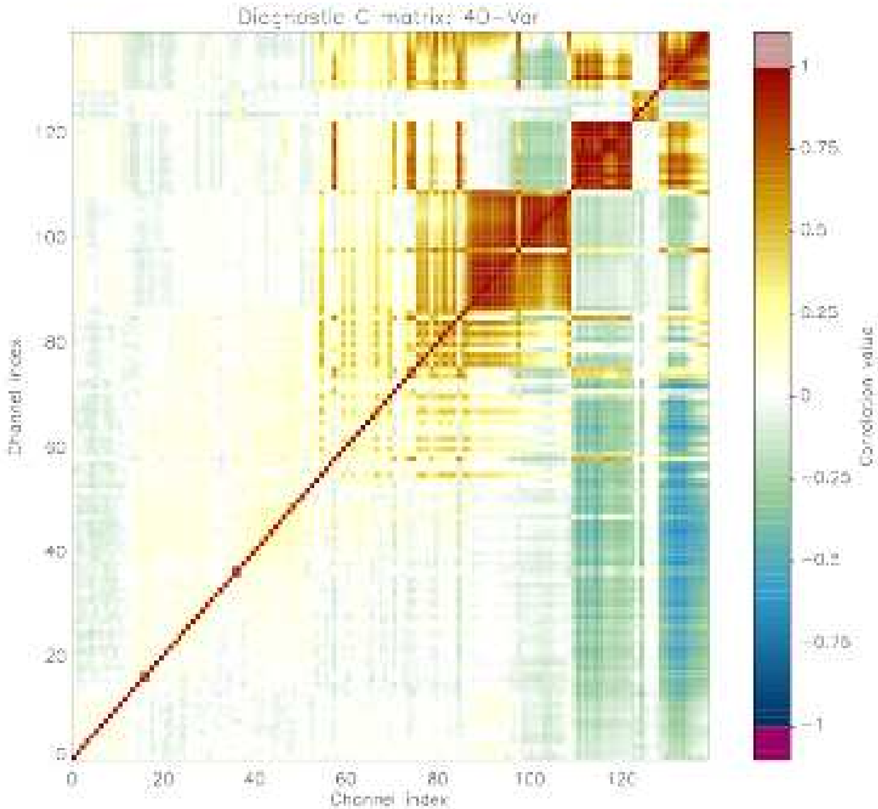
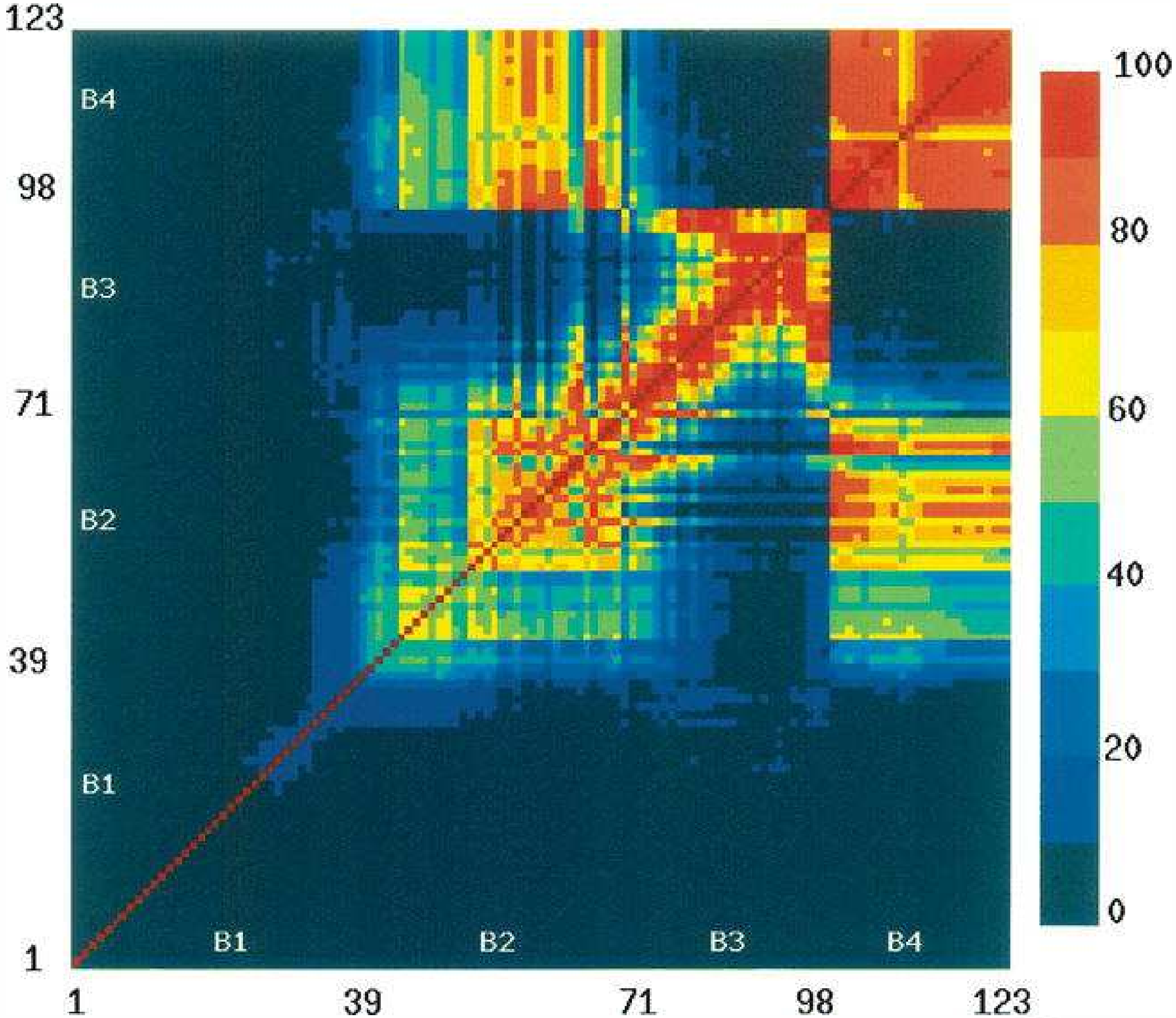


Figure: Error correlation matrix for 139 channels used in Var

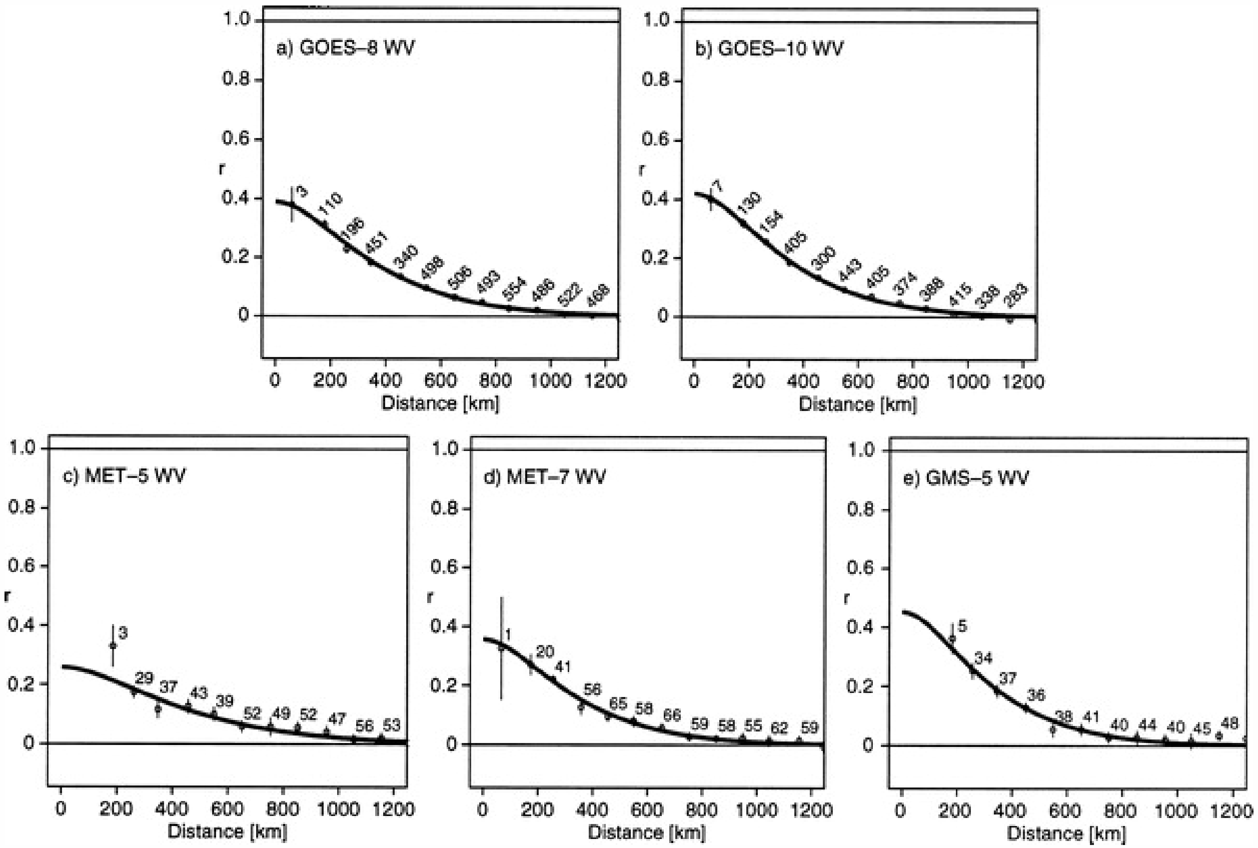
(Stewart, University of Reading, 2009)

AIRS inter-channel error correlations



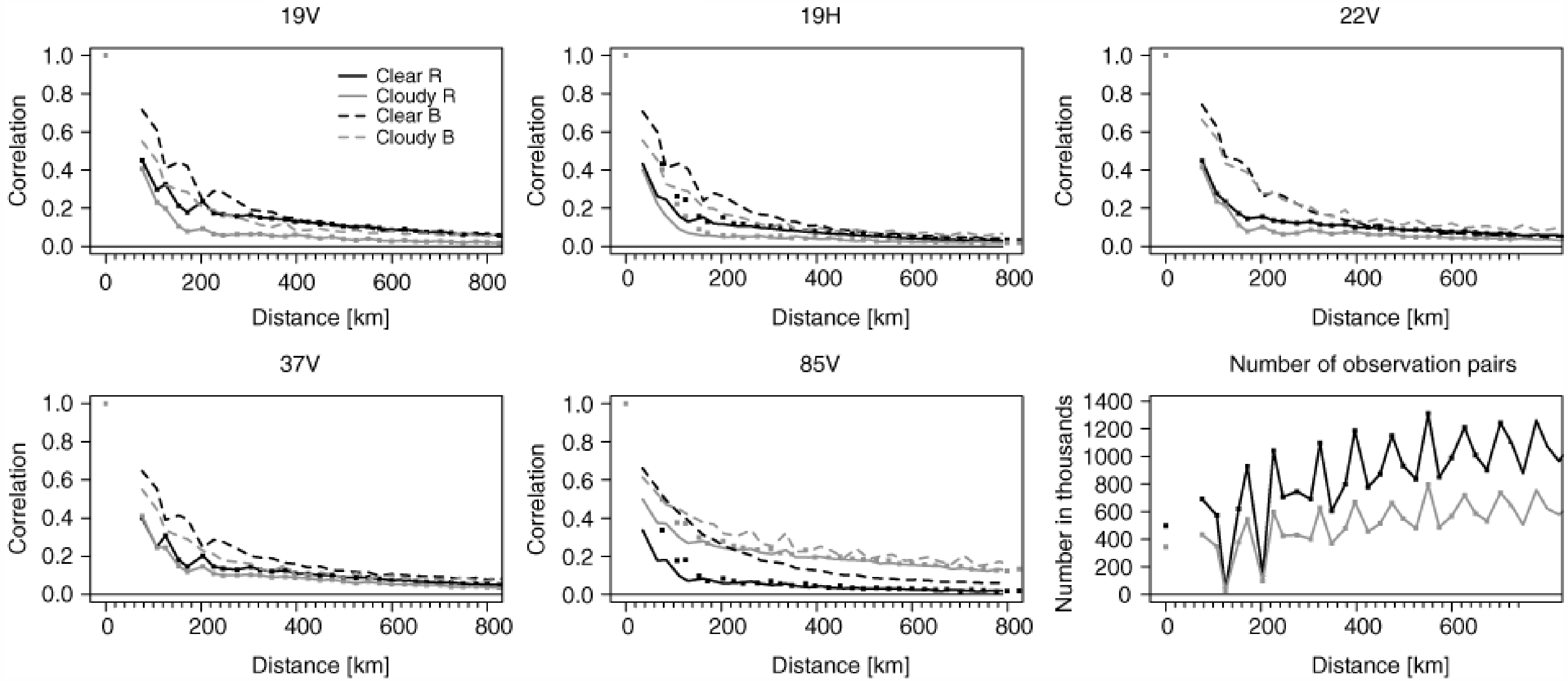
(Garand et al, Environment Canada, 2007)

AMVs spatial error correlations



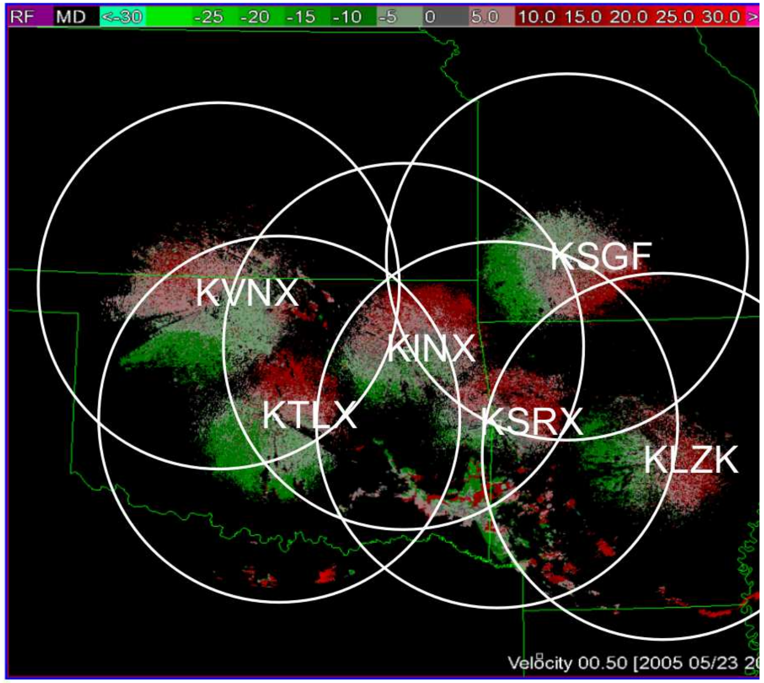
(Bormann et al, ECMWF, 2003)

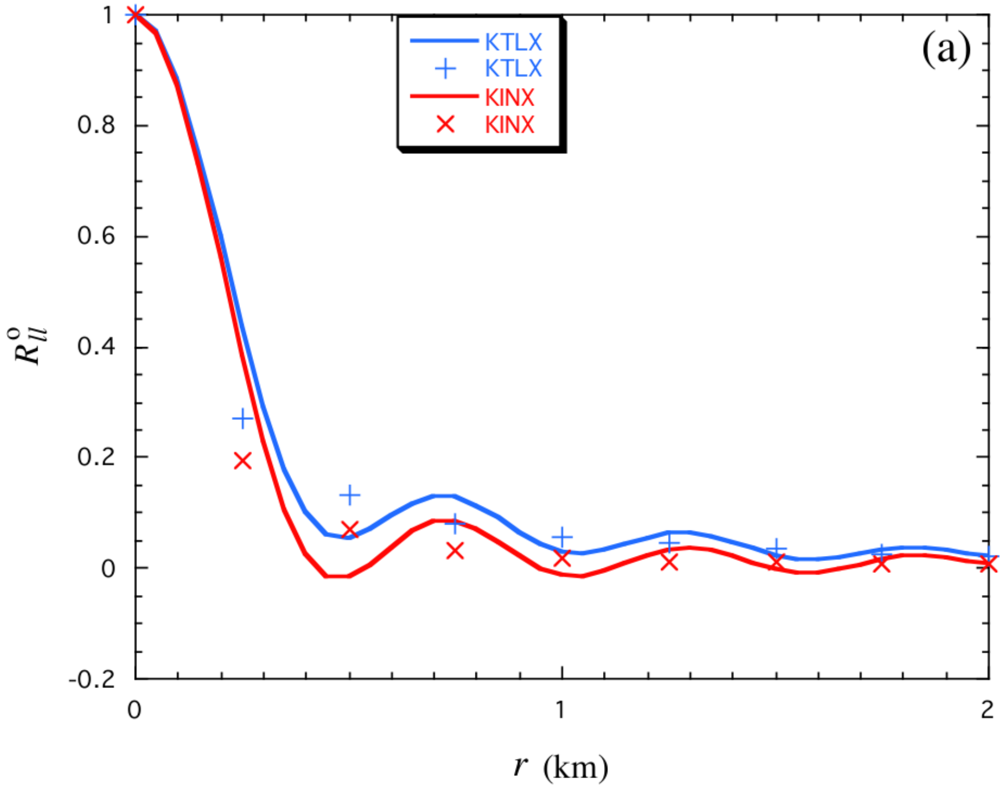
SSM/I spatial error correlations



Spatial error correlations for the F13 SSM/I
(solid lines; black: clear sample; grey: cloudy sample)
(Bormann et al, ECMWF, 2011)

Doppler radar wind spatial error correlations





Radial error correlation $R_{||}^{o}(r)$.

(Xu et al, NOAA, 2007)

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Representation of time-correlated errors in **R**

- Serial correlation for SYNOP and DRIBU in 4D-Var.
- Modelled by a continuous correlation function
  $c(t_1, t_2) = a \exp(-((t_1 - t_2) / b)^2)$ (with b = 6h).
- For observations $\mathbf{y}^o_i$ with uncorrelated observations errors,
  $J^o_i(d\mathbf{x}) = \mathbf{z}_i^T \mathbf{z}_i$, with $\mathbf{z}_i = \mathbf{S}_i^{-1}(\mathbf{y}^o_i - H_i(\mathbf{x}^b) - \mathbf{H}_i\, d\mathbf{x})$,
  (departures normalized by the standard-dev. of obs. errors).
- For observations $\mathbf{y}^o_i$ with time-correlated observations errors,
  computation of « effective » departures $\mathbf{z}^{eff}_i$,

  by solving the linear system of equations $\mathbf{z}^{eff}_i\, \boldsymbol{C} = \mathbf{z}_i$.

  (Järvinen et al, ECMWF, 1999)

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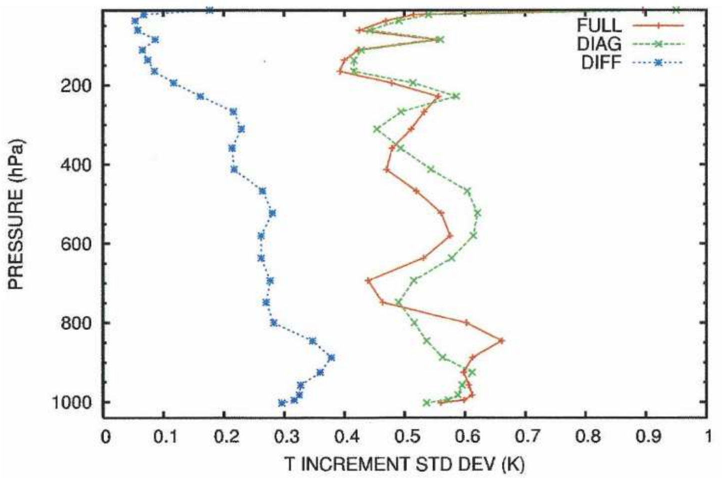
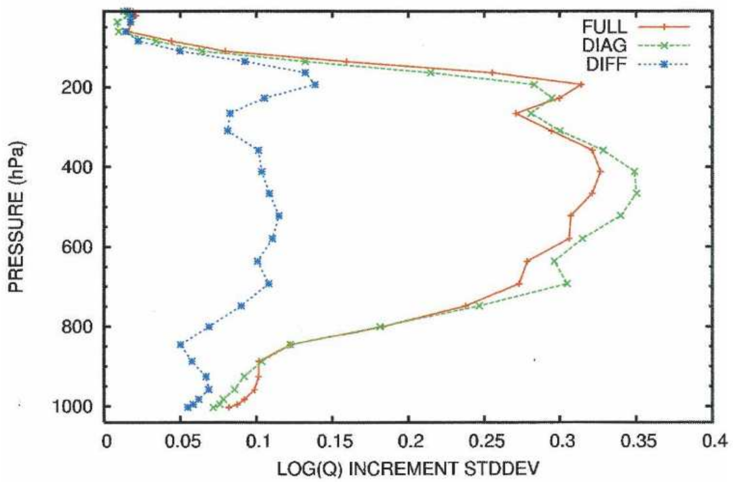
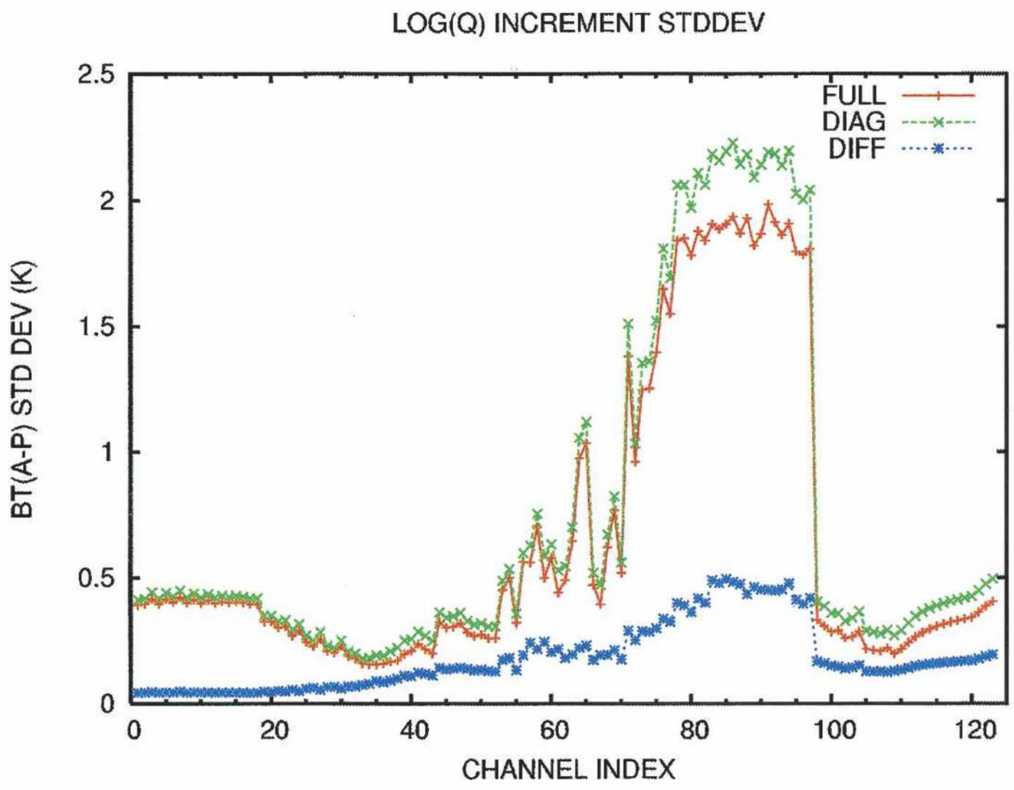
Representation of inter-channel error correlations in **R**







(Garand et al, Environment Canada, 2007)

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Representation of spatial error correlations in **R**

- Construction of a square-root correlation model for a block $\mathbf{R}_i = \Sigma_i^{-1} \mathbf{C}_i \Sigma_i^{-T}$ of **R** with horizontal correlations (Σ_i^{-1} normalization by standard-dev., $\mathbf{C}_i$ correlation matrix)

- $\mathbf{C}_i = \mathbf{U}_i \mathbf{U}_i^T$

- $\mathbf{U}_i = \mathbf{T}_i \mathbf{S}_i^{-1} \mathbf{G}_i^{1/2}$, where

  $\mathbf{G}_i$ is the spectral (Hankel) transform of the correlation function,
  $\mathbf{S}_i^{-1}$ is the inverse spectral transform (with a low, but sufficient resolution to represent the spatial correlation),
  $\mathbf{T}_i$ is an interpolator to observation locations.

(Fisher and Radnoti, ECMWF, 2006)

Representation of spatial error correlations in **R**

- Very useful, at this stage, to represent realistic perturbations for observation errors in EnKF / En Var assimilation:

  $\varepsilon^a = (\mathbf{I} - \mathbf{KH})\, \varepsilon^b + \mathbf{K}\, \varepsilon^o$, with

  $\varepsilon^o = \mathbf{R}^{t\ 1/2}\ \eta^o$ where η^o is a vector of random numbers,

  even if $\mathbf{R}^t$ is not used in **K**.

  (Fisher et al, ECMWF, 2003)

- Used in operational implementations of Ensemble Variational Assimilation

  (Berre et al, Météo-France, 2007; Isaksen et al, ECMWF, 2010).

Representation of spatial error correlations in **R**

- Approximation of $\boldsymbol{C}_i$ by

  $\boldsymbol{C}_i = \Sigma_1^K (\lambda_{i,k} - 1)\, \mathbf{v}_{i,k}\, \mathbf{v}_{i,k}^T$,

  where only a limited number K of eigenpairs ($\lambda_{i,k}$, $\mathbf{v}_{i,k}$) of $\boldsymbol{C}$ is used.

- The eigenpairs of $\boldsymbol{C}$ can be determined by a Lanczos algorithm.

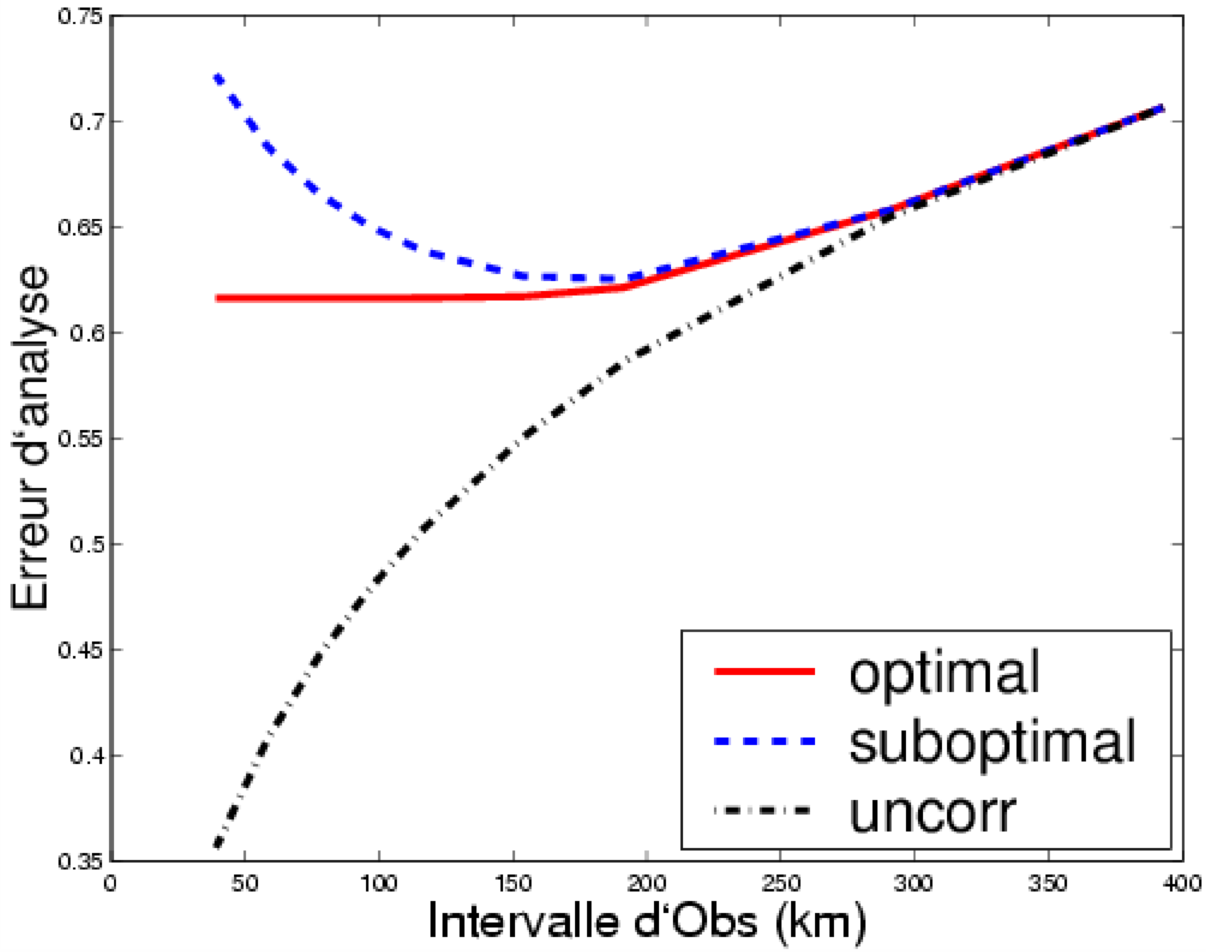
- Approximation of $\boldsymbol{C}^{-1}$ given by

  $\boldsymbol{C}_i = \Sigma_1^K (1/\lambda_{i,k} - 1)\, \mathbf{v}_{i,k}\, \mathbf{v}_{i,k}^T$.

- Computation of effective normalized departures $\mathbf{z}^{eff}_i$,
  with $\mathbf{z}^{eff}_i = \boldsymbol{C}_i^{-1} = 1/a_i\, \mathbf{z}_i + \mathbf{S}_1^K (1/\lambda_{i,k} - 1/a_i)\, \mathbf{v}_{i,k}\, (\mathbf{v}_{i,k}^T \mathbf{z}_i)$,
  where a_i is a parameter accounting for the truncation K.

(Fisher and Radnoti, ECMWF, 2006; Isaksen and Radnoti, ECMWF, 2010).

Representation of spatial error correlations in **R**



Δs = 100 km

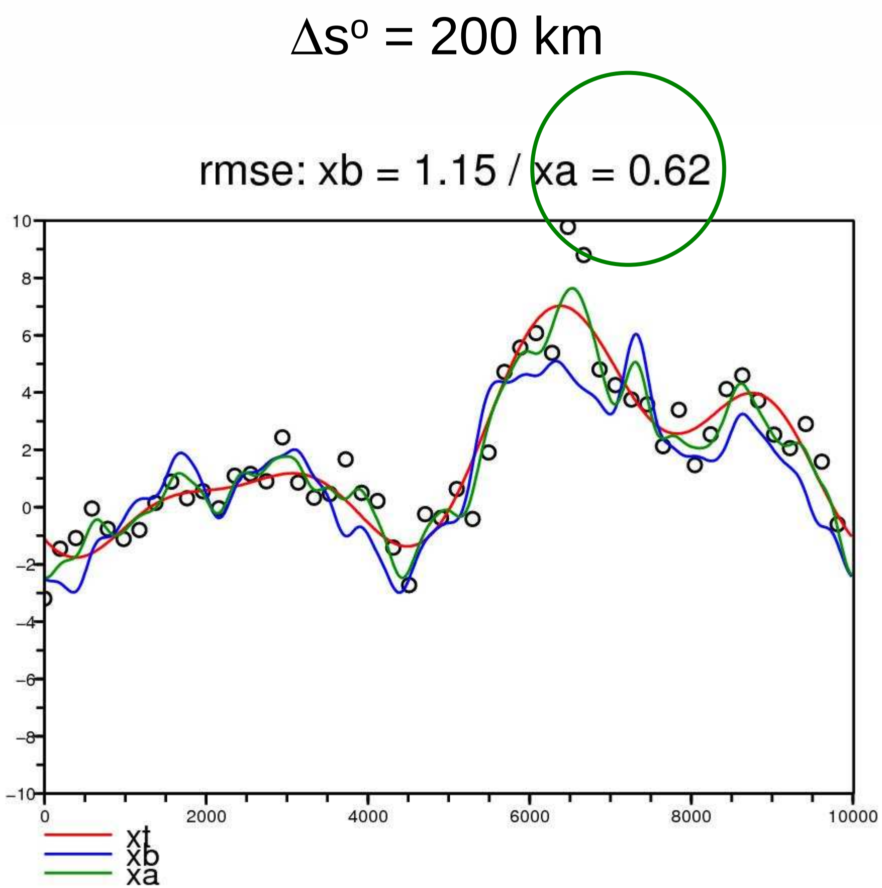
L^b = 200 km

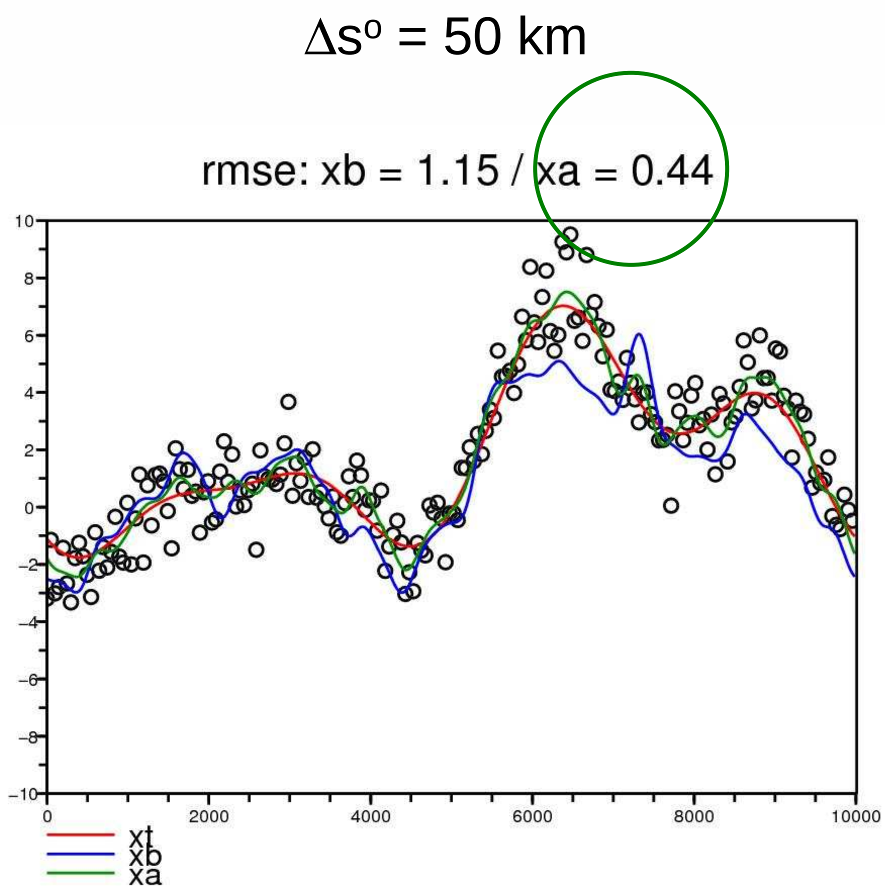
L^o = 100 km

$\sigma^b = \sigma^o = 1$

(Liu and Rabier, Météo-France, 2002)

Representation of spatial error correlations in **R**

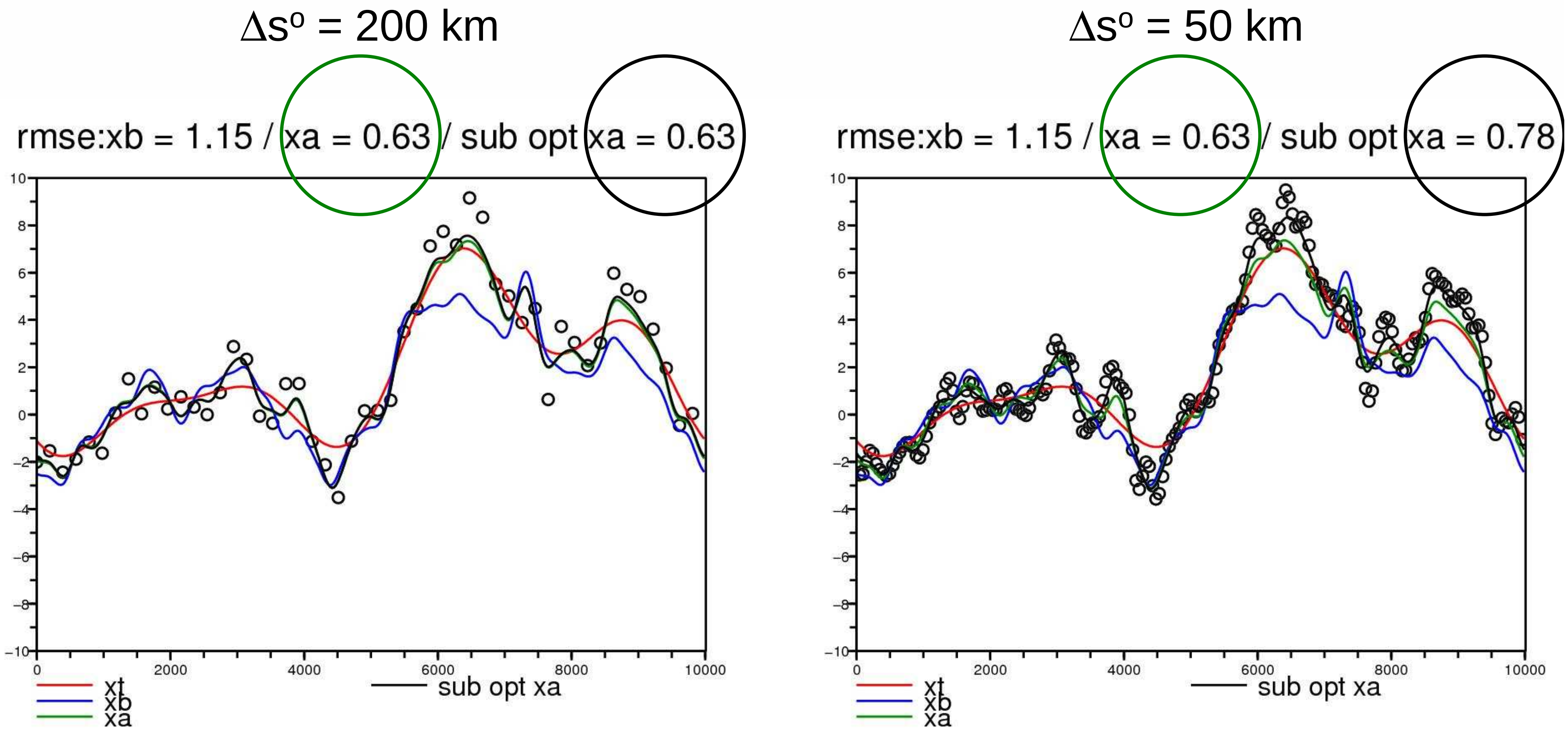




No spatial correlation in observation errors: L^b = 200 km, L^o = 0 km

$s^b = s^o = 1$, Δs = 25 km

Representation of spatial error correlations in **R**



Spatial correlation in observation errors: L^b = 200 km, L^o = 100 km
$s^b = s^o = 1$, Δs = 25 km

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Conclusion (I)

- Observation errors are not explicitely known.

- They can be inferred by a comparison with
  other observations or with the background (innovations).

- Available diagnostics of observation errors (variances and correlations),
  but relying on explicit or implicit hypotheses.

- Observation error variances are classicaly inflated.

- Correlation of observation errors can be found in many datasets:

✓ SYNOP time-correlations,
✓ AIRS, IASI inter-channel correlations,
✓ AMVs, SSM/I, radar spatial correlations.

Conclusion (II)

- Observation error correlations are often neglected,
  but with an empirical thinning and/or an inflation of error variance.
- Correlations can be more or less easily taken into account.
- A relevant formulation for spatial error correlation has been proposed and implemented in a real size system (ECMWF).
- Algorithms without $\mathbf{R}^{-1}$: PSAS, saddle-point formul. (Fisher, 2011)?
- One has to keep in mind that correlated observations are less informative than uncorrelated observations, even if **R** is well specified.
- It may thus appear inefficient to add too many correlated observations.
- The tuning of **R** must be consistent with the tuning of **B**.