Snowpack modelling and data assimilation

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1 Introduction

Snow on the ground, due to its high albedo and high latent heat of fusion, has a strong influence on surface energy balance and surface-atmosphere interactions. It is therefore desirable to have accurate analyses of snow properties for initialization of numerical weather predictions (Drusch et al. 2004). Available snow data present some challenges for assimilation: ground measurements of snow depth may only be representative of limited areas and are sparse in some regions, satellite measurements of snow extent are not strongly related to other important snow properties, and there are no sources of reliable and timely snow mass, temperature or liquid water content observations. This paper considers some simplifications of generic data assimilation equations when they are applied to the specific problems of assimilating snow depth or snow mass measurements in a land surface model. Results are illustrated using data from a highly instrumented site in arctic Finland.

2 Snow model state variables

Operational NWP has traditionally used very simple single-layer representations of snow thermodynamics, but multi-layer snow options with fixed or variable numbers of layers are now becoming available in land surface schemes used by operational models, e.g. HTESSEL at ECMWF (Dutra et al. 2012), JULES at the Met Office (Best et al. 2011) and ISBA-ES in SURFEX (Boone 2002). The thermodynamic state of a snow layer can be specified by variables representing its solid and liquid mass, temperature and density or thickness. Temperature and liquid water variables do not need to be stored separately because both can be diagnosed from layer heat content. A few sophisticated snow physics models include state variables for snow grain sizes and shapes in layers.

Mass and heat content state variables are governed by conservation equations. For the bulk solid mass S, liquid water mass W and temperature T of a snowpack of depth d, these might be written as

$$\frac{dS}{dt} = Sf - E_S - M, \tag{1}$$

$$\frac{dW}{dt} = Rf - E_W + M - Ro \tag{2}$$

and

$$C\frac{dT}{dt} = \frac{G_s - G_b}{d} - L_f M, \tag{3}$$

where Sf and Rf are snowfall and rainfall rates, E_S and E_W are sublimation and evaporation rates, M is melt rate, Ro is runoff rate at the base of the snow, C is heat capacity, G_s and G_b are heat fluxes at the surface and base of the snow, and L_f is the latent heat of fusion. Much of the complexity in snow modelling lies in parametrizing the flux terms within the snowpack and at its boundaries.

Parametrization of snow compaction is a different class of problem not constrained by a fundamental conservation law. The simplest approach, still used in the operational configuration of the Met Office model for example, is to assume a constant density; the optimum value for this constant will underestimate snow density early and overestimate late in the winter. Several models, including ISBA-FR (Douville et al. 1995) and TESSEL (van den Hurk et al. 2000), use an empirical parametrization

$$\frac{d\rho}{dt} = \tau^{-1}(\rho_{\max} - \rho) \tag{4}$$

for bulk density ρ with two parameters: a compaction timescale τ and a maximum attainable density $\rho_{\max}$. A more physical parametrization for density in layers, used in HTESSEL and ISBA-ES amongst other models, is given by

$$\frac{1}{\rho}\frac{d\rho}{dt} = \frac{Mg}{\eta(T,\rho)} + \xi(T,\rho), \tag{5}$$

where M is the mass of overlying snow, g is gravitational acceleration, η is a compactive viscosity and ξ parametrizes thermal metamorphosis of fresh snow. Models differ in how they parametrize the viscosity, but most are based on Kojima (1967).

3 Research and operational snow data assimilation

Many recent studies have investigated assimilation of data in snow models using Ensemble Kalman Filters (Andreadis and Lettenmaier 2006, Dechant and Moradkhani 2011, Durand and Margulis 2006, Durand et al. 2009, Kumar et al. 2008, Slater and Clark 2006, Su et al. 2008, Toure et al. 2011), while a few others have used Extended Kalman Filters (Dong et al. 2007, Sun et al. 2004) or Particle Filters (Dechant and Moradkhani 2011). In stark contrast, operational systems use methods for snow analyses that are much simpler than the state of the art in data assimilation and, indeed, simpler than methods often used for operational analyses of other components of the Earth system. For example, the current cycle (38r1) of the ECMWF Integrated Forecast System (IFS) uses 4D-Var for atmospheric analyses and a simplified Extended Kalman Filter for soil moisture analyses, but the snow analysis was only recently updated to use Optimal Interpolation (OI) with fixed background and observation error variances (ECMWF 2012). Snow depth reports from ground stations and satellite-derived snow extent are assimilated in IFS, but it is striking that no NWP centre currently assimilates either direct measurements or retrievals of snow mass.

Observed snow depths can be multiplied by a model forecast of snow density to obtain an estimate of snow mass per unit area for assimilation in the model. Figure 1 shows an example of ECMWF and HIRLAM snow depth and mass analyses in the winter of 2010-2011 for the grid box containing the Finnish Meteorological Institute (FMI) Arctic Research Centre at Sodankylä (67°22'N, 26°39'E, 179 m a.s.l.). Because there is a synoptic station making daily snow depth reports at Sodankylä, the snow depth analyses are very close to the observations. Snow density and mass are also measured at Sodankylä but not assimilated. Compaction of snow is slower in the cold and dry conditions of arctic Finland than in many other snowy environments, but the use of a constant model snow density clearly could not capture variations over the whole winter. The HIRLAM snow density, predicted using Equation (4), rapidly approaches a prescribed maximum of 300 kg m^{-3} for dry snow that is higher than observed at Sodankylä; the accurate depth but overestimated density give an overestimate of snow mass. Equation (4) was replaced by Equation (5) for ECMWF snow density in March 2009 (Dutra et al. 2010) and the snow analysis was updated to use OI in November 2010 (de Rosnay et al. 2012). The improved prediction of snow density gives better analyses of snow mass for most of the winter at Sodankylä.

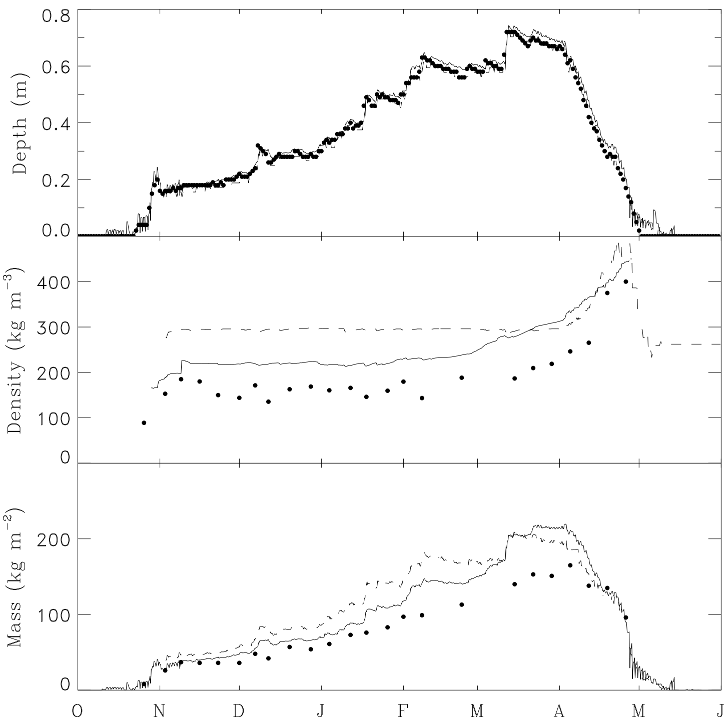

Figure 1: Snow depth, density and mass from ECMWF (solid lines) and HIRLAM (dashed lines) analyses and observations (dots) at Sodankylä in 2010-2011.

Bouttier and Courtier (1999) have given a useful review of data assimilation methods. A dynamical model of a system described by a vector of state variables $\mathbf{x}$ can be regarded as an operator $\mathbf{f}$ mapping between model ("background") estimates of the state vector $\mathbf{x}_b$ at times k and $k+1$ such that

$$\mathbf{x}_b(k+1) = \mathbf{f}[\mathbf{x}_b(k)]. \tag{6}$$

Compared with the true state of the system $\mathbf{x}_t$, the model state has errors $\varepsilon_b = \mathbf{x}_b - \mathbf{x}_t$ with average $\bar{\varepsilon}_b$ and covariance

$$\mathbf{B} = \overline{(\varepsilon_b - \bar{\varepsilon}_b)(\varepsilon_b - \bar{\varepsilon}_b)^T}, \tag{7}$$

partly due to model imperfections and partly due to imperfect initial conditions. Unless the model is perfect, it will have error $\varepsilon_t = \mathbf{f}[\mathbf{x}_t(k)] - \mathbf{x}_t(k+1)$ and error covariance

$$\mathbf{Q} = \overline{(\varepsilon_t - \bar{\varepsilon}_t)(\varepsilon_t - \bar{\varepsilon}_t)^T} \tag{8}$$

even in forecasts from perfect initial conditions. If there are N variables in the state vector, $\mathbf{B}$ and $\mathbf{Q}$ are symmetric $N \times N$ matrices.

Observations for data assimilation, gathered in an observation vector $\mathbf{y}$, need not be state variables but have to be related to state variables by an observation operator $\mathbf{h}$ such that

$$\mathbf{y} = \mathbf{h}(\mathbf{x}_t) \tag{9}$$

in the absence of errors. Observation errors $\varepsilon_o = \mathbf{y} - \mathbf{h}(\mathbf{x}_t)$ are then characterized by bias $\bar{\varepsilon}_o$ and covariance matrix

$$\mathbf{R} = \overline{(\varepsilon_o - \bar{\varepsilon}_o)(\varepsilon_o - \bar{\varepsilon}_o)^T}. \tag{10}$$

Data assimilation attempts to make an optimal estimate of the true state (the "analysis") given the model background, observations and knowledge of their error statistics. For unbiased Gaussian errors, the maximum likelihood estimate of $\mathbf{x}_t$ is $\mathbf{x} = \mathbf{x}_a$ which minimizes the cost function

$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b) + [\mathbf{y} - \mathbf{h}(\mathbf{x}_b)]^T \mathbf{R}^{-1} [\mathbf{y} - \mathbf{h}(\mathbf{x}_b)]. \tag{11}$$

The minimization may be performed iteratively or by solving $\nabla J(\mathbf{x}_a) = 0$, which gives

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}[\mathbf{y} - \mathbf{h}(\mathbf{x}_b)] \tag{12}$$

where

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T (\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1} \tag{13}$$

is the Kalman gain matrix and

$$\mathbf{H} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \tag{14}$$

is the Jacobian of the observation operator.

Several data assimilation methods use Equations (12) and (13) but differ in how they handle background errors. The Extended Kalman Filter (EKF) uses the forecast equation

$$\mathbf{B}(k+1) = \mathbf{F}\mathbf{B}(k)\mathbf{F}^T + \mathbf{Q} \tag{15}$$

and analysis error covariance

$$\mathbf{A} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B}, \tag{16}$$

where

$$\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}. \tag{17}$$

If the model is expensive to run then Equation (15) is very expensive to run for a large state vector, and if the model is a complicated piece of code then the adjoint model giving $\mathbf{F}^T$ is also a complicated piece of code that has to be maintained. The Simplified Extended Kalman Filter (SEKF) instead uses a finite difference approximation

$$F_{ij} = \frac{f_i(\mathbf{x} + \delta x_j) - f_i(\mathbf{x})}{\delta x_j} \tag{18}$$

with a perturbed forecast for each state variable. The same method can be used to approximate $\mathbf{H}$ for complex non-linear observation operators. Without the benefit of perfect observations to determine model error, the elements of $\mathbf{Q}$ are usually calibrated to optimize the filter performance in some way (Reichle et al. 2002, Seuffert et al. 2004).

Data assimilation need not, and often does not, operate on the whole model state variable; $\mathbf{x}$ is then a "control vector" containing a subset or functions of the state variables. The analysis equations in matrix form, reproduced in many publications, reduce greatly in complexity for the simple case of assimilating one observable variable that is also a control variable. Matrices and vectors are replaced by scalars: $\mathbf{B} = \sigma_b^2$, $\mathbf{F} = F$, $\mathbf{H} = H$, $\mathbf{Q} = \sigma_q^2$, $\mathbf{R} = \sigma_r^2$, $\mathbf{x} = x$ and $\mathbf{y} = x_o$. The Jacobian of the observation operator has the trivial form $H = 1$, and the state analysis equation reduces to

$$x_a = x_b + \left(\frac{\sigma_b^2}{\sigma_b^2 + \sigma_r^2} \right) (x_o - x_b), \tag{19}$$

which shows that the analysis increment just depends on the difference between the model and observed state, and the ratio of the background and observation error variances. Observations are not used if the model is perfect ($x_a = x_b$ if $\sigma_b = 0$), and a perfect observation directly replaces the model state ($x_a = x_o$ if $\sigma_r = 0$). For EKF, the background error forecast equation is

$$\sigma_b^2(k+1) = F^2 \sigma_b^2(k) + \sigma_q^2 \tag{20}$$

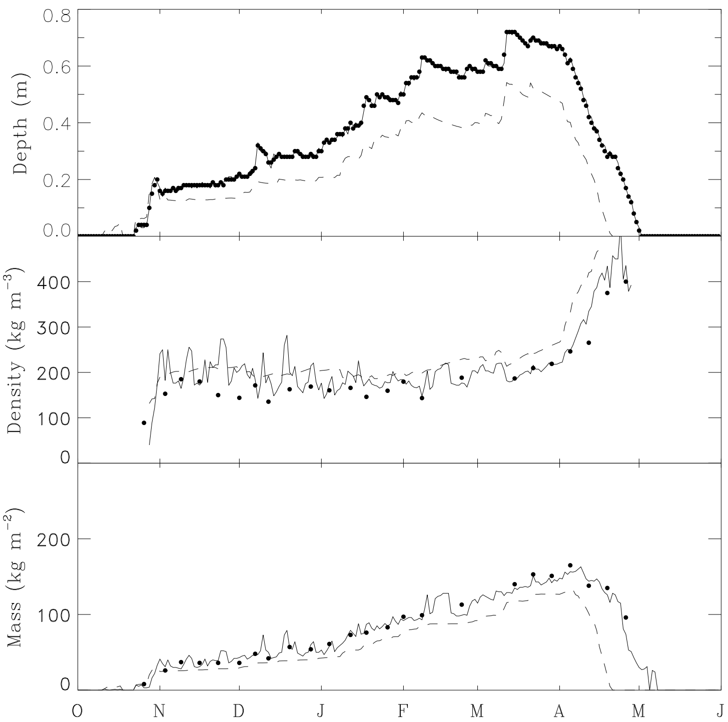

Figure 2: Snow depth, density and mass simulations in open-loop (dashed lines) and with direct insertion of observations (solid lines) analyses, compared with independent observations (dots) at Sodankylä in 2010-2011.

and the analysis error

$$\sigma_a^2 = \left(\frac{\sigma_r^2}{\sigma_b^2 + \sigma_r^2} \right) \sigma_b^2 \tag{21}$$

provides the initial condition for the background error forecast after an analysis step. For a linear model, F is a constant and the Extended Kalman Filter reduces to the linear Kalman Filter. Moreover, the background error at analysis times then tends to a constant after repeated applications of Equations (20) and (21) if analyses are performed at regular intervals.

4 Assimilation of point snow data

In addition to the regular manual measurements, automatic instruments at Sodankylä measure snow depth by ultrasonic ranging and snow mass by gamma radiation absorption. These data can be assimilated in a snow model, reserving the manual measurements for independent evaluations of snow depth and mass simulations. A 3-layer snow model (Essery et al. 2013) using Equation (5) for compaction has been found to give good simulations of the snowpack at Sodankylä when driven with the high-quality in situ meteorological data available from the site. Here, the model is instead driven with NWP data from HILRLAM: temperature, humidity, wind speed and pressure analyses, and 3 to 6 hour forecasts of shortwave radiation, longwave radiation, snowfall and rainfall. Figure 2 shows results from an open-loop simulation without assimilation and a direct-insertion assimilation with the model snow depth and mass replaced by the automatic measurements once a day. The HIRLAM short-range forecasts for Sodankylä, which has synoptic and upper-air stations in an area of limited relief, are generally good, but the open-loop simulation underestimates snow depth and mass due to underestimates of forecast snow-

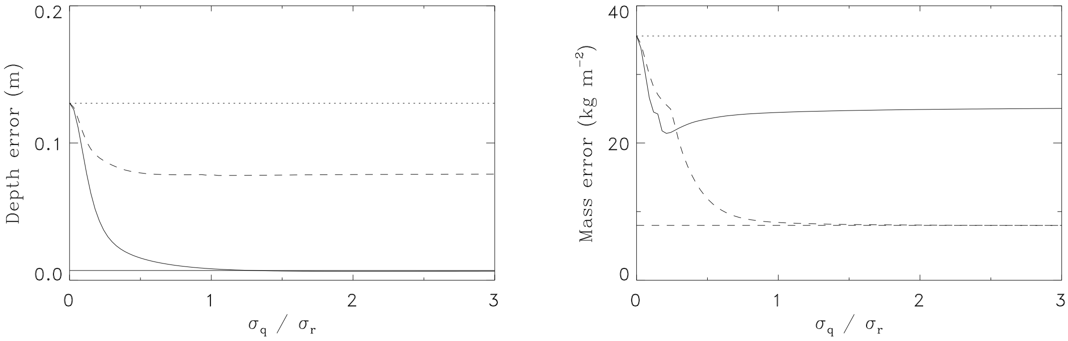

Figure 3: Snow depth and mass errors with assimilation of either snow depth (solid lines) or snow mass (dashed lines) observations. Horizontal lines show errors for open-loop simulations (upper) and direct insertion of observations (lower).

fall and overestimates of longwave radiation compared with in situ measurements. The automatic snow depth measurements are very close to the manual measurements and give low errors when used to correct the model by direct insertion. The automatic snow mass sensor was calibrated to previous manual measurements and so has a low bias, but measurement noise introduces some spurious variations in the model snow mass and density when inserted in the model.

Figure 3 shows how rms errors in snow depth and mass simulations with SEKF assimilation of either daily snow depth or snow mass observations vary as the model error parameter is adjusted. As expected, errors tend towards the errors for open-loop simulations when σ_q is small and tend towards the errors for direct insertion when it is large. The differences between automatic measurements used for assimilation and manual measurements used for evaluation are sufficiently small that there is no parameter value for which SEKF outperforms direct insertion. To investigate the influence of observations errors, the assimilation experiments were repeated with observations degraded by the addition of random errors. The results plotted in Figure 4 now show clear minima for which analysis errors are lower than both open-loop forecast errors and observation errors.

For deep snow, sublimation and melt rates can be expected to be insensitive to small mass perturbations. The mass balance Equation (1) then gives

$$F = \frac{\partial S(k+1)}{\partial S(k)} \approx 1, \qquad (22)$$

which is confirmed by model results in Figure 5. With daily assimilation of snow mass observations, the background error is almost constant; Equations (20) and (21) give this constant as

$$\sigma_b^2 = \frac{\sigma_q^2}{2}\left[1 + \left(1 + \frac{4\sigma_r^2}{\sigma_q^2}\right)^{1/2}\right]. \qquad (23)$$

If the background error is constant, there is no need for it to be updated by the SEKF; it can instead be taken as a parameter and adjusted to give an optimized interpolation between modelled and observed snow mass that minimizes the rms analysis error. In fact, it is found that the minimum errors obtained with assimilation of degraded observations are very similar for SEKF and OI (15.4 and 14.1 kg m^{-2} respectively).

It is less clear that Equation (5) will give a model that is linear in snow depth. For the slow compaction in the cold conditions at Sodankylä, however, the model turns out to be nearly linear for much of the winter. F and σ_b are then nearly constant for assimilation of snow depth measurements, as seen in Figure

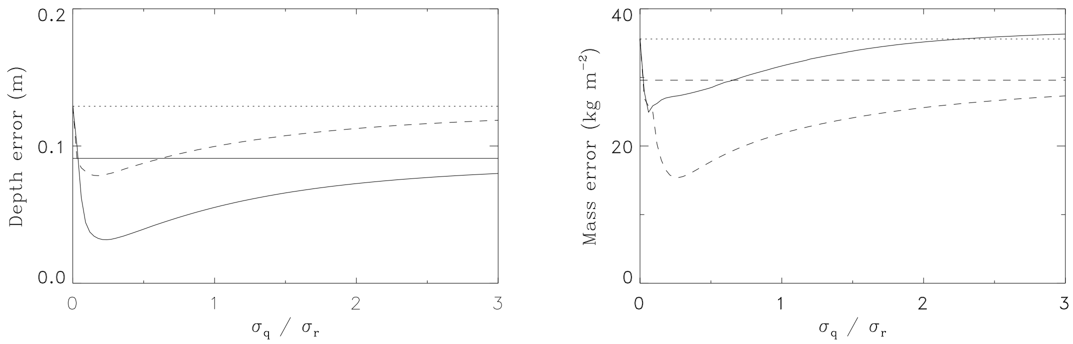

Figure 4: As Figure 3, but for assimilation of degraded observations.

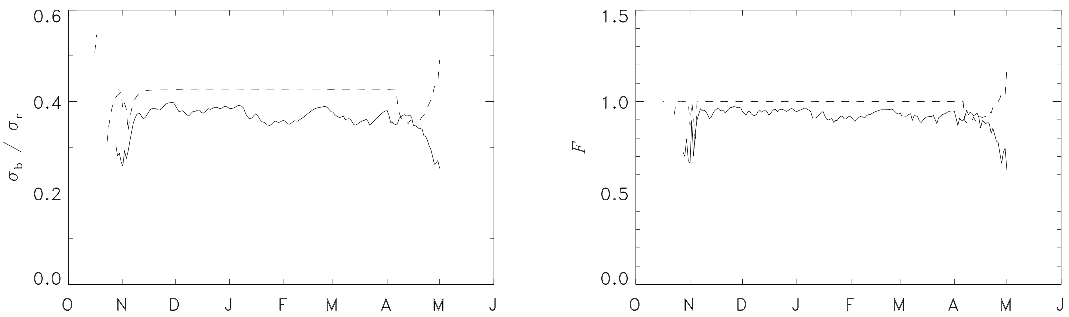

Figure 5: Background error and F for assimilation of degraded snow depth (solid lines) or snow mass (dashed lines) observations.

5. Again, the minimum analysis errors that can be achieved by adjusting the model error parameter in SEKF and OI are very similar (3.2 and 3.0 cm).

5 Conclusions

Assimilation of either snow depth or snow mass observations requires a model estimate of snow density if information from increments in one of these variables is to be used to update the other. Multi-layer snow models with reasonable parametrizations of density already exist, but they are not yet widely used in operational NWP.

The use of data from a highly-instrumented site such as FMI-ARC at Sodankylä allows detailed investigations of assimilation performance and assumptions but does not represent all of the challenges of data assimilation in practice. Although independent measurements are available for assimilation and evaluation of the resulting analyses, they are so close that the optimum strategy is simply to replace the modelled snow depth or mass with an observation whenever one is available. With degraded observations, it is possible to make a better analysis by combining model and observation estimates. In the simple case considered here of predicting snow depth and mass at a cold site, where snow accumulates gradually through the winter and melts rapidly in the spring, the model error turns out to be nearly constant for most of the winter and there is no benefit from using an assimilation method that forecasts the error. The use of a Kalman Filter will still be beneficial if information can be propagated to unobserved state variables through off-diagonal elements in the gain matrix, either due to correlation between state variables in the model or the use of a complex observation operator such as a microwave emission model for assimilation of radiance data.

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