

# Representing convection in models - How stochastic does it need to be?

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## 1. Introduction

The representation of convection in large-scale models remains one of the most difficult problems in atmospheric science. This is due to the need for this representation to be of conceptual nature as convective processes act on scales much smaller than the grid-spacing of global weather and climate models. As a consequence, convective processes need to be parametrized, i.e., the behaviour of variables on the (relatively) small convective scales must be described in relation to that of the variables on the (relatively) large scales resolved by the model. The ideas and concepts that underpin parametrizations of convection date back more than 40 years and were originally developed for descriptions of tropical cyclones (e.g., Ooyama (1969), Kuo (1965)) before being introduced in a more general context (e.g., Arakawa and Schubert (1974), Kuo (1974)). The basic ideas of traditional convection parametrization as well as recent developments are summarized in the review by Arakawa (2004).

The parametrization of convection is inherently complex and it is perhaps not surprising that many of the existing shortcomings in contemporary global models ranging from errors in the mean climate (e.g., Zhang et al. (2007)) to the models inability to simulate modes of tropical variability (e.g., Lin et al. (2006)) have been linked to shortcomings in the representation of convection. Many improvements in model behaviour are also often associated with changes to the convection parametrization. Recent examples range from improved El Niño / Southern Oscillation behaviour (Neale et al. (2008)) to improved extratropical forecast performance (Bechtold et al. (2008)).

A recent criticism leveled at the model descriptions of convection is that they are behaving in what is termed a “deterministic” fashion, in that identical large-scale states (if they could be found) would result in identical corresponding convective states, when this is unlikely to be true in the real world. All traditional parametrizations, not just those of convection, behave in this way leaving no room for variability around the parametrized mean relationship between scales. Several studies using cloud-resolving models (CRMs) have investigated the large- to small-scale relationships and have identified significant variability around the mean relationships traditionally used in parametrizations (e.g., Cohen and Craig (2006), Shutts and Palmer (2007), Plant and Craig (2008)). Attempts to alleviate the limitations of traditional deterministic parametrizations date back to the introduction to what is usually referred to as “stochastic physics” in the ECMWF model by Buizza et al. (1999). They showed that

the application of a multiplicative noise to the tendencies from physical processes improved the spread-skill relationship of the ECMWF ensemble prediction system. Several approaches to stochastic parametrizations have since been proposed most of which target in particular the behaviour of convection. Those approaches range from the introduction of a stochastic backscatter from small to large scales (e.g. Berner et al. (2009)), the use of empirical relationships to adjust the behaviour of the convection parametrization (e.g., Lin and Neelin (2003)), the use of a Markov chain lattice to stochastically describe the evolution of convective cloud types in a grid-cell (e.g., Khouider and Majda (2006)) to a fully stochastic description of convection (e.g., Plant and Craig (2008)). It is worth noting that all of the efforts so far rely on either ad-hoc empirical relationships or on the use of other models, most prominently CRMs, to study the degree of stochasticity in the large-scale to convection relationship. To our knowledge, none of the existing stochastic physics approaches has used observations in its derivation or evaluation. It is the main purpose of this study to alleviate this severe limitation in the field of stochastic physics.

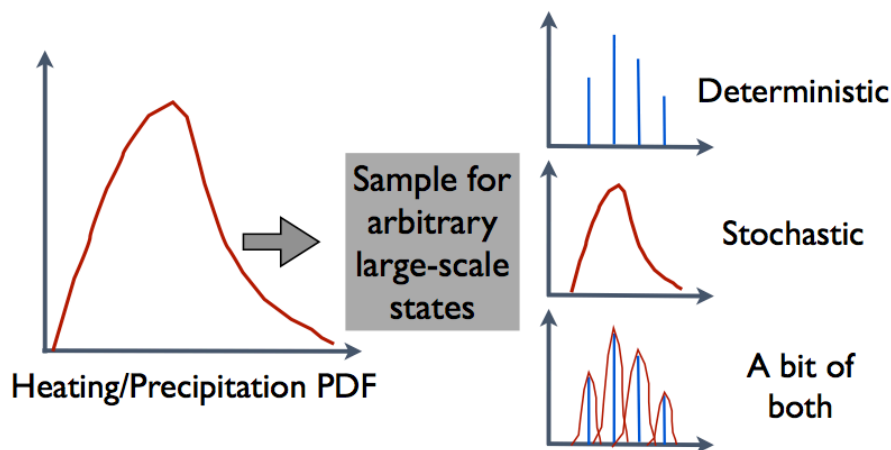


Figure 1: Schematic of the definition of the terms “deterministic” and “stochastic” for the purpose of this study. See text for details.

Before deriving an observational data set that is suitable to investigate the relationships between convective and large scales it is worth clarifying what exactly we mean when classifying behaviour into deterministic and stochastic. While the words “stochastic physics” are now in wide-spread use, their meaning is not always clear as many if not all “stochastic” physics schemes acknowledge the presence of deterministic relationships. For instance the multiplicative approach of Buizza et al. (1999) relies first on the existence of a non-zero tendency from physical processes derived with a traditional, and hence deterministic, set of parametrizations. Figure 1 schematically shows how the terms “deterministic” and “stochastic” will be interpreted throughout this study. The left hand panel of the figure shows a distribution function of a small-scale variable, such as convective precipitation or convective heating. It is assumed that this distribution function has been derived from a large sample of cases collected in either space or time, each being representative for an area equivalent to a model grid-box. To identify if this distribution is the result of a deterministic or stochastic relationship with larger scale processes means to sub-sample the distribution for classes of large-scale state. In the case of convection one can imagine that the large-scale state might be described by dynamic and/or thermodynamic characteristics, such as convergence/divergence or measures of atmospheric stability,

such as Convective Available Potential Energy (CAPE). It is assumed in this schematic that each large-scale state can be sampled many times. If the scale-relationship is deterministic, each large scale state would correspond to exactly one small scale state and the overall distribution would be the result of a large number of delta functions, one for each large-scale state (top right). If on the other hand, the relationship was entirely stochastic, the (normalized) distribution for each large-scale state would resemble the overall distribution, indicating that the large-scale state has no influence at all on the small-scale behaviour (middle right). A third possibility is that there is a mixture of deterministic and stochastic behaviour in the scale-relationship, so that each large-scale state corresponds to a distribution of small-scale states which changes with the large-scale state itself (bottom right).

It is a key purpose of this paper is to investigate from observations which, if any, of the conceptual models outlined in Figure 1 is appropriate to describe the large-scale to convection relationship in the tropics. We also aim to quantify some of the aspects of the relationships and in particular we investigate the validity of some common assumptions in existing stochastic physics approaches. Section 2 describes the derivation of the observational data sets necessary to carry out this study. Section 3 provides insight into some basic relationships between large and small scales in a convecting atmosphere. Section 4 focuses on identifying just how much stochasticity there is in the scale-relationships. A particular focus of this section will be to highlight how the relative importance of deterministic to stochastic behaviour can be a strong function both of the choice of model to describe that relationship, as well as the large-scale state itself. Section 5 will provide the main conclusions of this study.

## **2. A long-term data set to study large-scale to small-scale relationships in a convecting atmosphere**

The use of observations to study the relationships between large and small scales requires concurrent long-term data sets of the relevant variables on both scales. For the study of convection such data sets need to include both thermodynamic and dynamic variables, in particular large-scale convergence and divergence patterns, as well as a description of the statistical distribution of convective properties within the large-scale area.

Providing reliable estimates of the large-scale variables required to study tropical convection has proven challenging. The most common and most readily available source of large-scale information are analyses provided by Numerical Weather Prediction (NWP) centres, such as the commonly used reanalysis products (e.g., Uppala et al. (2005); Kanamitsu et al. (2002)). However, due to the lack of both observations and large-scale constraints in the analysis process, the quality of these analyses is poorest in tropical latitudes and for variables that are key to convection, such as convergence.

Another common source of large-scale information are analyses of arrays of radiosondes and associated observations as they are frequently applied during field experiments (e.g., Yanai et al. (1973); Houze and Betts (1981); Ciesielski et al. (1997)). A particularly powerful method to analyze such data is the variational budget analysis approach developed by Zhang and Lin (1997), as it combines the use of radiosonde information with surface and top-of-the atmosphere (TOA) observations that are used to constrain the vertically integrated heat and moisture budgets. It has been shown that in particular the use of surface precipitation as a constraint improves the analysis quality significantly (Zhang et al. (2001)). Unfortunately, much of the large-scale information derived using radiosonde arrays is limited to short periods of infrequent field experiments.

Xie et al. (2004) developed a hybrid approach that used NWP analysis data as a surrogate for radiosonde observations and combined this information with surface and top-of-the-atmosphere (TOA) observations taken at the Atmospheric Radiation Measurement program's (ARM, Ackerman and Stokes (2003)) site in the U.S. Southern Great Plains (SGP) using the variational technique of Zhang and Lin (1997). They demonstrated that for this extratropical location this hybrid approach can successfully provide estimates of the large scale state of the atmosphere for long and continuous periods of time, provided that long-term observations of surface precipitation and TOA radiation are available.

Here, we extend the hybrid approach of Xie et al. (2004) to a tropical location. We make use of the availability of long-term, high-quality radar observations at Darwin, Australia. The radar observations are taken by a C-band polarimetric research radar (CPOL, Keenan et al. (1998)) located at Gunn Point and have been converted to surface rainfall estimates making full use of the polarimetric observations by using the algorithm of Bringi et al. (2004). The advantage of the use of radar data over rain gauges lies in the superior spatial sampling. Its disadvantage is in the need to retrieve rainfall from the radar observations, which in the case of CPOL is somewhat mitigated by the additional measurements from the polarimetric capabilities of the radar.

To test the suitability of our methodology, we first apply it to the data taken during the Tropical Warm Pool International Cloud Experiment (TWP-ICE, May et al. (2008)), which took place in the Darwin area in 2006. This allows us to compare the results of an analysis that uses the extensive TWP-ICE radiosonde array (detailed in Xie et al. (2010)) with those from an experiment where the radiosonde data, specifically vertical profiles of zonal and meridional wind, temperature and specific humidity, are replaced by grid-point information from the ECMWF operational analysis.

Figure 2 shows time-height slices of the area-averaged vertical velocity from the radiosonde-based TWP-ICE analysis (top), the hybrid variational approach using ECMWF profiles and radar-derived rainfall (middle), and the ECMWF operational analysis (bottom). It is evident that the ECMWF operational analysis does not provide a good match with the best-estimate of vertical velocity derived from the full TWP-ICE data set. While there is some qualitative agreement in the early period, which was dominated by the presence of a strong monsoon trough, the less active middle period as well as the strongly diurnally varying final period of the experiment are not well-captured. Using ECMWF profiles as a radiosonde surrogate as well as the radar rainfall observations in the variational technique leads to very good agreement with the analysis using all the observations. Encouraged by these results we apply the hybrid variational approach to three wet seasons at Darwin (2004/05; 2005/06; and 2006/07) providing us with reliable estimates of the large scale atmospheric state for approximately 1900 six-hourly samples.

To study the large-scale to convection relationships requires a data set that describes the statistical properties of convection in the area for which the large-scale is known. Here we take advantage of the CPOL radar once again. First, we use the algorithm of Steiner et al. (1995) to separate the rainfall field into its convective and stratiform components. Having done so we can calculate area-averaged convective and stratiform rain rates, the convective and stratiform fractional rain area as well as convective and stratiform rainfall intensities (defined as rainfall per unit raining area). As convective rainfall and convective area (through its inclusion in the convective mass-flux) are key variables in convective parametrization schemes, we proceed to investigate the large- to small-scale relationships using those variables. Note that we consider this a starting point of our investigation and we are

actively pursuing the derivation of further small-scale variables from the radar observations, including the vertical velocity in convective updraughts.

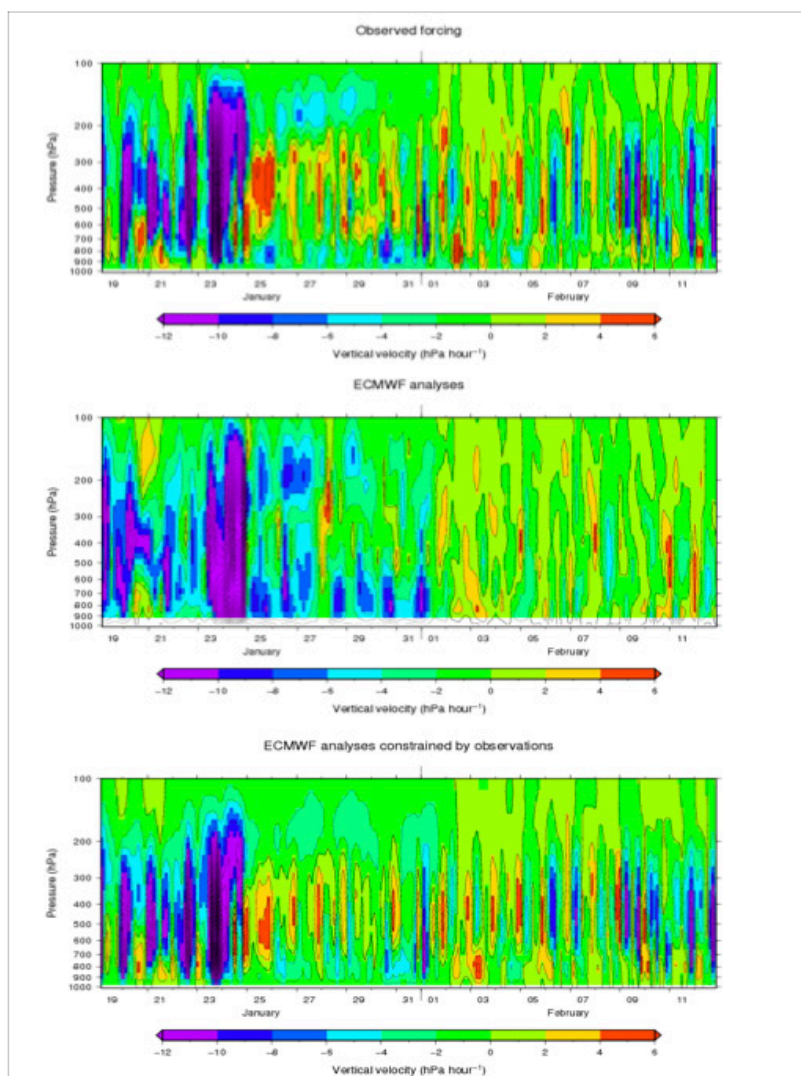


Figure 2: Time-height cross-section of vertical velocity using all TWP-ICE observations (top), from direct output of the ECMWF analysis (middle) and using the hybrid approach (bottom). See text for details.

### 3. Some basic relationships between large and small scales in a convecting atmosphere

We use the concurrent data sets of large and small scales derived in the previous section to study some basic relationships between processes acting on the two scales. These relationships will then guide our discussion on the relative importance of deterministic (“signal”) and stochastic (“noise”) elements in these relationships in the next section. Figure 3 shows the relationship between area-mean convective rainfall and vertically integrated moisture convergence and CAPE.

It is evident that there is quite a strong relationship between moisture convergence and convective rainfall while the relationship with CAPE is rather poor. Strong moisture convergence is associated with larger amounts of convective rainfall, while little to no rainfall is observed when moisture convergence is negative, indicating a net divergence of moisture from the domain and hence very likely subsiding conditions. While the relationship is strong, there is considerable scatter indicating at least some stochastic elements in the relationship. It is worth noting that the relationships identified in Figure 3 must not be interpreted as causal in any way. In the case of convergence and convection it is well known that strong feedbacks between the two do exist, with convective heating inducing convergence and convergence inducing convection. The fact that the relationship between convective rainfall and CAPE is poor, while not entirely surprising, is of some importance, as many of today’s cumulus parametrization schemes heavily rely on the existence of such a relationship. We will return to this issue in Section 4.

Before doing so we further investigate the nature of the relationship between rainfall and moisture convergence. The area-averaged convective rainfall,  $R_c$ , used in Figure 3 can be written as

$$R_c = \sigma I_c,$$

where  $\sigma$  is the fractional area covered by convection and  $I_c$  is the local rainfall intensity. Figure 4 shows the two quantities on the right hand side as a function of vertically integrated moisture convergence. It is evident that most of the relationship in Figure 3 is the result of a relationship between convective area and convergence, while the relationship to convective rainfall intensity is weak. It has been postulated in the past based on CRM simulations that convection may react to an increase in the strength of the large-scale “forcing” through an increase in convective area (e.g., Cohen and Craig (2006)). Figure 4 provides observational support for this hypothesis. The physical mechanisms that control this behaviour are not fully understood and will be subject to further examination.

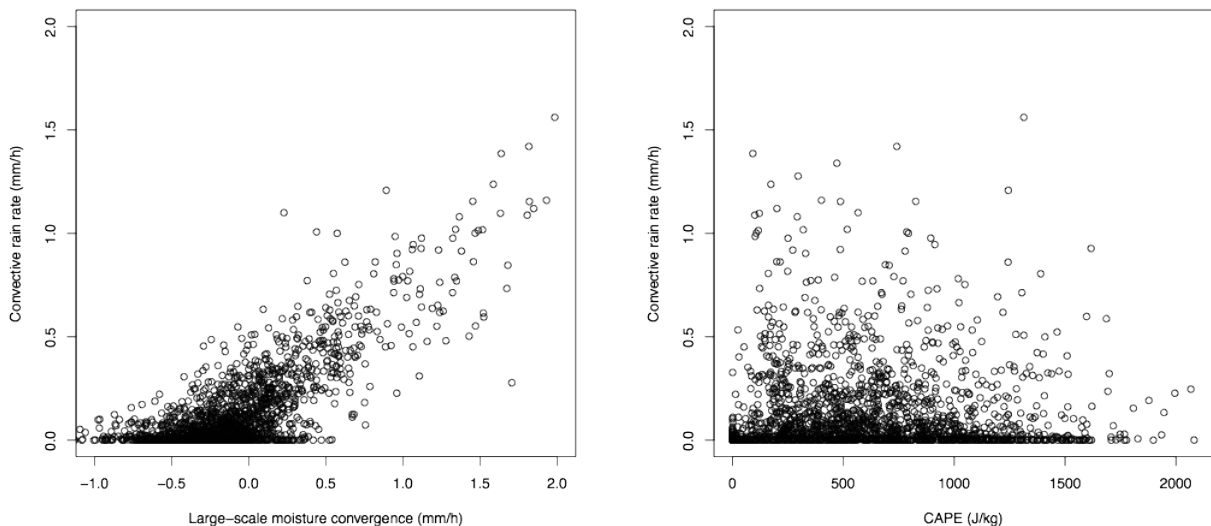


Figure 3: The relationships of 6-hourly convective rainfall and vertically integrated large-scale moisture convergence (left) and CAPE (right) for the Darwin area.

#### 4. How stochastic is convection?

In the previous section we demonstrated the utility of concurrent observations of large and convective scales derived for a GCM grid-box size area around Darwin in Northern Australia. Figures 3 and 4 illustrate that i) there are potentially strong relationships between the large and convective scales; and ii) that none of these relationships is entirely free of some “stochastic” variations around the “mean” relationship. This justifies the notion that conceptual models of convection as they are used in parametrizations should contain a stochastic component. However, the figures also illustrate that the strength of the “deterministic” component in the relationship is a strong function of the model variables. In Figure 3 it is immediately obvious that a model based on CAPE behaves more stochastically than one based on moisture convergence. We will investigate this issue further by considering the relationships between vertical motion in pressure coordinates, whose vertical derivative is directly related to convergence through the continuity equation, and the apparent heat source,  $Q_1$ , whose vertical integral is directly related to rainfall and which has been shown to be strongly related to convective heating. Here,  $Q_1$  is defined in the traditional way (e.g., Yanai et al. (1973)).

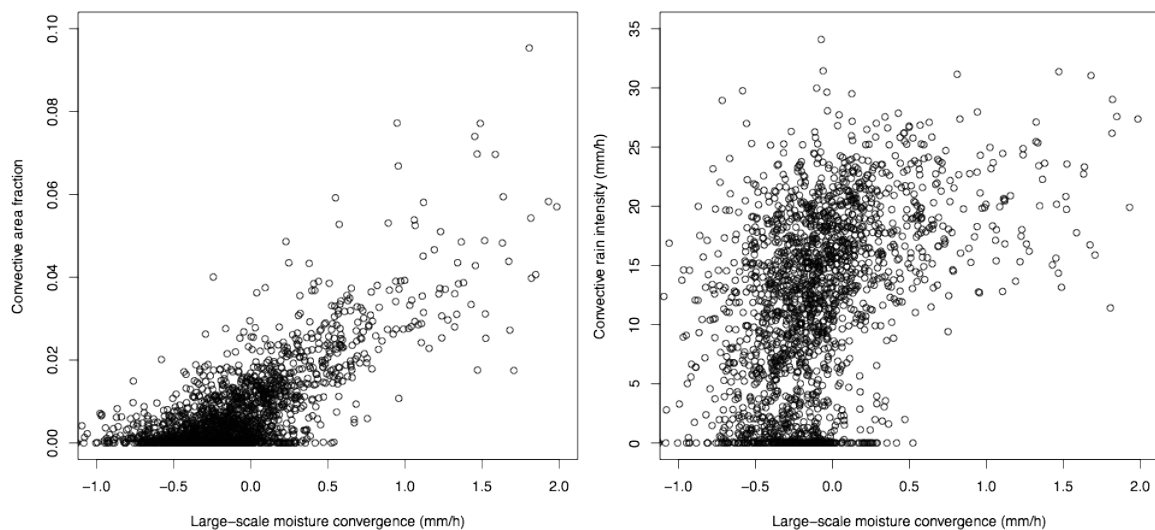


Figure 4: The relationships of 6-hourly vertically integrated large-scale moisture convergence and fractional area covered by convection (left) and local convective rainfall intensity (right) for the Darwin area.

Figure 5 shows a selection of so called beanplots (Kampstra (2008)), which are used to qualitatively illustrate the shape of a sample distribution. In particular the plot allows to depict a number of such distributions at the same time. We show distributions of apparent heat source  $Q_1$  as a function of CAPE (top panels) and pressure vertical velocity at 500 hPa,  $\omega_{500}$  (bottom panels). The left panels of Figure 5 show the distributions of  $Q_1$  as a function of pressure for the lower (red) and upper (blue) terciles of CAPE and upward (red) and downward (blue)  $\omega_{500}$  respectively. When  $Q_1$  is separated into high and low CAPE, there is no distinguishable difference in its distributions at any level of the atmosphere. In contrast, if  $Q_1$  is separated using  $\omega_{500}$  terciles a strong separation between upward and downward motion is immediately apparent. Apart from the lowest level, downward  $\omega_{500}$  is associated with diabatic cooling, most likely from radiation. Upward  $\omega_{500}$  on the other hand shows distinctly different distributions of heating, with strong heating in the middle and upper troposphere and a low

level distribution that is slightly skewed towards cooling, most likely from convective downdrafts and the evaporation of precipitation.

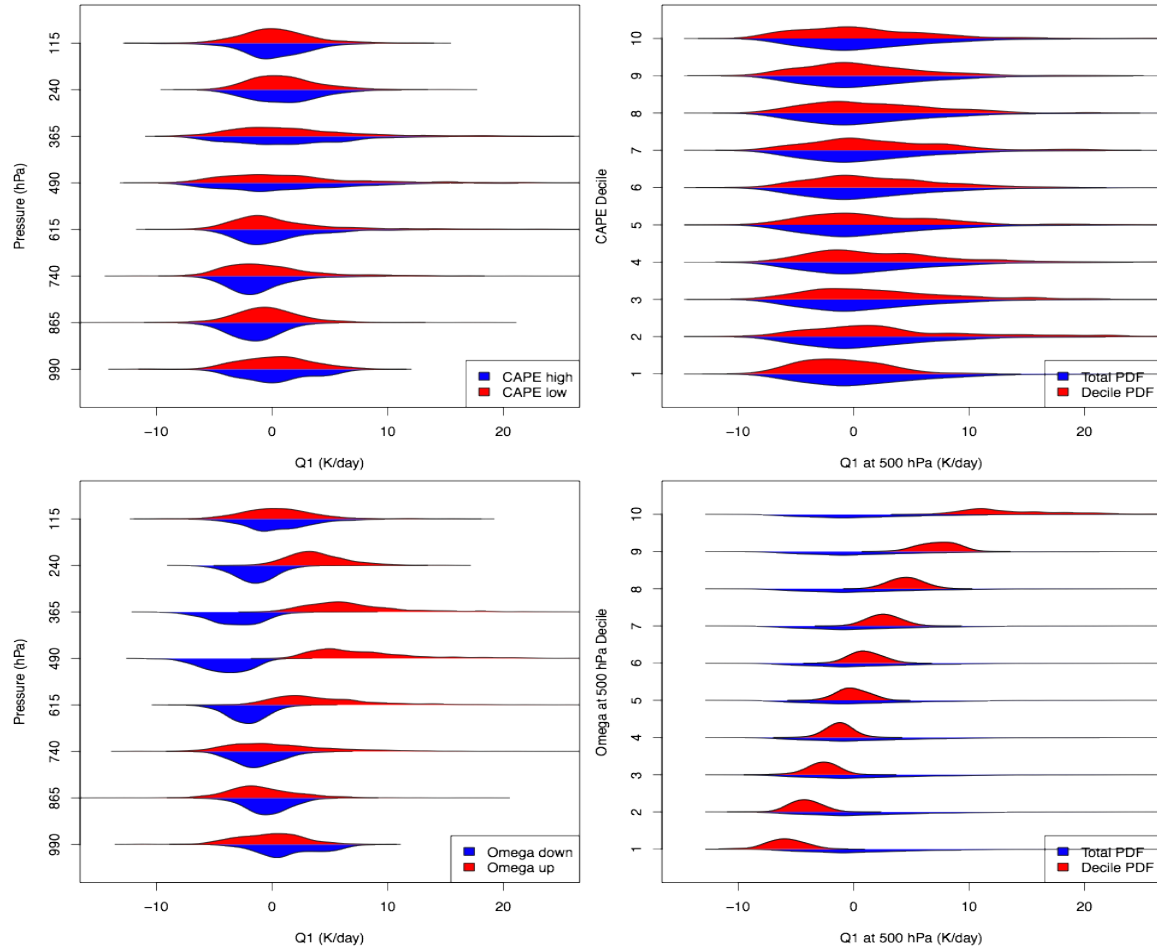


Figure 5: Beanplots of the relationships of the apparent heat source  $Q_1$  and CAPE (top panels) and pressure vertical velocity at 500 hPa,  $\omega_{500}$ , (bottom panels). The left panels depict the estimated shape of the  $Q_1$ -distribution as a function of pressure for the high (blue) and low (red) CAPE terciles (top) and downward (blue) and upward (red)  $\omega_{500}$  terciles (bottom). The right panels depict  $Q_1$  at 500 hPa as a function of CAPE (top) and  $\omega_{500}$  (bottom) deciles. The total  $Q_1$ -distribution is depicted in blue, while the distribution within each decile is depicted in red.

The right panels on Figure 5 show  $Q_1$  at 500 hPa separated into deciles of CAPE (top) and  $\omega_{500}$  (bottom). In these panels, the lower (blue) distributions represent the total distribution of  $Q_1$  at 500 hPa independent of the large-scale variables, while the upper (red) distributions represent the distribution in each decile individually. It is evident that using CAPE each individual decile-distribution looks almost identical to the overall distribution. In other words, the sample distribution of  $Q_1$  is entirely independent of CAPE, giving the impression that the relationship between the large scales (represented by CAPE) and convection (represented by  $Q_1$ ) is entirely stochastic similar to the second panel in Figure 1. Using  $\omega_{500}$  as the large-scale variable gives an entirely different result. There is a very strong separation between the individual decile and the overall distributions with a clear progression from cooling for the lower deciles to strong heating in the upper deciles. Note that the sign



of  $\omega_{500}$  has been switched so that lower deciles represent downward (positive) and upper deciles upward (negative)  $\omega_{500}$ . This behaviour is much more akin to the third panel in in Figure 1 and hence represents quasi-deterministic behaviour with a certain amount of stochastic variability

Having identified at least some stochastic behaviour in the relationship between convection and the large scales, it is worth posing the question if the “noise” in the relationship is itself a function of the “signal”. In the multiplicative approaches to stochastic model physics by Buizza et al. (1999) by definition, the “signal-to-noise” ratio is constant and hence independent of the signal itself. We investigate the validity of this assumption by distributing convective rainfall into bins of  $\omega_{500}$  and calculating the mean and standard deviation in each bin. The results are shown in Figure 6. The mean rainfall (blue line) is small for downward  $\omega_{500}$  but increases strongly with decreasing (upward) values of  $\omega_{500}$ . The standard deviation (red line) of convective rainfall in each bin also increases steadily as  $\omega_{500}$  becomes more and more strongly upward. However, the increase in the mean proceeds significantly faster than that in the standard deviation. At downward and weakly upward  $\omega_{500}$  the standard deviation is larger than the mean, indicative of the strongly stochastic character of convection when it is weak and/or weakly forced. For medium and strong upward motion, the standard deviation becomes significantly smaller than the mean indicating a much more deterministic behaviour of convection in that regime. The overall behaviour is summarized in the ratio of standard deviation and mean (black line), which decreases rapidly from values above 1.5 at downward motion to 0.5 at strong upward motion.

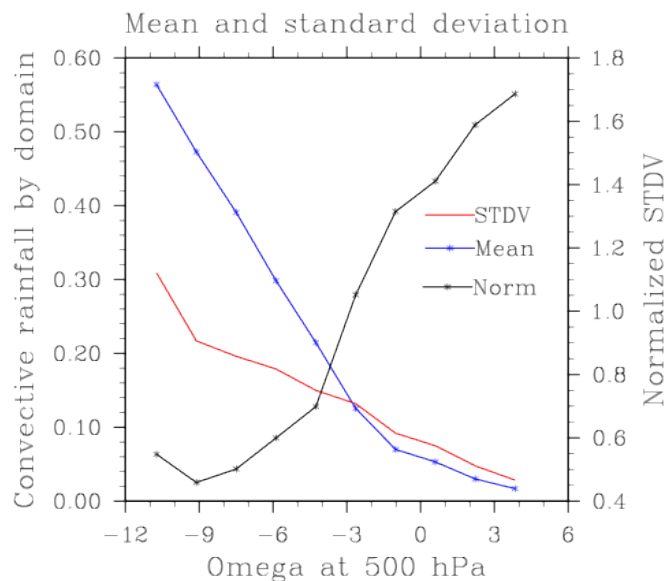


Figure 6: Mean (blue) and standard deviation (red) of convective rainfall as a function of  $\omega_{500}$ . Both lines refer to the left y-axis. Also shown is the ratio of standard deviation and the mean (black), using the right y-axis.

## 5. Conclusions

There has been significant recent debate on the degree to which the relationship between large and convective scales is governed by deterministic relationships and how much of a stochastic component is required in representations of convection in models. Several approaches to introduce stochastic elements into models have been proposed and implemented over the last few years. Most if not all of the current approaches rely on ad-hoc relationships or information obtained from CRMs, while

observations have been largely ignored in the debate. It was the main purpose of this study to overcome this gap by constructing a long-term data set of concurrent large- and convective scale information and to use this data set to investigate the behaviour of the scale relationship. In doing so we assessed several of the commonly used assumptions in stochastic physics implementations in models. The main conclusion of our work are:

1. While complex, the construction of the data sets that are required to investigate the stochastic aspects of convection with respect to large-scale behaviour is possible at least at some sites. Building such data sets requires frequent radar observations from which both rainfall and other small-scale quantities can be derived as well as standard NWP analyses. Few such sites in the tropics exist to date.
2. Early results of analyzing the data at the Darwin site indicate poor relationships between stability-based measures and the small-scale behaviour but much stronger links between convergence-based variables and convective heating. Care must be taken when interpreting these relationships as cause and effect as convection is both affected by and affects the convergence field. Nevertheless, there are indications that all strong convective events are associated with significant convergence, while weaker events occur under a wider range of convergence/divergence conditions.
3. A key finding of this study is that the strength of stochastic behaviour in the relationship between large- and convective scales is strongly dependent on the “model” chosen in connecting the two scales. Here, CAPE is shown to be a poor predictor of convection and as a consequence we would postulate convection to be a very stochastic process, while the use of  $\omega_{500}$  largely alleviates the poor relationship and provides an altogether different picture of convection. It is worth noting that many of the convection parametrizations in use today as well as some of the proposed stochastic approaches heavily rely on CAPE. Our results indicate that this may i) lead to poor descriptions of the convective scales and ii) overstate the case of the stochastic behaviour of convection.
4. Stochastic physics schemes using a multiplicative noise approach assume that the signal-to-noise ratio in the large- to small-scale relationship is independent of the large-scale state itself. We demonstrate that at least for the location of our study this is not the case. We find a strong dependence of this ratio on the large-scale state, with weak upward or downward mid-tropospheric vertical motion showing small signal and significant noise, while strong upward vertical motion is associated with stronger signals and significantly reduced scatter (in the relative sense). These results imply that stronger convection has a more deterministic link to larger scales, while weaker convective events show much more stochastic behaviour, a fact well-known to forecasters in tropical locations when predicting rainfall on a daily basis.

In using observations in the stochastic physics debate this study also raises several interesting questions for future research. As the strength of the large- to small-scale relationships is clearly a function of the variables chosen, how do we know when a strongly stochastic signal indicates a bad model assumption and when the problem is truly stochastic? Can we use the observations to derive better parametrization approaches than those in use today? How do we optimally blend information from observations, theory and process models, such as CRMs, to advance the field of convection parametrization? Are the results presented here specific to the Darwin location or do they hold more broadly? Several efforts are underway to answer at least some of these questions. However, this study

and the workshop it was part of demonstrate the need for a more comprehensive, strategic and determined community approach to stochastic physics in models.

## Acknowledgements

The research presented in this article is supported by the U.S. Department of Energy under Grant DE-FG02-09ER64742 as part of the Atmospheric Systems Research Program as well as the Australian Research Council Linkage Project Grant LP0883961, the Discovery Project Grant DP0985665 and the ARC Centre of Excellence for Climate System Science CE110001028. We thank the R Project for Statistical Computing as well as the NCL project, whose freely available software was used to generate many of the plots in this paper.

## Bibliography

- Ackerman, T. P., and G. M. Stokes, 2003: The Atmospheric Radiation Measurement Program. *Physics Today*, **January 2003**, 38-44.
- Arakawa, A., 2004: The Cumulus Parameterization Problem: Past, Present, and Future. *J. Clim.*, **17**, 2493-2525.
- Arakawa, A., and W. H. Schubert, 1974: Interaction of a Cumulus Cloud Ensemble with the Large-Scale Environment, Part I. *J. Atmos. Sci.*, **31**, 674-701.
- Bechtold, P., and Coauthors, 2008: Advances in simulating atmospheric variability with the ECMWF model: From synoptic to decadal time-scales. *Quart. J. Royal Meteor. Soc.*, **134**, 1337-1351.
- Berner, J., G. J. Shutts, M. Leutbecher, and T. N. Palmer, 2009: A Spectral Stochastic Kinetic Energy Backscatter Scheme and Its Impact on Flow-Dependent Predictability in the ECMWF Ensemble Prediction System. *J. Atmos. Sci.*, **66**, 603-626.
- Bringi, V. N., T. Tang, and V. Chandrasekar, 2004: Evaluation of a New Polarimetrically Based Z-R Relation. *Journal of Atmospheric and Oceanic Technology*, **21**, 612-623.
- Buizza, R., M. Miller, and T. N. Palmer, 1999: Stochastic representation of model uncertainties in the ECMWF Ensemble Prediction System. *Q. J. R. Meteorol. Soc.*, **125**, 2887-2908.
- Ciesielski, P. E., L. M. Hartten, and R. H. Johnson, 1997: Impacts of merging profiler and rawinsonde winds on TOGA COARE analyses. *J. Atmos. Oceanic Technol.*, **14**, 1264-1279.
- Cohen, B. G., and G. C. Craig, 2006: Fluctuations in an Equilibrium Convective Ensemble. Part II: Numerical Experiments. *J. Atmos. Sci.*, **63**, 2005-2015.
- Houze, R. A., Jr., and A. K. Betts, 1981: Convection in GATE. *Rev. Geophys. Space Phys.*, **19**, 541-576.
- Kampstra, P., 2008: Beanplot: A Boxplot alternative for visual comparison of distributions. *Journal of Statistical Software, Code Snippets*, **28**, 1-9.
- Kanamitsu, M., W. Ebisuzaki, J. Woollen, S.-K. Yang, J. J. Hnilo, M. Fiorino, and G. L. Potter, 2002: NCEP-DOE AMIP-II Reanalysis (R-2). *Bull. Amer. Meteor. Soc.*, **83**, 1631-1643.
- Keenan, T., K. Glasson, F. Cummings, T. S. Bird, J. Keeler, and J. Lutz, 1998: The BMRC/NCAR C-Band Polarimetric (C-POL) Radar System. *Journal of Atmospheric and Oceanic Technology*, **15**, 871-886.
- Khouider, B., and A. Majda, 2006: Multicloud Convective Parametrizations with Crude Vertical Structure. *Theor. Comput. Fluid Dyn.*, **20**, 351-375.

- Kuo, H. L., 1965: On Formation and Intensification of Tropical Cyclones Through Latent Heat Release by Cumulus Convection. *J. Atmos. Sci.*, **22**, 40-63.
- Kuo, H. L., 1974: Further Studies of the Parameterization of the Influence of Cumulus Convection on Large-Scale Flow. *J. Atmos. Sci.*, **31**, 1232-1240.
- Lin, J.-L., and Coauthors, 2006: Tropical Intraseasonal Variability in 14 IPCC AR4 Climate Models. Part I: Convective Signals. *J. Clim.*, **19**, 2665-2690.
- Lin, J. W.-B., and J. D. Neelin, 2003: Toward stochastic deep convective parameterization in general circulation models. *Geophys. Res. Lett.*, **30**, 1162.
- May, P. T., J. H. Mather, G. Vaughan, C. Jakob, G. M. McFarquhar, K. Bower, and G. G. Mace, 2008: The Tropical Warm Pool International Cloud Experiment (TWP-ICE). *Bull. Amer. Meteorol. Soc.*, **89**, 629-645.
- Neale, R., J. H. Richter, and M. Jochum, 2008: The impact of convection on ENSO: From a delayed oscillator to a series of events. *J. Clim.*, **21**, 5904-5924.
- Ooyama, K., 1969: Numerical Simulation of the Life Cycle of Tropical Cyclones. *J. Atmos. Sci.*, **26**, 3-40.
- Plant, R. S., and G. C. Craig, 2008: A Stochastic Parameterization for Deep Convection Based on Equilibrium Statistics. *J. Atmos. Sci.*, **65**, 87-105.
- Shutts, G. J., and T. N. Palmer, 2007: Convective forcing fluctuations in a cloud-resolving model: Relevance to the stochastic parameterization problem. *J. Clim.*, **20**, 187-202.
- Steiner, M., R. A. Houze, and S. E. Yuter, 1995: Climatological Characterization of Three-Dimensional Storm Structure from Operational Radar and Rain Gauge Data. *Journal of Applied Meteorology*, **34**, 1978-2007.
- Uppala, S. M., and Coauthors, 2005: The ERA-40 re-analysis. *Quart. J. Royal Meteor. Soc.*, **131**, 2961-3012.
- Xie, S. C., R. T. Cederwall, and M. H. Zhang, 2004: Developing long-term single-column model/cloud system-resolving model forcing data using numerical weather prediction products constrained by surface and top of the atmosphere observations. *J. Geophys. Res.*, **109**, D01104, doi:01110.01029/02003jd004045.
- Xie, S. C., T. Hume, C. Jakob, S. A. Klein, R. McCoy, and M. Zhang, 2010: Observed Large-Scale Structures and Diabatic Heating and Drying Profiles during TWP-ICE. *J. Clim.*, **23**, 57-79.
- Yanai, M., S. K. Esbensen, and J. H. Chu, 1973: Determination of bulk properties of tropical cloud clusters from large-scale heat and moisture budgets. *J. Atmos. Sci.*, **30**, 611-627.
- Zhang, M. H., and J. L. Lin, 1997: Constrained variational analysis of sounding data based on column-integrated conservations of mass, heat, moisture and momentum: Approach and application to ARM measurements. *J. Atmos. Sci.*, **54**, 1503-1524.
- Zhang, M. H., J. L. Lin, R. T. Cederwall, J. J. Yio, and S. C. Xie, 2001: Objective analysis of ARM IOP data: Method and sensitivity. *Mon. Wea. Rev.*, **129**, 295-311.
- Zhang, X., W. Lin, and M. Zhang, 2007: Toward understanding the double Intertropical Convergence Zone pathology in coupled ocean-atmosphere general circulation models. *J. Geophys. Res.*, **112**, D12102, doi:12110.11029/12006JD007878.