Simulations of the solar magnetic cycle with EULAG-MHD

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1. The solar magnetic field and its cycle
2. Magnetic cycles with EULAG-MHD
3. Why does it work ?
4. What is next…
5. …talks by Strugarek and Cossette

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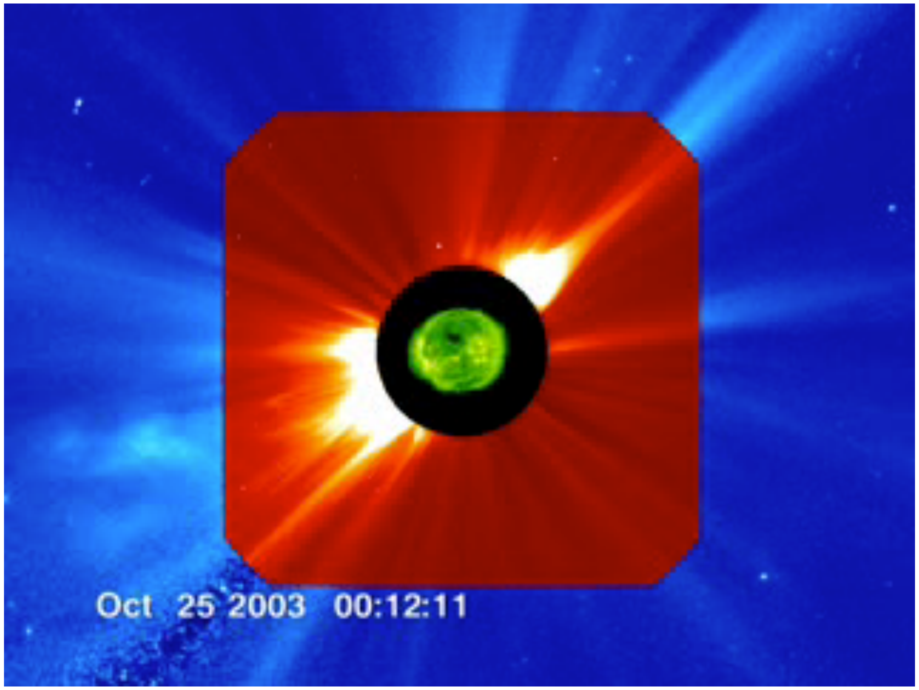
Space weather = solar magnetism

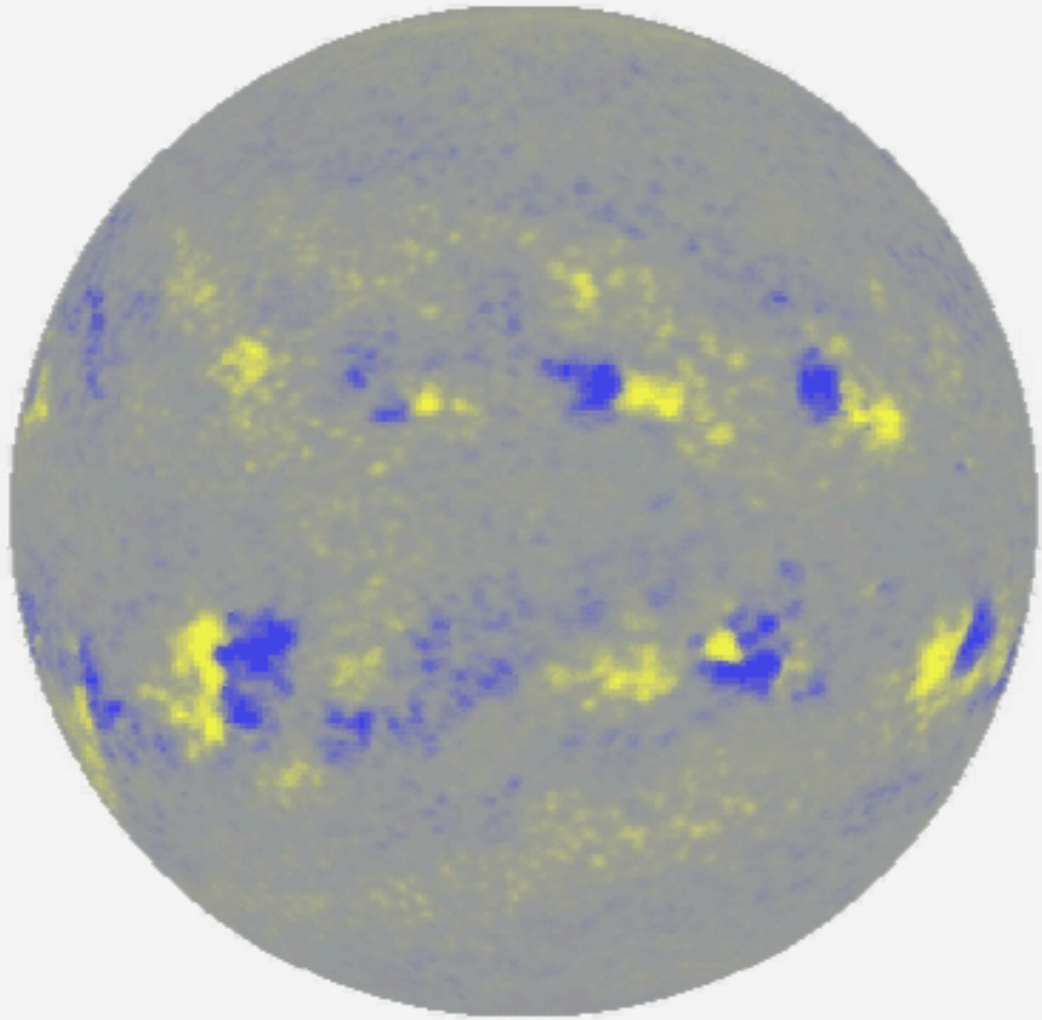
Data from SoHO/EIT LASCO, NASA+ESA



NSWP: Numerical Space Weather Prediction

Solar magnetic field is engine

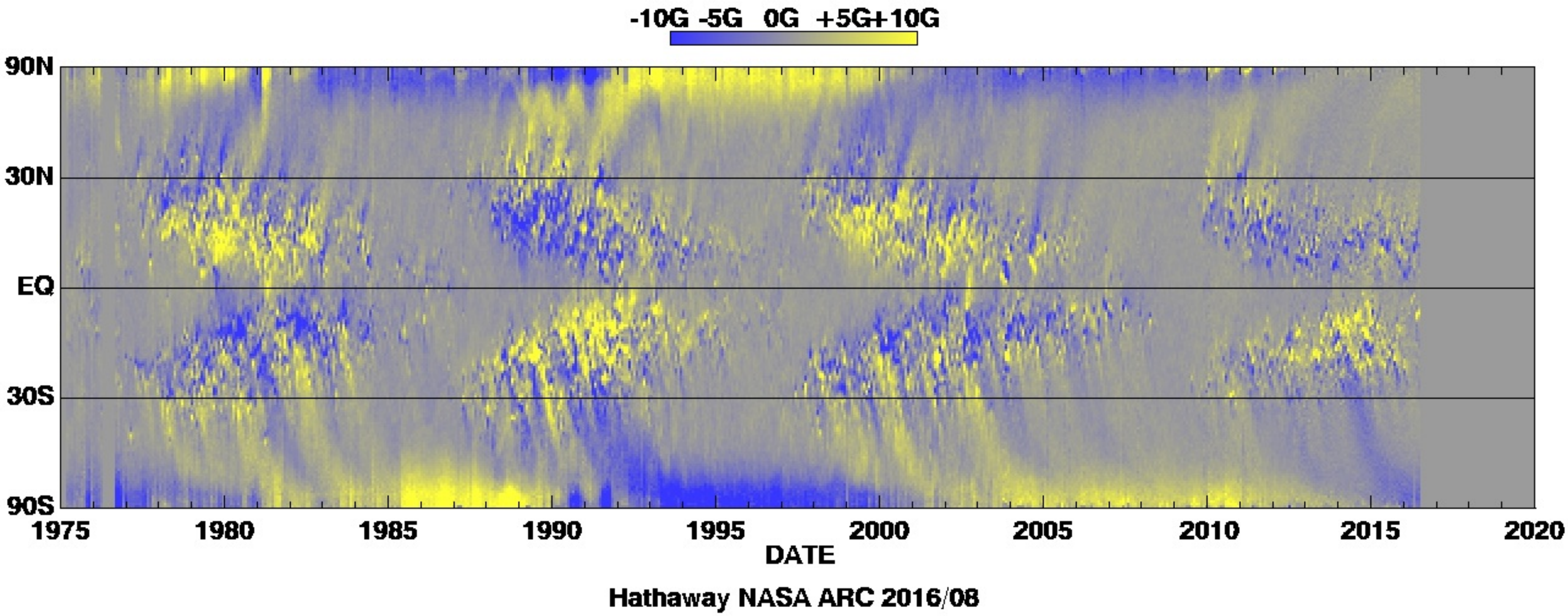
Energetics not problematic; ~10^{-5} of solar luminosity



Line-of-sight magnetogram animation by D. Hathaway, NASA/Ames

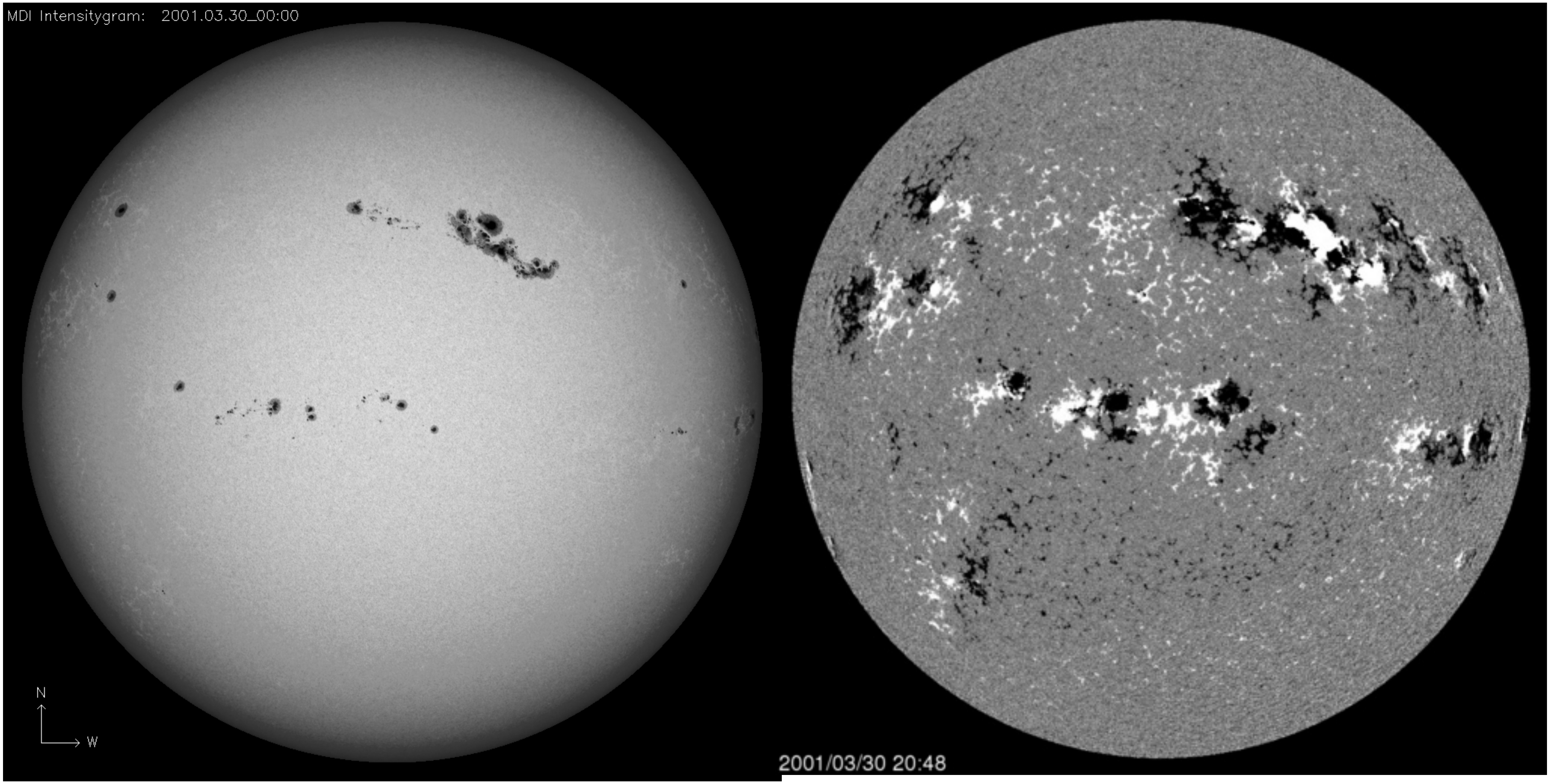
Solar magnetism is observed

Zonal average of surface radial magnetic component



Magnetic polarity reversal every ~11 yr,
full magnetic cycle period ~22 yr

Sunspots as tracers of magnetism



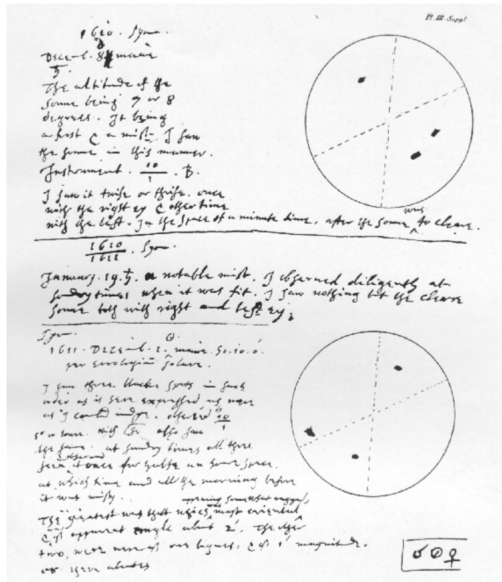
2001, cycle peak

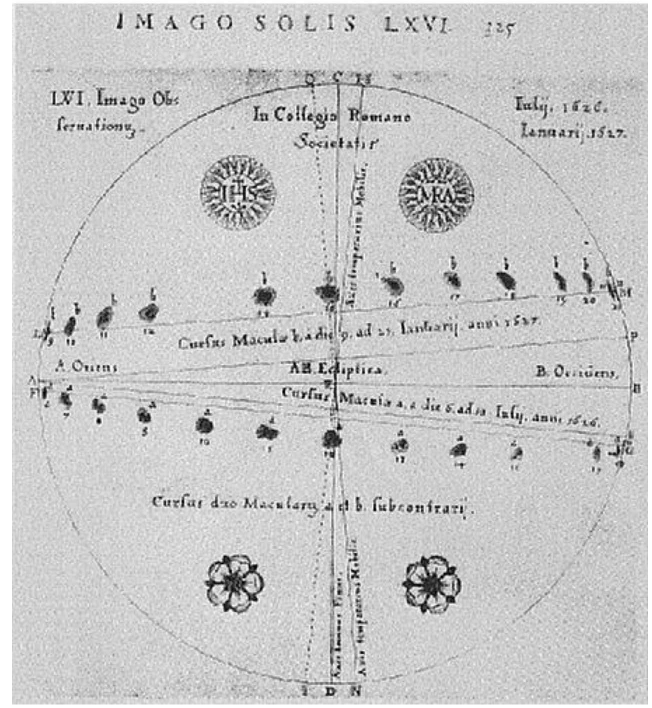
Magnetogram

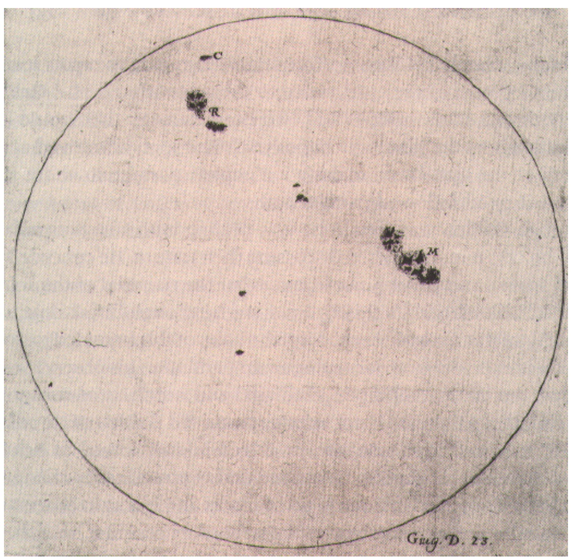
Data: SoHO/MDI (NASA+ESA)

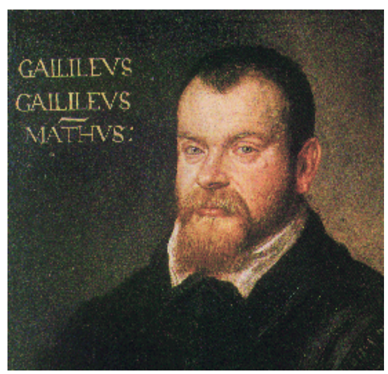
Harriot, Fabricius, Galileo, Scheiner,...


JOH. FABRICII PHRYSII
De
MACULIS IN
SOLE OBSERVATIS, ET APPARENTE
earum cum Sole conversione,
NARRATIO.
cui
Adjecta est de modo eductionis specierum visibilium dubitatio.
WITEBERGAE.
ANNO M. DC. XI



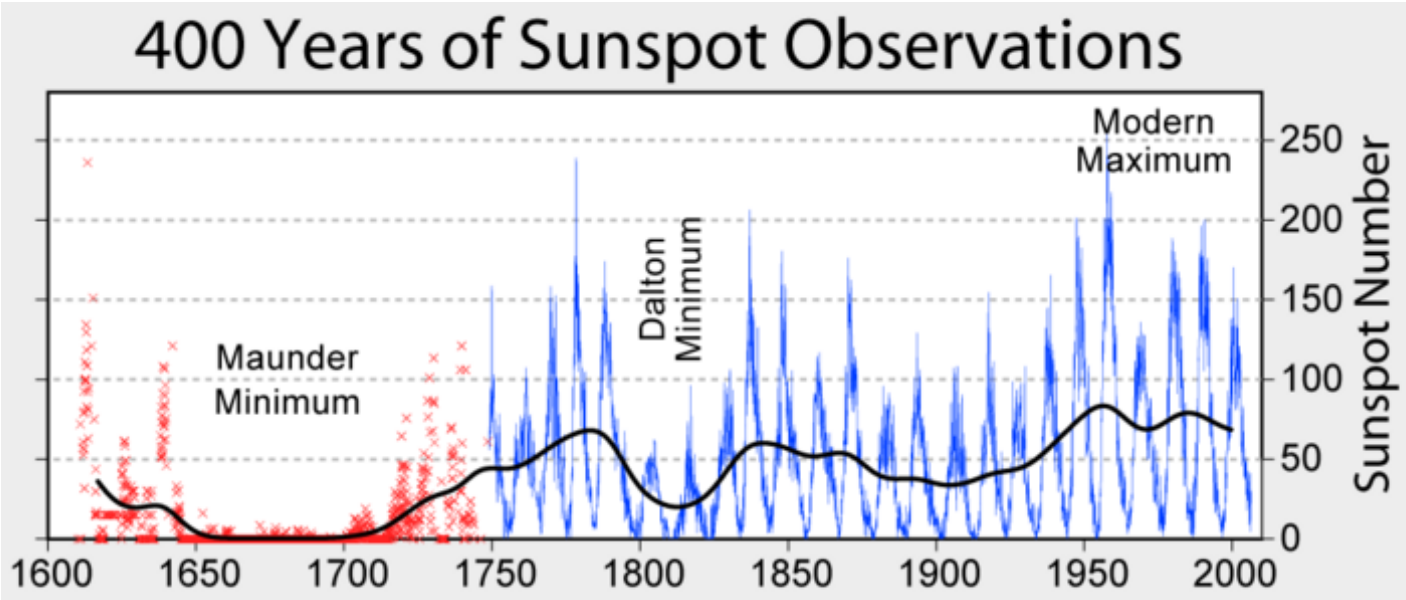






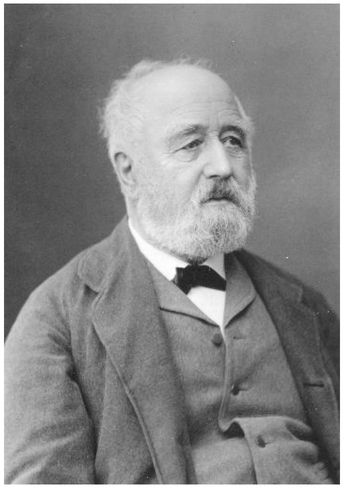
The sunspot cycle

Discovered in 1843 by an amateur astronomer, after 17 years of nearly continuous sunspot observations.



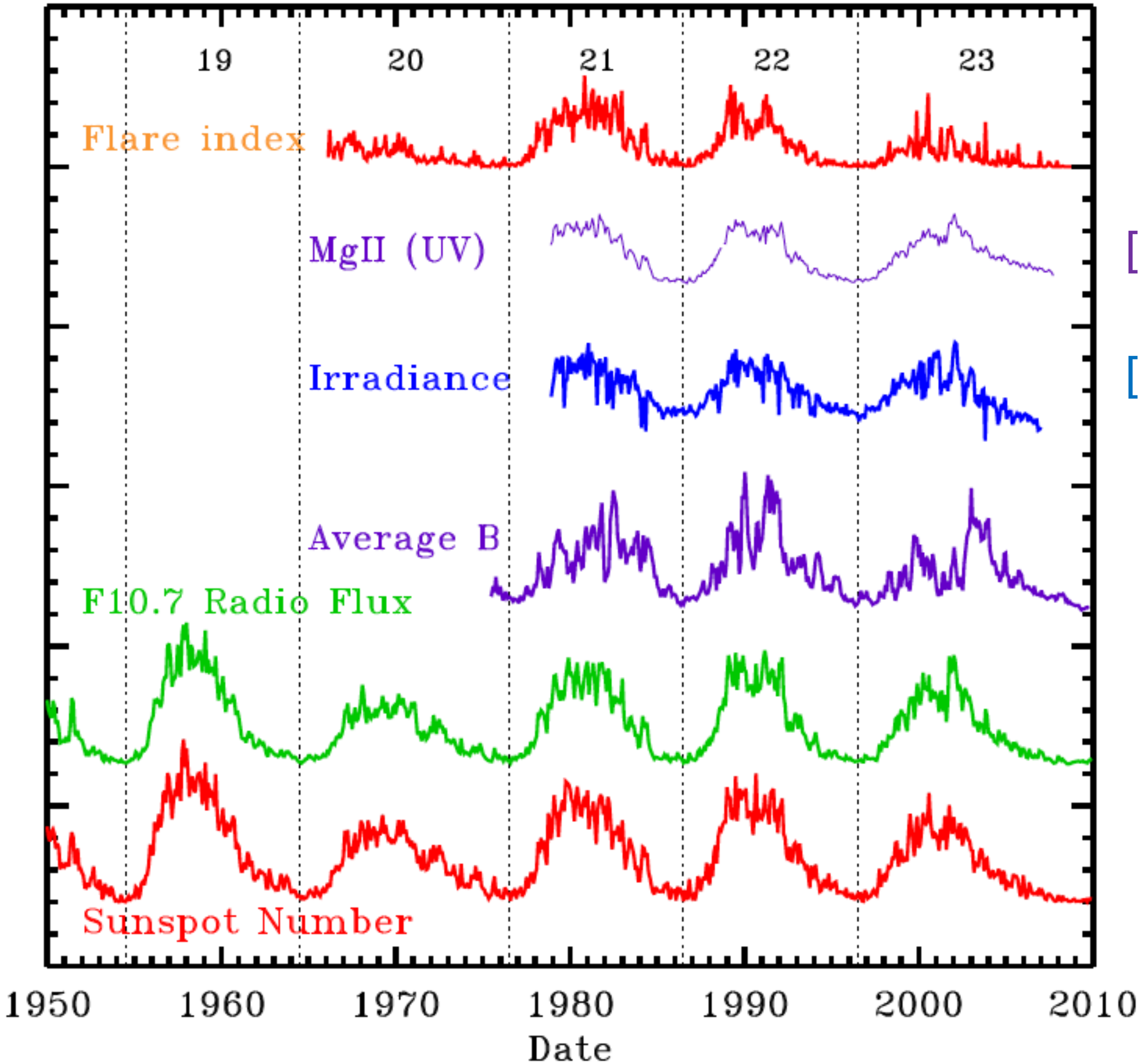
The sunspot cycle shows large cycle-to-cycle fluctuations in amplitude and, to a lesser extent, duration, as well as extended episodes of apparent halt (1645-1715 Maunder Min)

Heinrich Schwabe



Rudolf Wolf

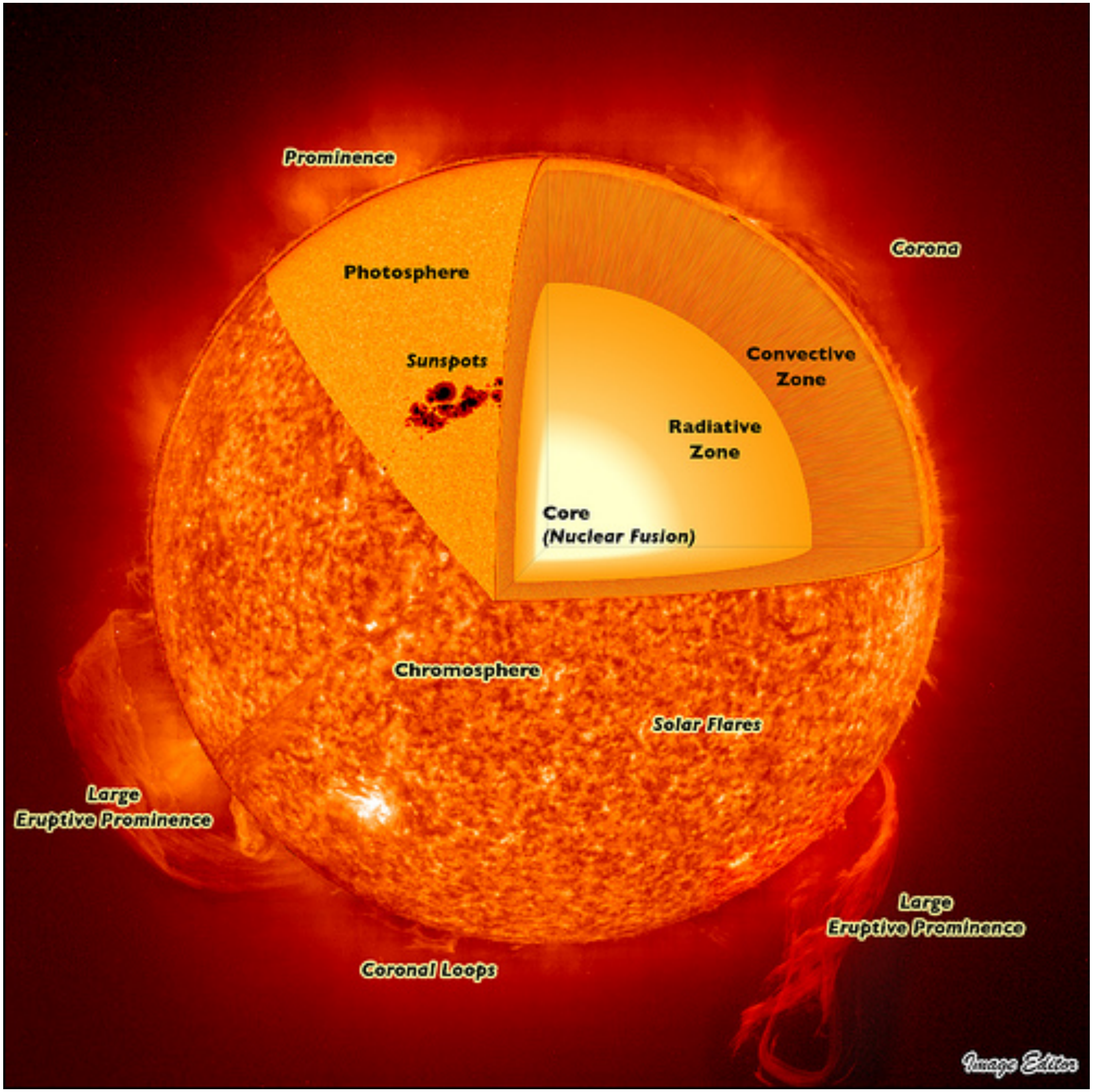
Magnetic cycle = pulse of solar activity



[~10-100%]

[~0.1%]

Solar internal structure



Atmospheric GCMs vs solar convection

Earth's atmosphere	Solar convection zone
Thin layer, constant g	Thick layer, $g \sim r^{-2}$
Rotation+stratification	Rotation+stratification
Oceans+topography	Sitting on one big « ocean »
Chemistry/phase changes	Ionization of H and He
Heating varying in space/time	Steady heating from below
Intermittent convection	Ever-present convection
B dynamically unimportant	**B** dynamically important
Ro < 1, Re >> 1, Ek << 1	Ro < 1, Re >>> 1, Ek << 1
Multiscale **u**	Very multiscale (**B** ~ $Rm^{-1/2}$)

The MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \ ,$$

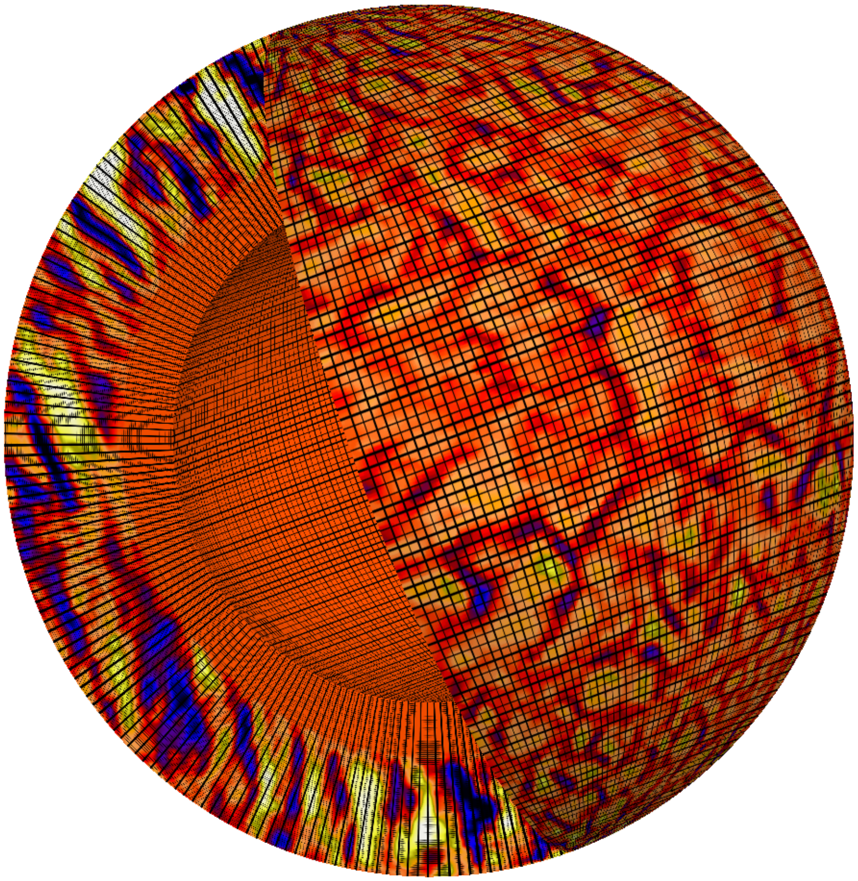
$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \boxed{\frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B}} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \ ,$$

$$\frac{\mathrm{D}e}{\mathrm{D}t} + (\gamma - 1)e\nabla \cdot \mathbf{u} = \frac{1}{\rho}\Big[\nabla \cdot \Big((\chi + \chi_r)\nabla T\Big) + \phi_\nu \boxed{+ \phi_B}\Big] \ ,$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \ .}$$

Simulation design: EULAG-MHD

[Smolarkiewicz & Charbonneau, *J. Comput. Phys*. **236**, 608-623 (2013)]



Simulate anelastic convection in thick, **rotating** and unstably **stratified** fluid shell of electrically conducting fluid, overlaying a stably stratified fluid shell.

Recent such simulations manage to reach Re, Rm ~10^2-10^3; a long way from the solar/stellar parameter regime (10^8-10^{10}).

Throughout the bulk of the convecting layers, **convection is influenced by rotation** (Ro<1), leading to alignment of convective cells parallel to the rotation axis.

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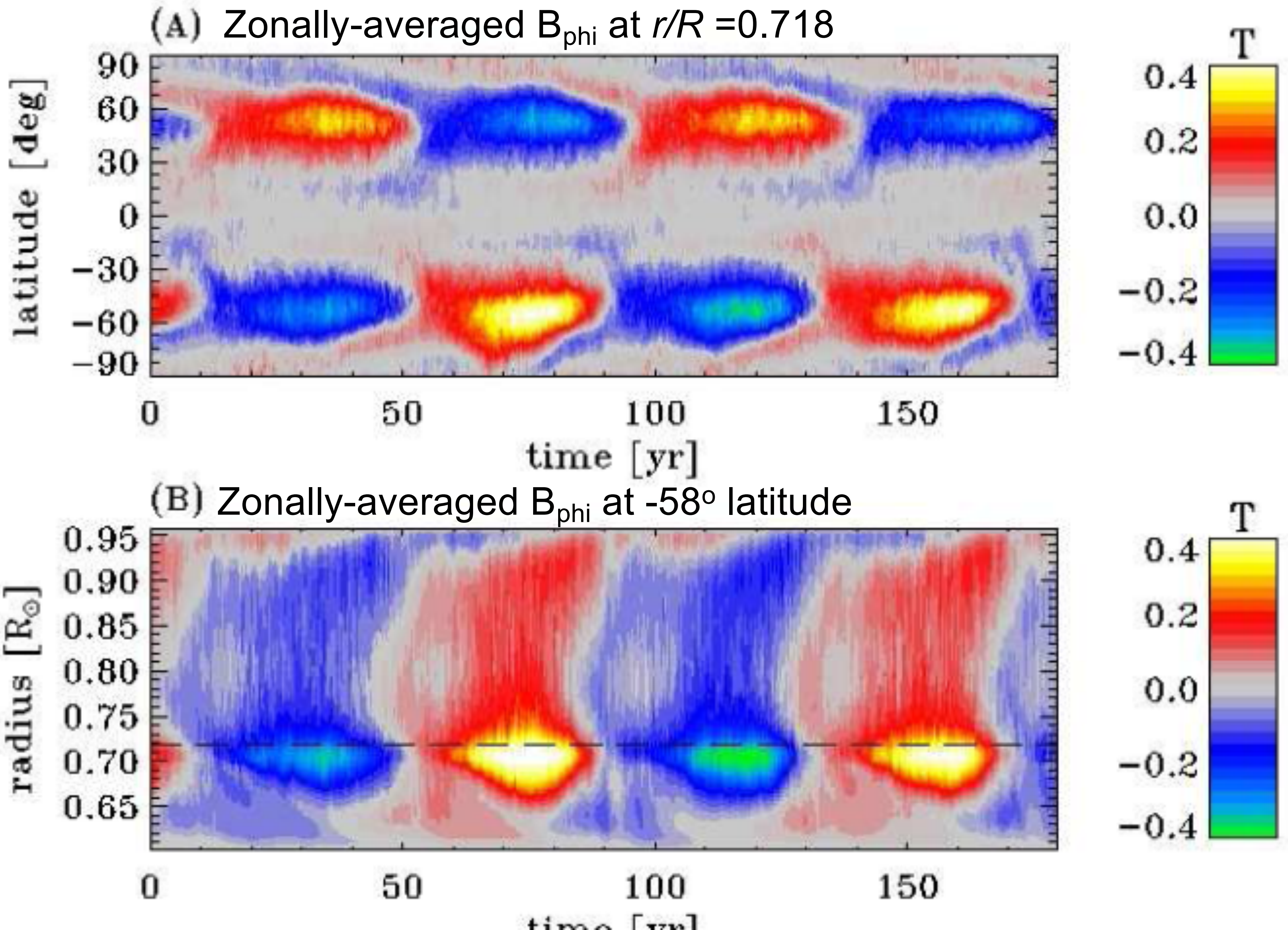
Run EULAG-MHD in **ILES mode** with **volumetric thermal forcing** driving convection, and absorbers at base of stable fluid layer

Simulated magnetic cycles

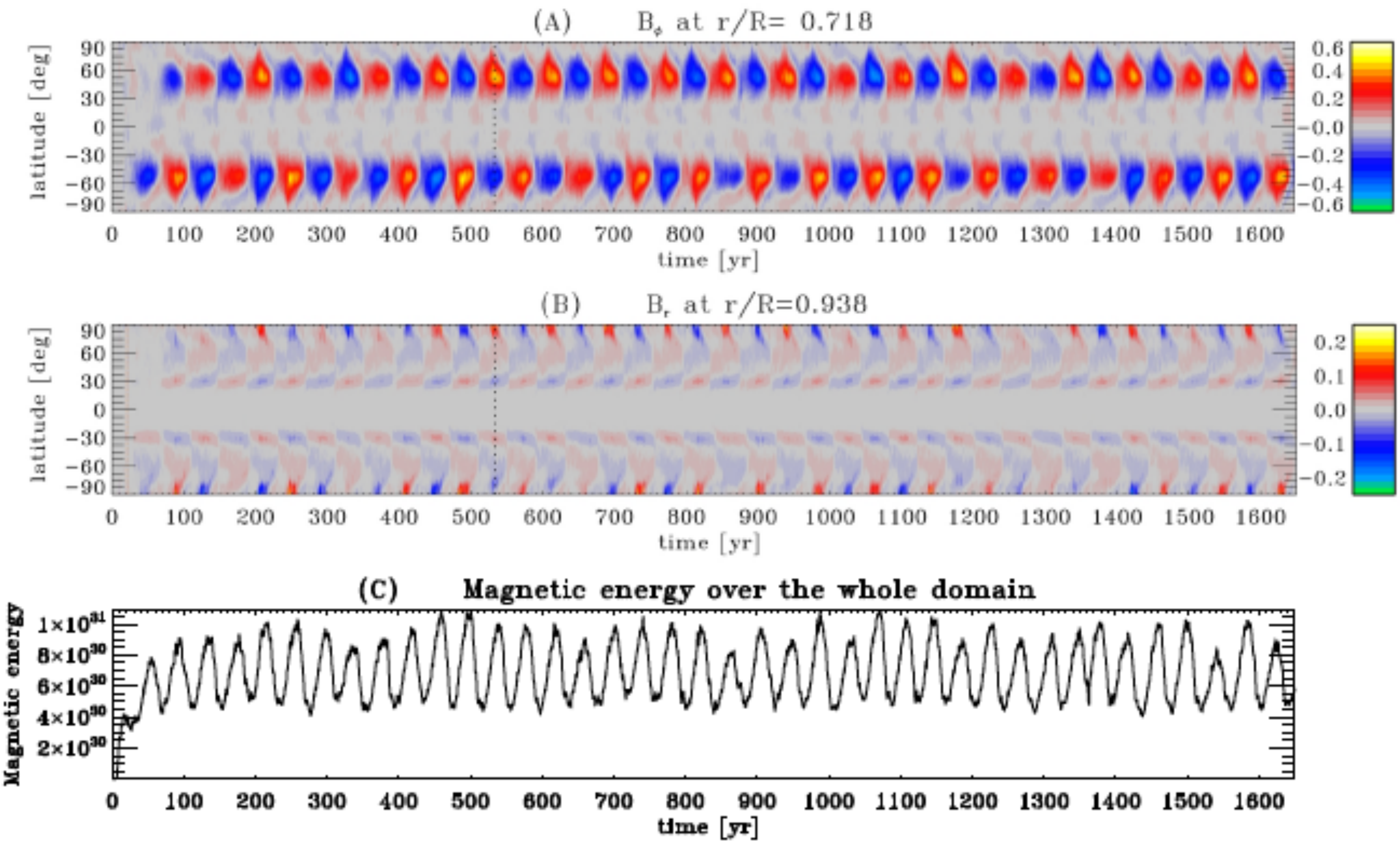
Large-scale organisation of the magnetic field takes place primarily at and immediately below the base of the convecting fluid layers.

Magnetic field amplification through a **dynamo mechanism**: converting flow kinetic energy into (electro)magnetic energy.


(A) Zonally-averaged B phi at r/R =0.718
latitude [deg]
90
60
30
0
−30
−60
−90
0
50
100
150
time [yr]
T
0.4
0.2
0.0
−0.2
−0.4
(B) Zonally-averaged B phi at -58o latitude
radius [R⊙]
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0
50
100
150
time [yr]
T
0.4
0.2
0.0
−0.2
−0.4

The « millenium simulation »

[Passos & Charbonneau 2014, *Astron. & Ap.*, **568**, 113]



Why does it work ?

Mechanisms of MHD induction

In an electrically conducting fluid, magnetic field lines behave like vortex lines in an inviscid fluid: they are frozen into the fluid (« flux freezing »). MHD induction can be viewed as stretching existing magnetic fieldlines

Classical solar dynamo models based on two primary inductive mechanisms:

1. Shearing of large-scale poloidal magnetic field by differential rotation;

2. Turbulent electromotive force associated with the action of cyclonic convection on large-scale magnetic field.

Mean-field electrodynamics (1)

Mean-field electrodynamics is built around the idea of **scale separation**. Assume that the total flow ***u*** and magnetic field ***B*** can be decomposed into a large-scale average and a small-scale fluctuating component (this is **not** a linearization!):

$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}' \ , \qquad \boldsymbol{B} = \langle \boldsymbol{B} \rangle + \boldsymbol{B}' \ ,$$

The procedure hinges on there existing a spatial scale over which the small-scale components vanish upon averaging:

$$\langle \boldsymbol{u}' \rangle = 0 \ , \qquad \langle \boldsymbol{B}' \rangle = 0 \ .$$

and the large-scale components are deemed constant over this intermediate averaging scale.

Mean-field electrodynamics (2)

Substituting into the induction equation and averaging yields an evolutionary equation for the mean magnetic field:

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \boldsymbol{\nabla} \times \left[\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \boldsymbol{\mathcal{E}} - \eta \boldsymbol{\nabla} \times \langle \boldsymbol{B} \rangle \right],$$

Where an additional source term has appeared: the **mean electromotive force**:

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$$

Closure is achieved by expressing the emf as a tensorial development in terms of the mean field:

$$\mathcal{E}_i = \alpha_{ij} \langle B_j \rangle + \beta_{ijk} \partial_j \langle B_k \rangle + \text{ higher order derivatives },$$

Mean-field electrodynamics (3)

Given the spatiotemporal behavior of the magnetic field building up in our simulation the « natural » averaging operator is a **zonal average**:

$$\langle \boldsymbol{B} \rangle (r, \theta, t) = \frac{1}{2\pi} \int_0^{2\pi} \boldsymbol{B}(r, \theta, \phi, t) \mathrm{d}\phi$$

This leads to the **definition** of the small-scale components as:

$$\boldsymbol{u}'(r, \theta, \phi, t) = \boldsymbol{u}(r, \theta, \phi, t) - \langle \boldsymbol{u} \rangle (r, \theta, t) \ ,$$

$$\boldsymbol{B}'(r, \theta, \phi, t) = \boldsymbol{B}(r, \theta, \phi, t) - \langle \boldsymbol{B} \rangle (r, \theta, t) \ .$$

This now allows the **calculation** of the mean electromotive force from the simulation output:

$$\boxed{\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle}$$

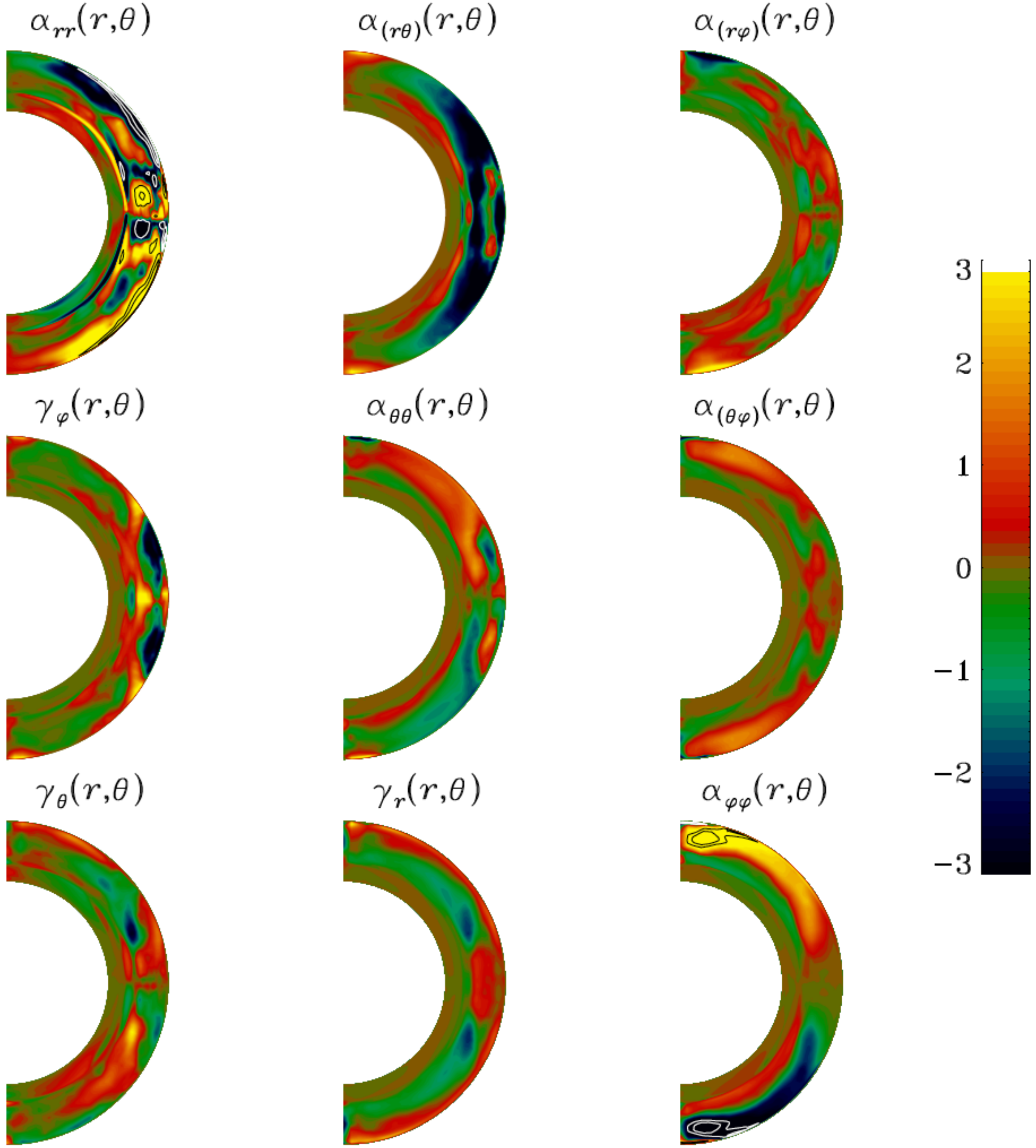
Extracting the alpha-tensor

Retain for now only first term in the tensorial development for the EMF:

$$\mathcal{E}_i(t,r,\theta) = \alpha_{ij}(r,\theta)\langle B_j\rangle(t,r,\theta)$$

With all EMF and large-scale magnetic field components extracted from the simulation, this can be recast as a least-squares fitting problem for the tensorial components of the alpha-tensor at each grid point in a meridional plane; We tackle this fitting problem using Singular Value Decomposition. Other extraction schemes are available (e.g., test-field method).

MHD: the alpha-tensor



Mean-field electrodynamics (4)

NO

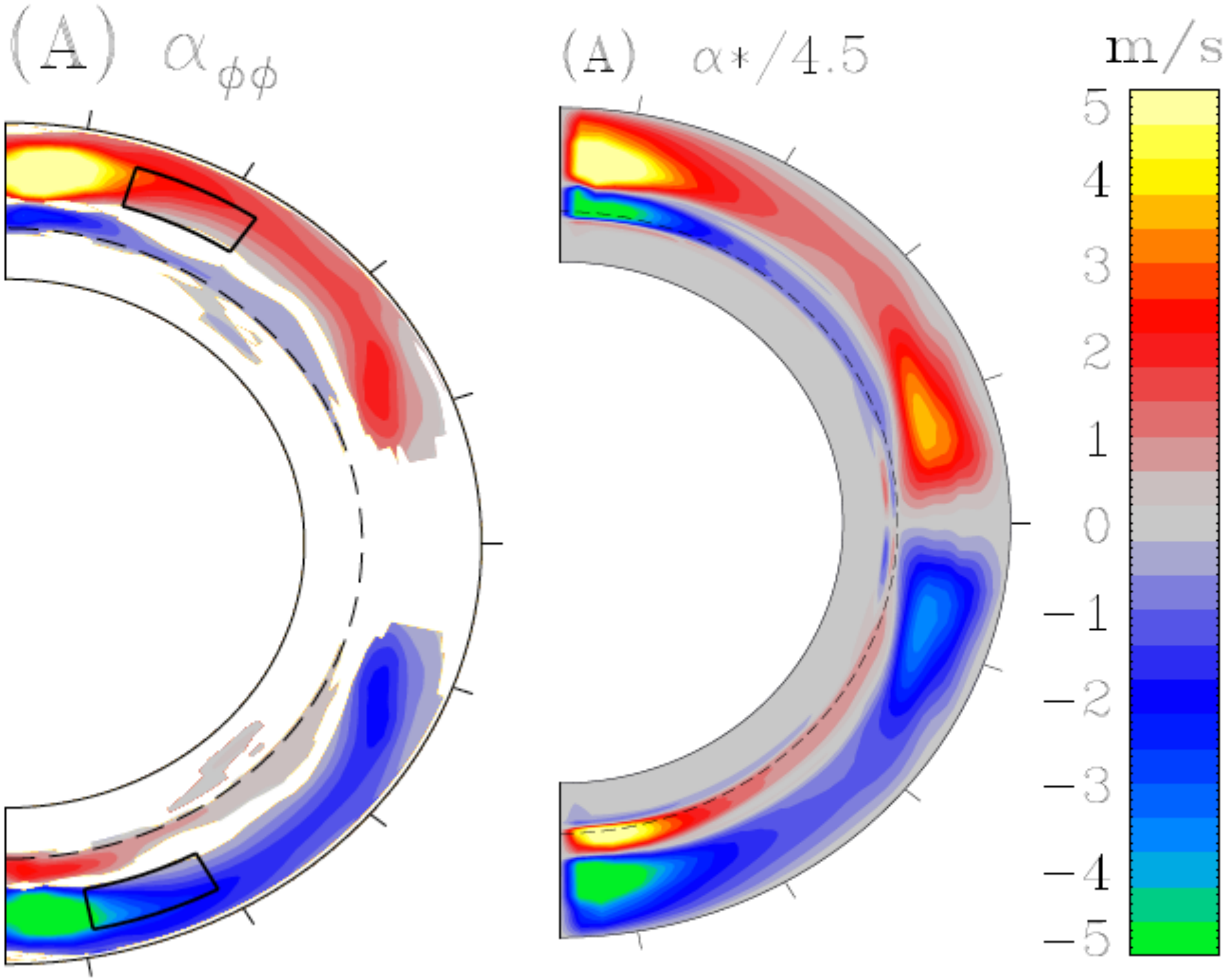
For stratified rotating MHD turbulence that is $\boxed{\text{homogeneous}}$ NO and $\boxed{\text{isotropic}}$ but lacks reflectional symmetry and is only $\boxed{\text{weakly influenced by the magnetic field,}}$ the alpha-tensor NO becomes diagonal, with coefficient proportional to the fluid's kinetic helicity:

$$\alpha = -\frac{\tau_c}{3} h_v \ , \qquad h_v = \langle \boldsymbol{u}' \cdot \nabla \times \boldsymbol{u}' \rangle \ .$$

Where τ_c is the correlation time of the turbulent eddies. This is a very hard quantity to extract from simulations; here we follow astrophysical mixing-length prescription: equating correlation time to estimate of convective turnover time.

MHD: alpha vs kinetic helicity

[Simard et al., Adv. Sp. Res. **58**, 1522 (2016)]



Turbulent diffusivity

Turn now to the second term in the EMF development:

$$\mathcal{E}_i^{(2)} = \beta_{ijk}\partial_k \langle B\rangle_j$$

In cases where **u** is isotropic, we have $\beta_{ijk} = \beta\epsilon_{ijk}$, and thus:

$$\nabla \times \mathcal{E}^{(2)} = \nabla \times \left(-\beta\nabla \times \langle \mathbf{B}\rangle\right) = \beta\nabla^2\langle \mathbf{B}\rangle.$$

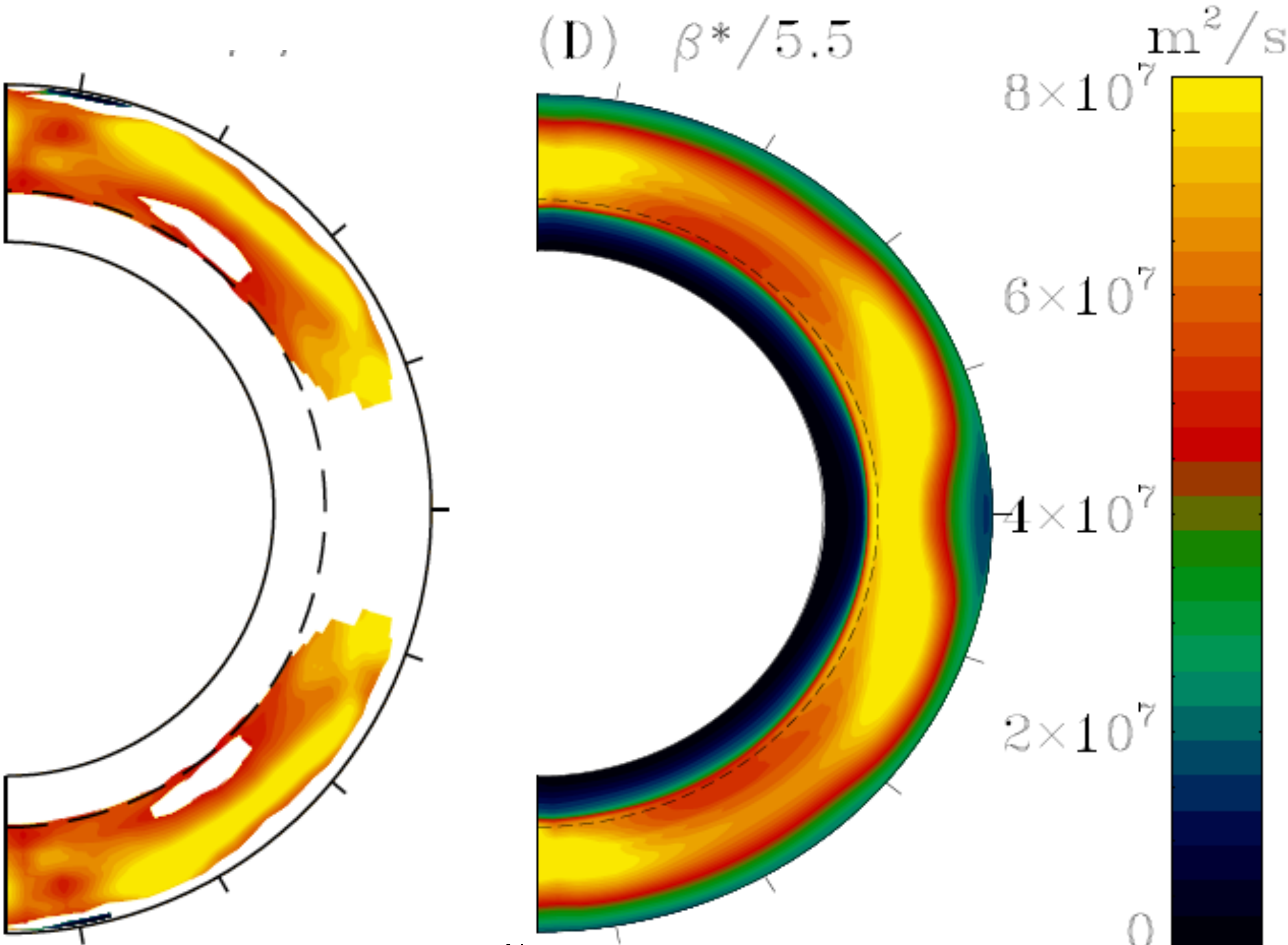
The mathematical form of this expression suggests that β can be interpreted as a **turbulent diffusivity** of the large-scale field. for homogeneous, isotropic turbulence with correlation time τ_c it can be shown that

$$\beta = \frac{1}{3}\tau_c\langle \mathbf{u}^2\rangle \ , \qquad [\mathrm{m}^2\mathrm{s}^{-1}]$$

This result is expected to hold also in mildly anisotropic, mildly inhomogeneous turbulence. In general, $\beta \gg \eta$

MHD: beta vs turbulent intensity

[Simard et al., Adv. Sp. Res. **58**, 1522 (2016)]



Low coherence time turbulence ?

$$\alpha * = -\frac{\tau_c}{3}\langle \mathbf{u}' \cdot \nabla \times \mathbf{u}' \rangle \ , \qquad (/ 4.5)$$

$$\beta * = \frac{\tau_c}{3}\langle (u')^2 \rangle \ , \qquad (/ 5.5)$$

Simulations and analytical (SOCA) theory match, but with an amplitude error by a factor of ~5; can be « explained » if coherence time of turbulent eddies is smaller than turnover time by a comparable factor.

Conjecture: the scale-dependent implicit dissipation introduced by MPDATA leads to low correlation time turbulence in the physical regime of our solar simulations.

Where do we go next ?

Understand what sets the cycle period(s)

[Talk by A. Strugarek]

Understand physical underpinnings of the cyclic modulation of the convective energy flux

[Talk by J.-F. Cossette]

Comparative benchmark with ASH simulations

[Talk by A. Strugarek]

Understand role of tachocline instabilities in long term behavior of simulations, and possible role in triggering Maunder-Minimum-like period of strongly reduced activity

[Lawson et al., *Astrophys. J.*, **813**, 95 (2015)]

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Fonds de recherche sur la nature et les technologies
Québec

Collaborators: Piotr Smolarkiewicz, Dario Passos, Antoine Strugarek, Jean-François Cossette, Patrice Beaudoin, Corinne Simard, Nicolas Lawson, Amélie Bouchat, Mihai Ghizaru, Caroline Dubé.