Behind the scenes: Benchmarking sub-grid scales models in global simulations of stellar convective dynamo

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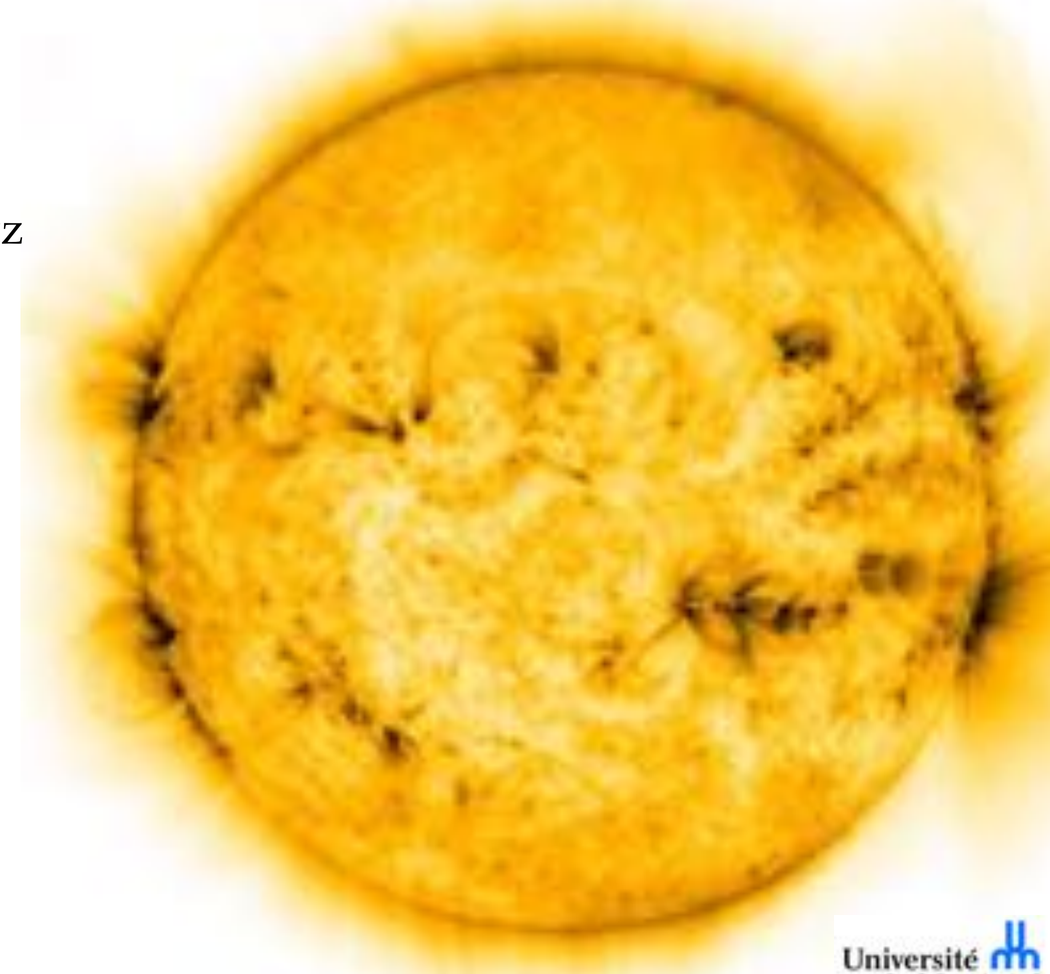

1. Introduction

2. Implicit *vs* Explicit dissipation: modeling subgrid-scales effects

3. Numerical simulations of stellar dynamos: a new take on cyclicality

4. Conclusions

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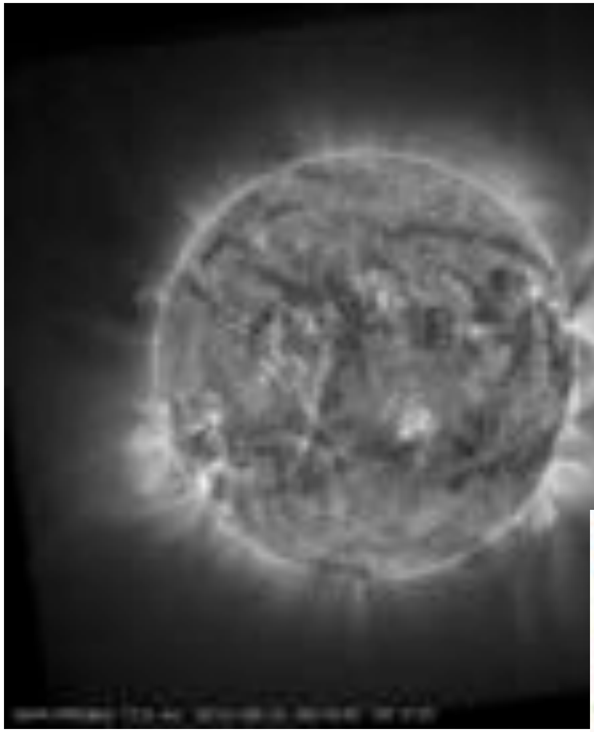
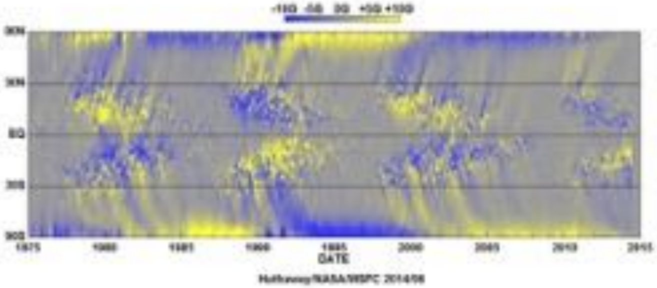
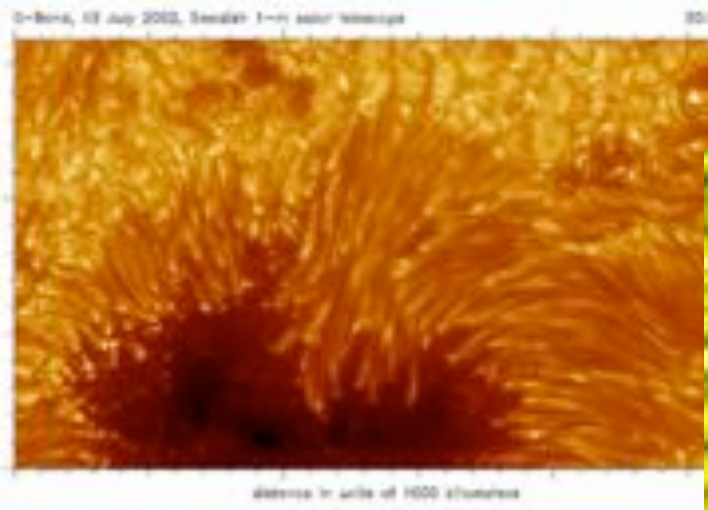
The many scales of solar magnetism

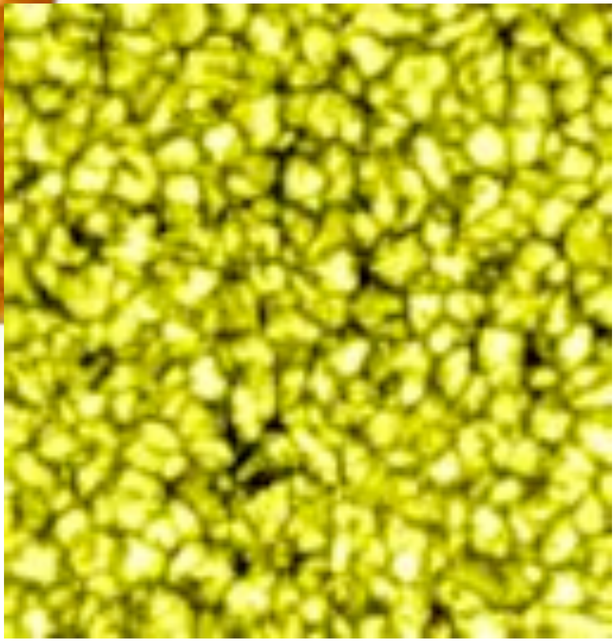




Sun
Size ~ 700 Mm
Rotation ~ month
Cycle ~ 11 years

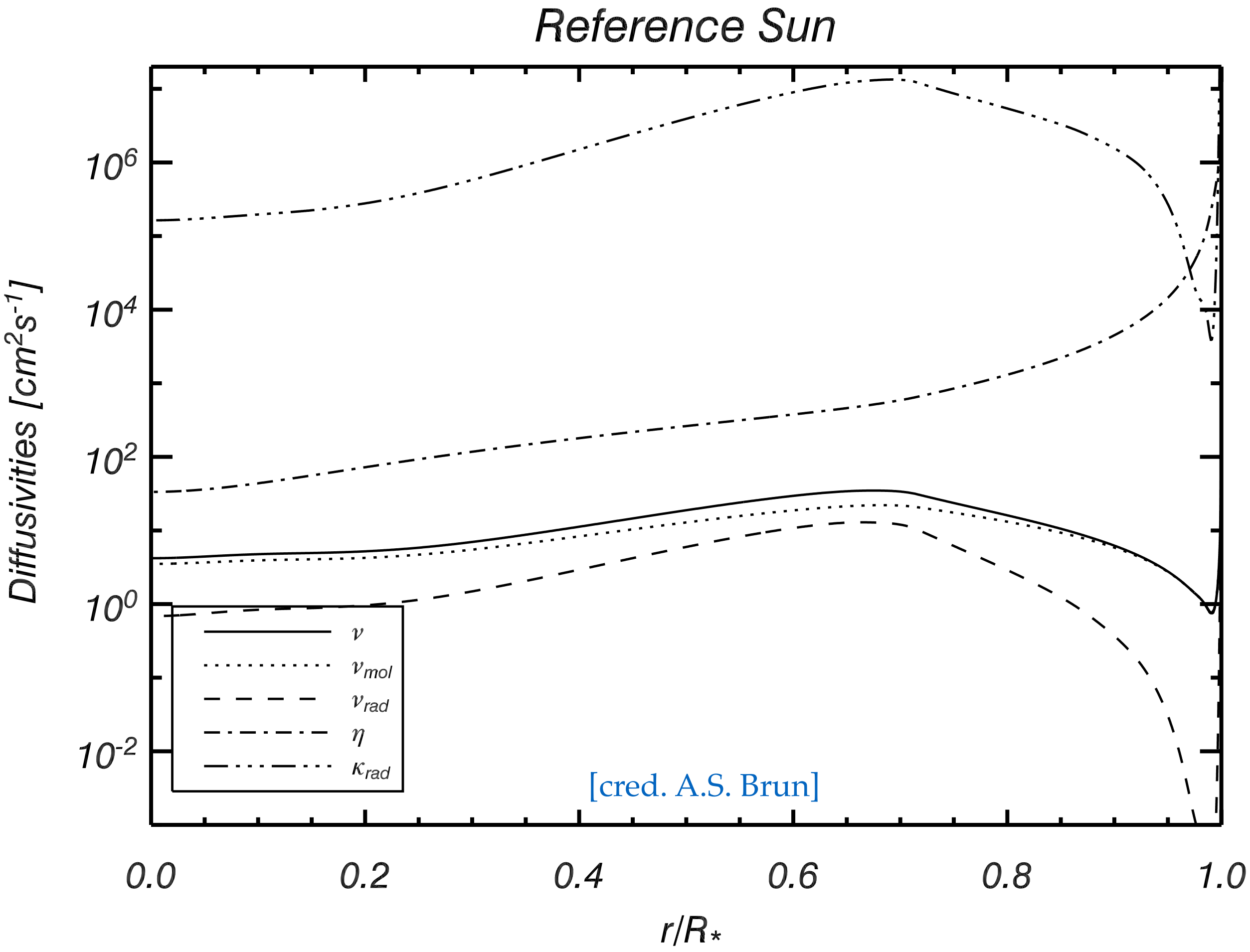
Granules
Size ~ 1 Mm
Life ~ 10 minutes





Spots
Size ~ 10 Mm
Life ~ days

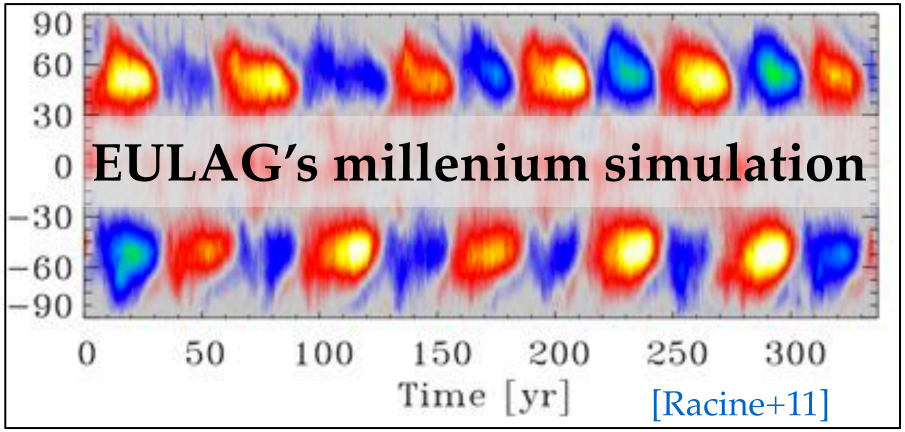
Challenge: ab-initio models of convective dynamos



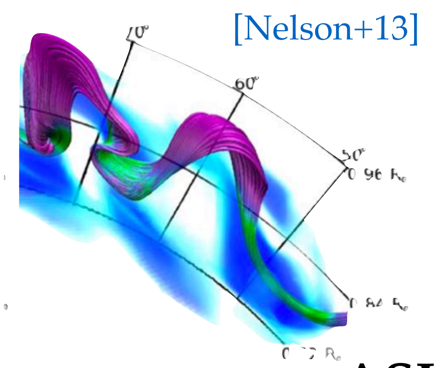
[cred. A.S. Brun]

Tremendously high
Reynolds (flow),
Taylor (rotation),
and Rayleigh (buoyancy,heat)
numbers

Variety of 'stellar' convective dynamos today

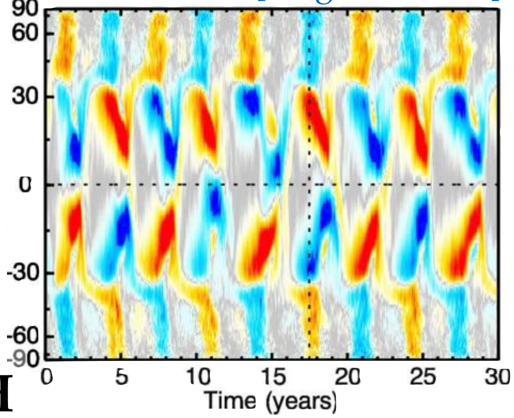


[Racine+11]



[Nelson+13]

[Augustson+15]



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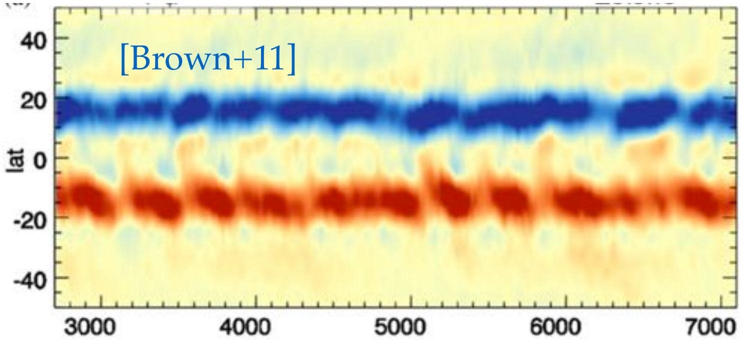
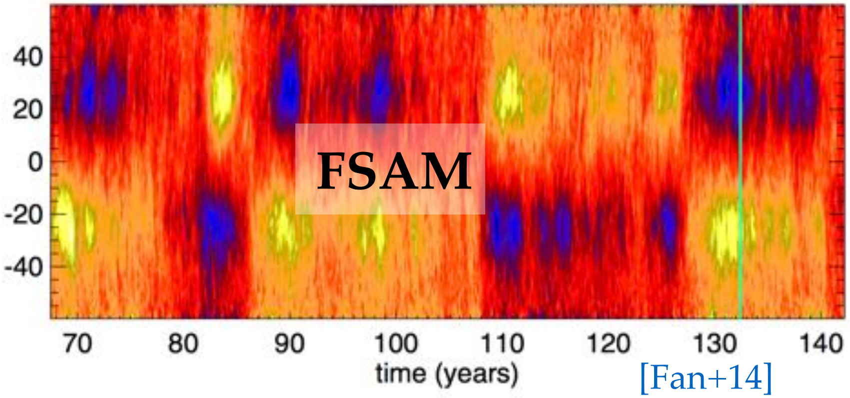
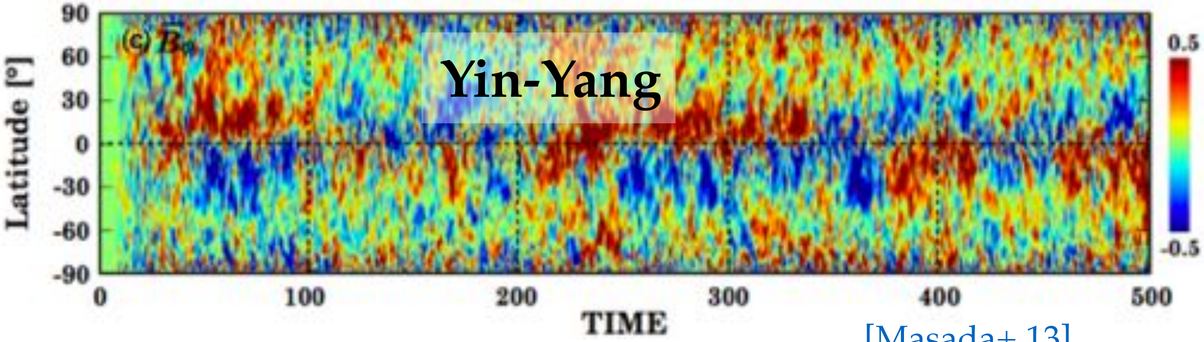


[Brown+11]

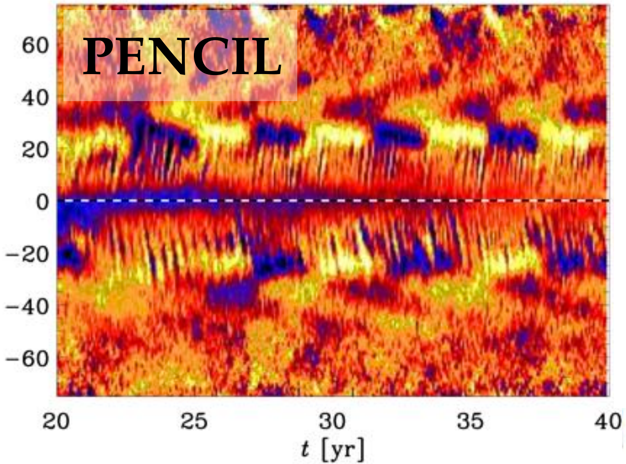


[Fan+14]

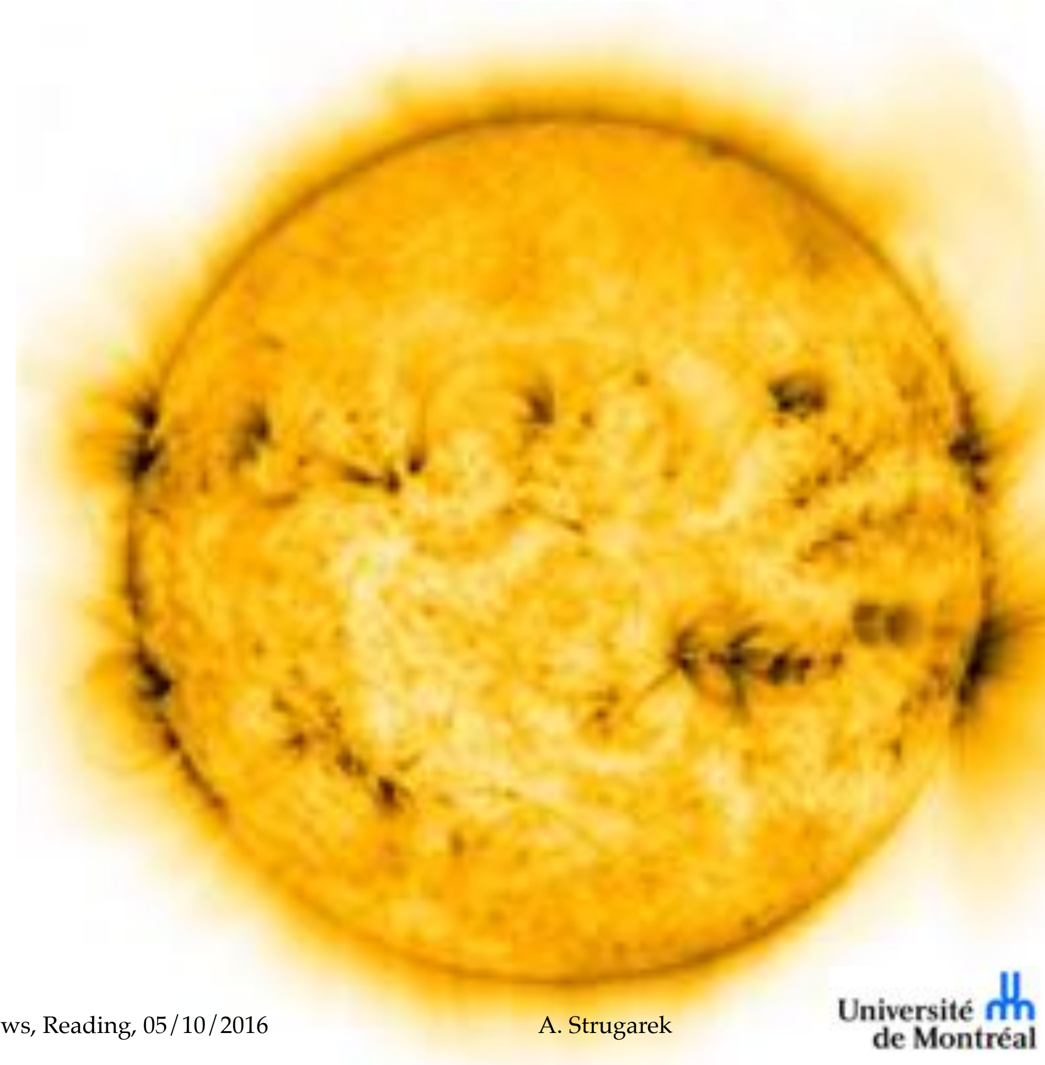
[Kapyla+12,Warnecke+14]



[Masada+ 13]



Benchmarking convective dynamo simulations: a first take on convection



Strugarek+ 2016, Advances in Space Research

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A set of anelastic MHD equations

$$\mathrm{d}_t \mathbf{u} = -\nabla\left(\frac{p}{\bar{\rho}}\right) - \frac{S}{c_p}\mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{u} + \frac{1}{\bar{\rho}}\mathbf{J}\times\mathbf{B} + \frac{1}{\bar{\rho}}\boldsymbol{\nabla}\cdot\mathcal{D}_\nu$$

$$\mathrm{d}_t S = -(\mathbf{v}\cdot\boldsymbol{\nabla})S_a - \frac{S}{\tau} + Q_{\kappa,\nu,\eta}$$

$$\mathrm{d}_t \mathbf{B} = (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{u} - (\boldsymbol{\nabla}\cdot\mathbf{u})\mathbf{B} + \boldsymbol{\nabla}\cdot\mathcal{D}_\eta$$

τ ~ 20 rotations

$$\nabla\cdot(\bar{\rho}\mathbf{u}) = 0$$

$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0$$

Dissipation

+

Sub-grid scales effects

☛ Background state based on the anelastic benchmark of Jones+ 2011

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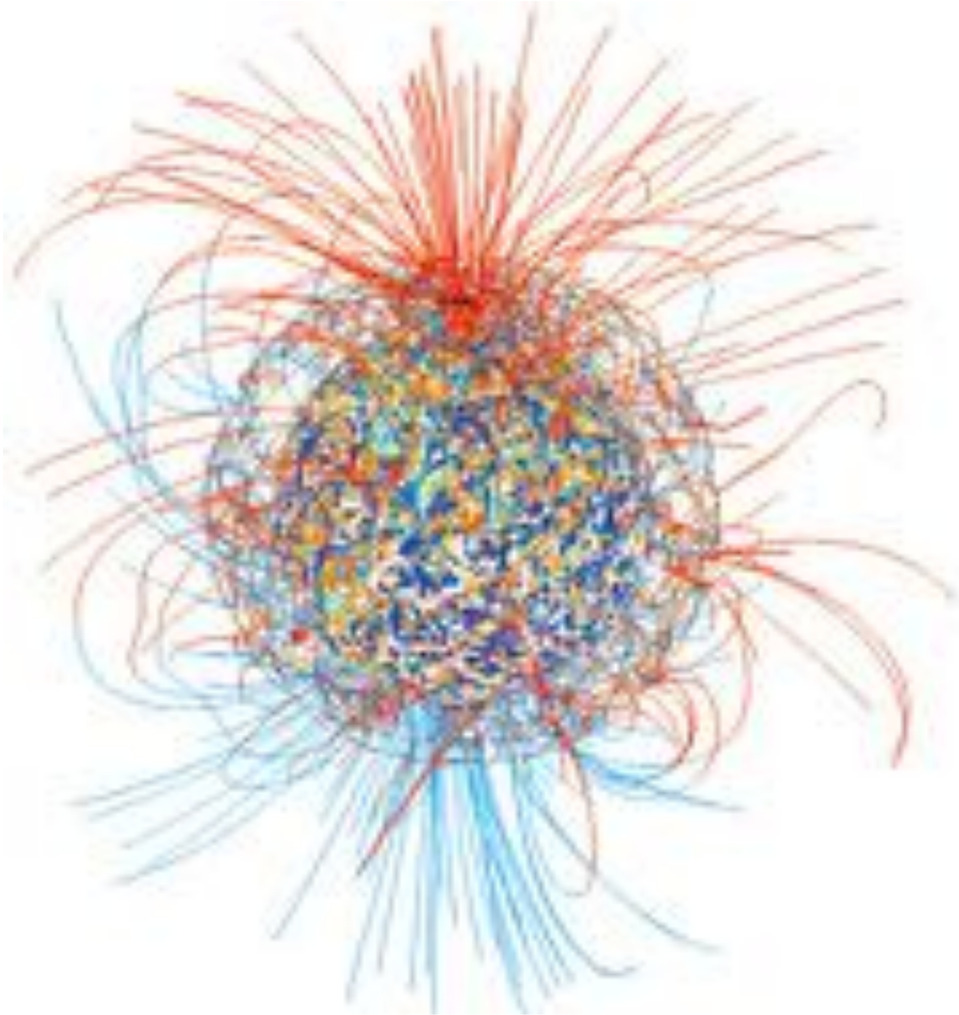
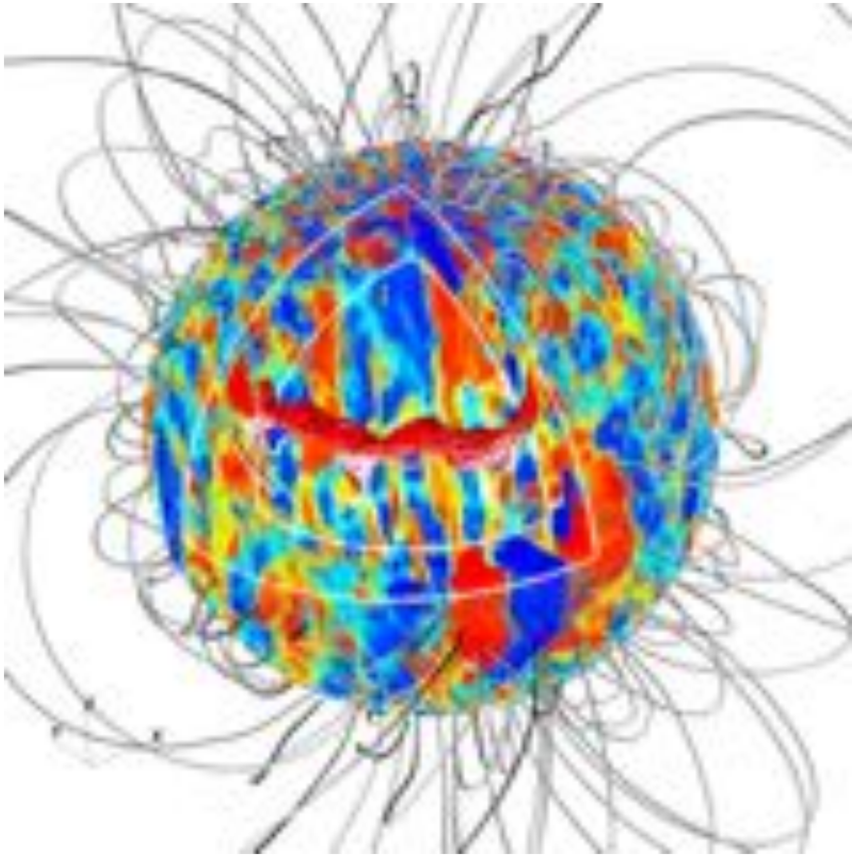
EULAG-MHD





Enhanced diffusion

(& dynamic Smagorinsky, SLD)

Pseudo-Spectral

Implicit dissipation

(& explicit diffusion)

Finite volumes

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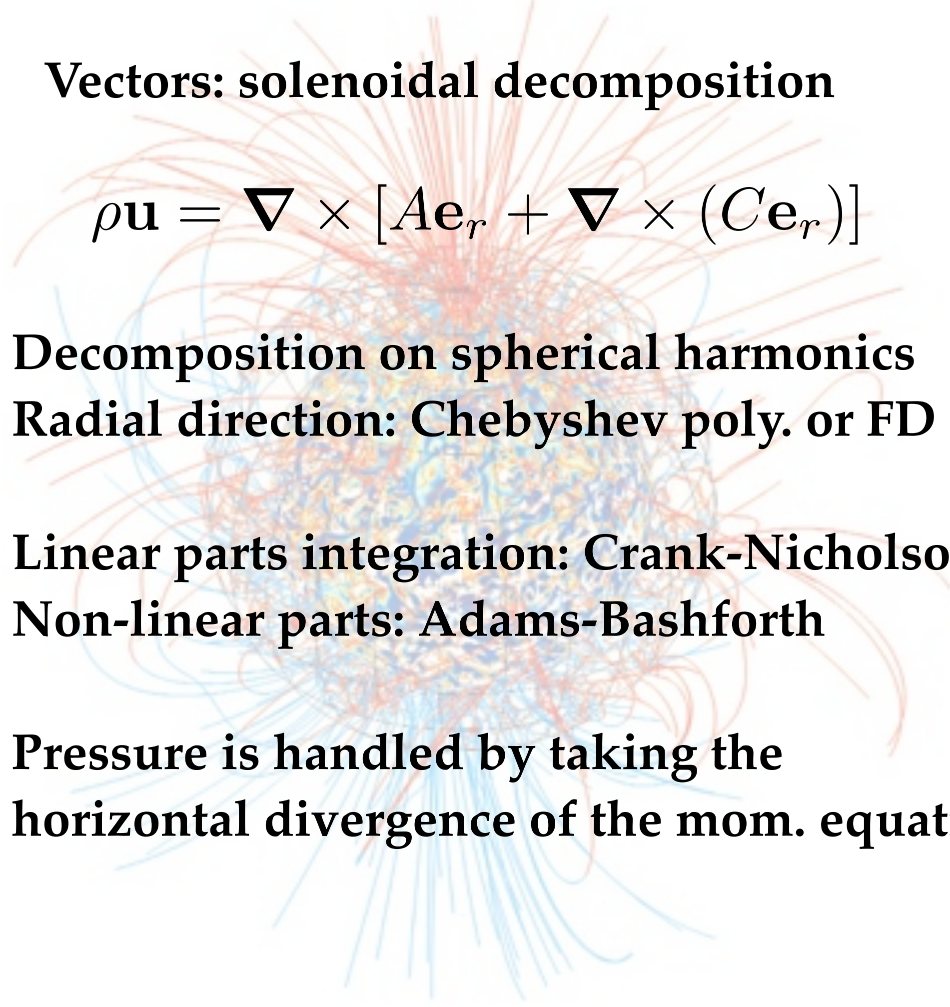


Vectors: solenoidal decomposition

$$\rho\mathbf{u} = \boldsymbol{\nabla} \times \left[A\mathbf{e}_r + \boldsymbol{\nabla} \times (C\mathbf{e}_r)\right]$$

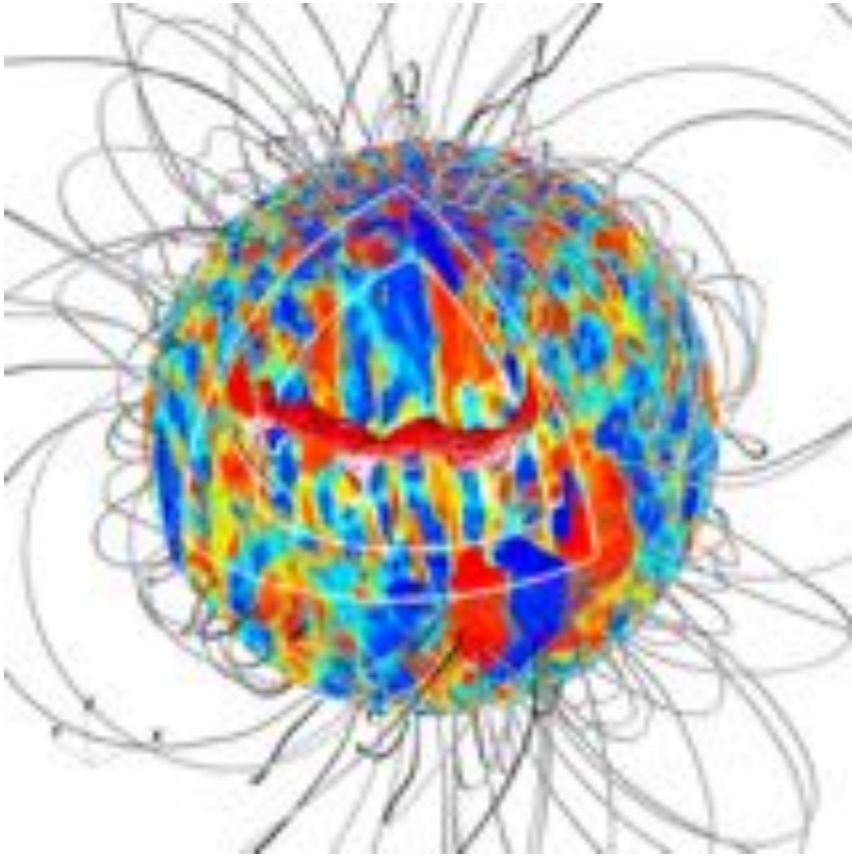
Decomposition on spherical harmonics
Radial direction: Chebyshev poly. or FD

Linear parts integration: Crank-Nicholson
Non-linear parts: Adams-Bashforth

Pressure is handled by taking the horizontal divergence of the mom. equation

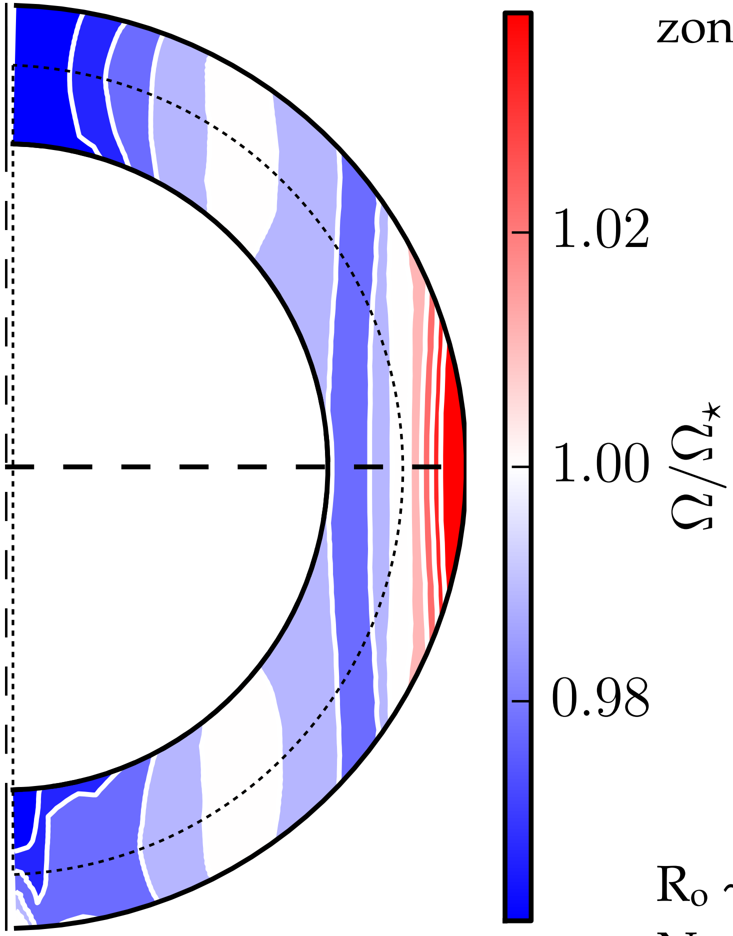
Enhanced diffusion
(& dynamic Smagorinsky, SLD)
Pseudo-Spectral

EULAG-MHD



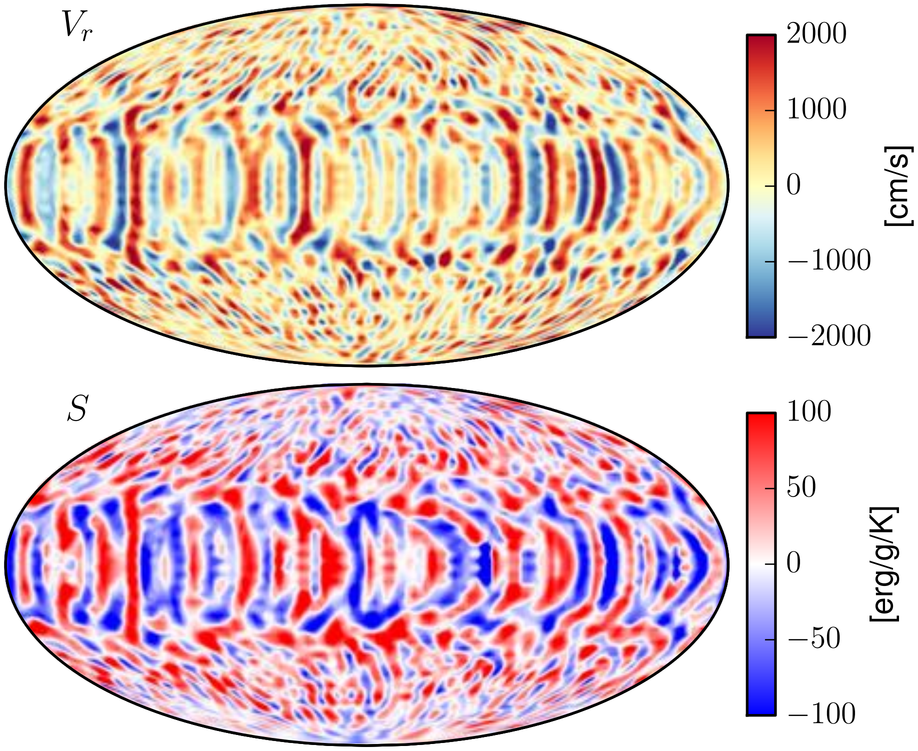
Implicit dissipation
(& explicit diffusion)
Finite volumes

A simple convection simulation with EULAG (I)



[cf Jones+ 2011, Icarus]

A ‘benchmark’ simulation of a turbulent convection zone generating cylindrical Ω-contours



$R_o \sim 0.06$

$N_\varrho = 1.5$

$\Delta S = 2\ 10^3\ \mathrm{erg/K/g}$

Kinetic energy balance: scale-by-scale budget

Coriolis

Buoyancy

Reynolds stress

$$\partial_t E_L^K = \mathcal{C}_{L\pm1} + \mathcal{P}_L + \mathcal{G}_L + \mathcal{V}_L + \sum_{L_1,L_2} \mathcal{R}_{L_1,L_2}$$

Pressure Gradient

Viscous & subgrid model

Clebsch-Gordan coefficients

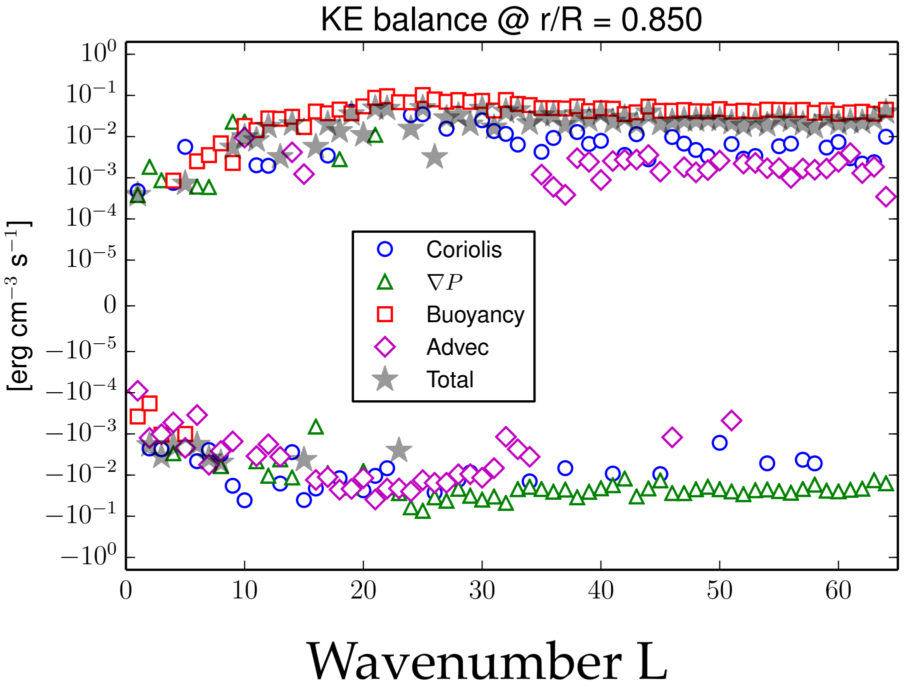
Statistical steady-state:

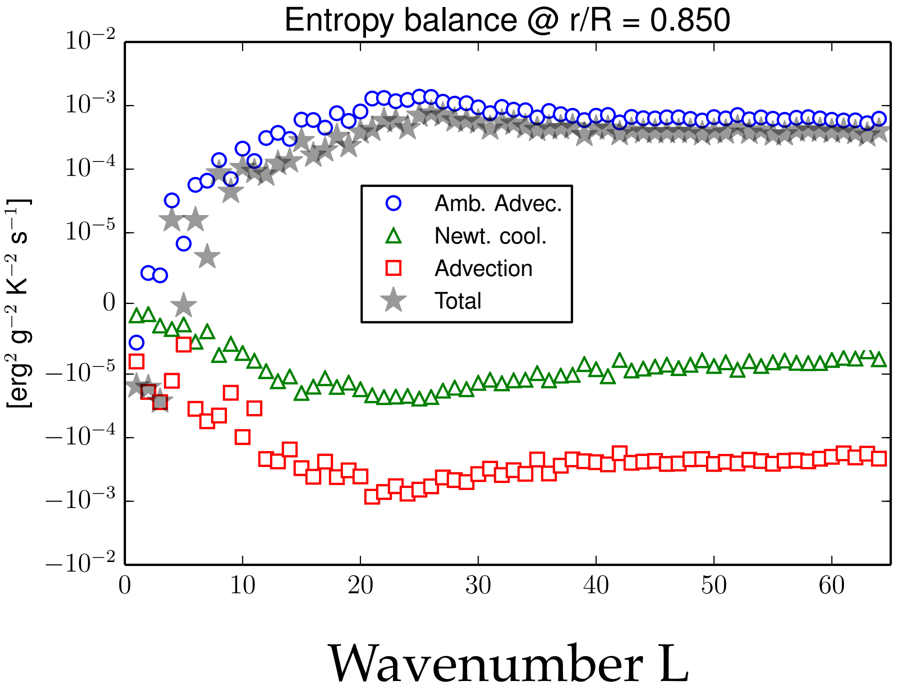
$$-\mathcal{V}_L = \mathcal{C}_{L\pm1} + \mathcal{P}_L + \mathcal{G}_L + \sum_{L_1,L_2} \mathcal{R}_{L_1,L_2}$$

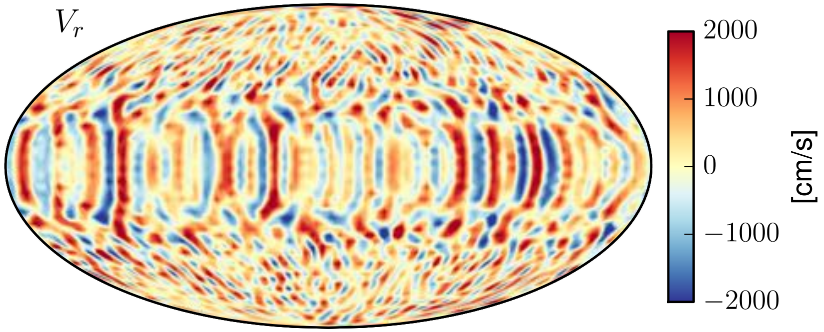
The same procedure can be repeated for the heat equation

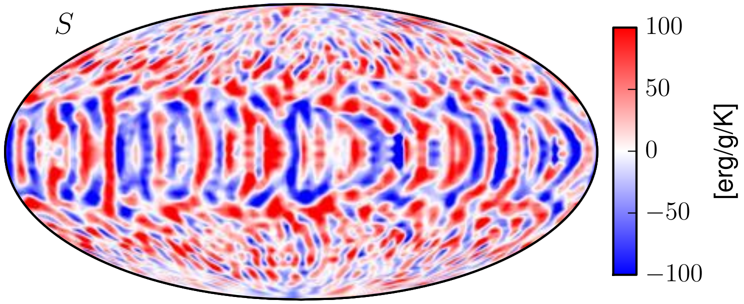
[*cf* Strugarek+13]

Spectral energy transfer in EULAG simulation

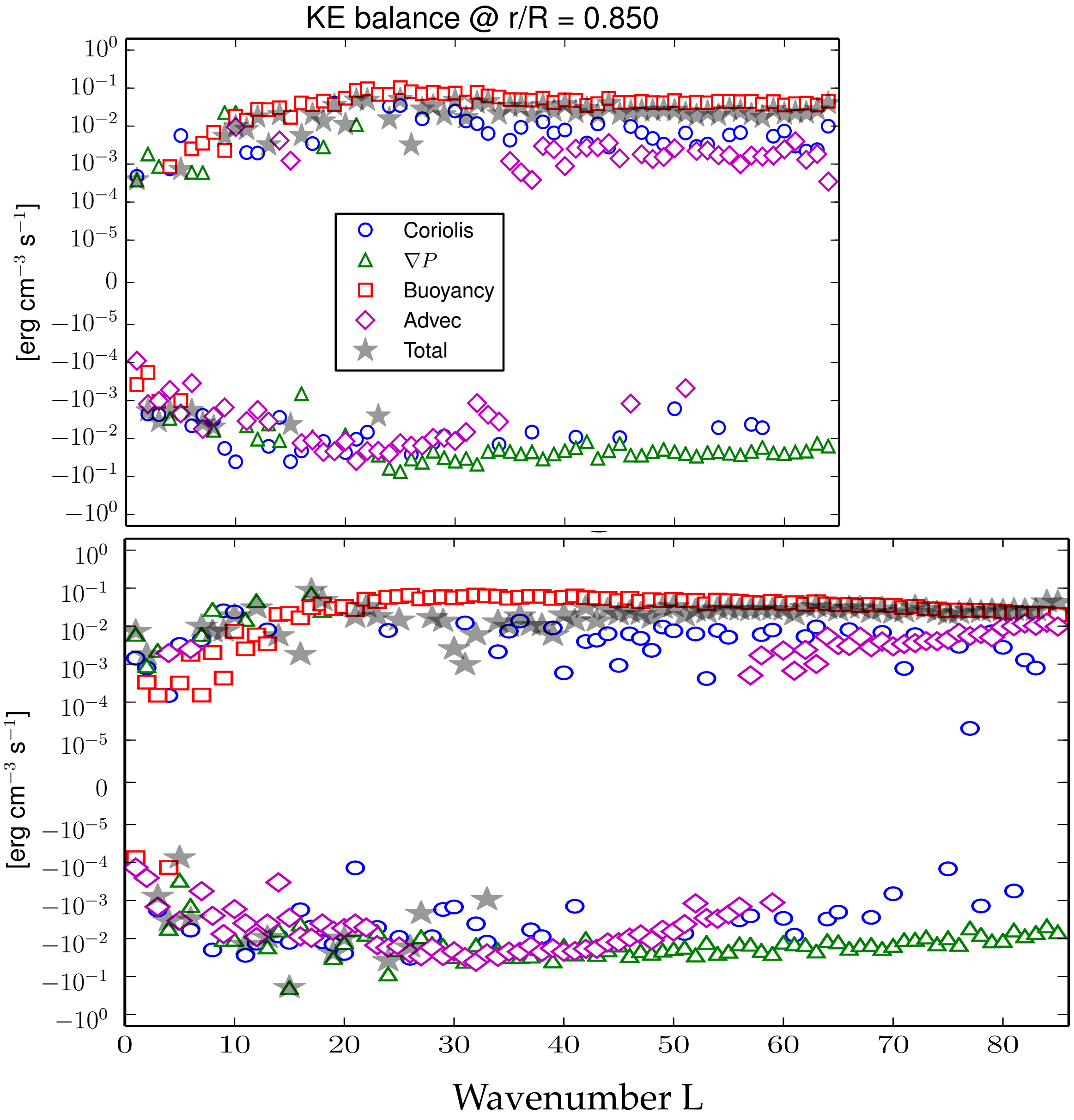


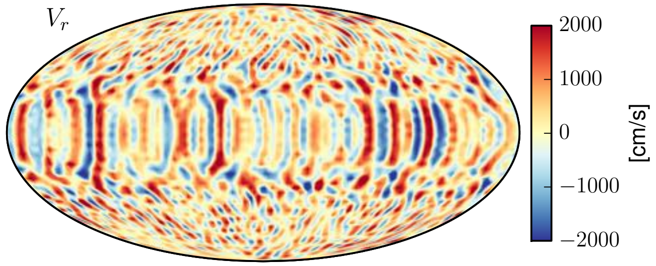


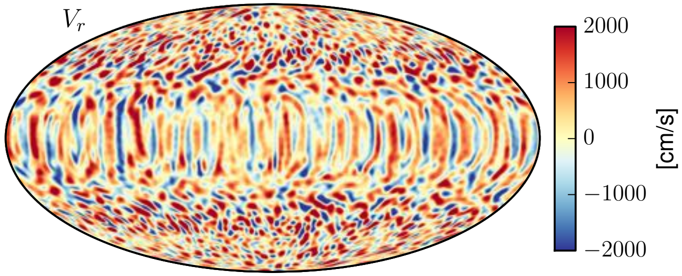




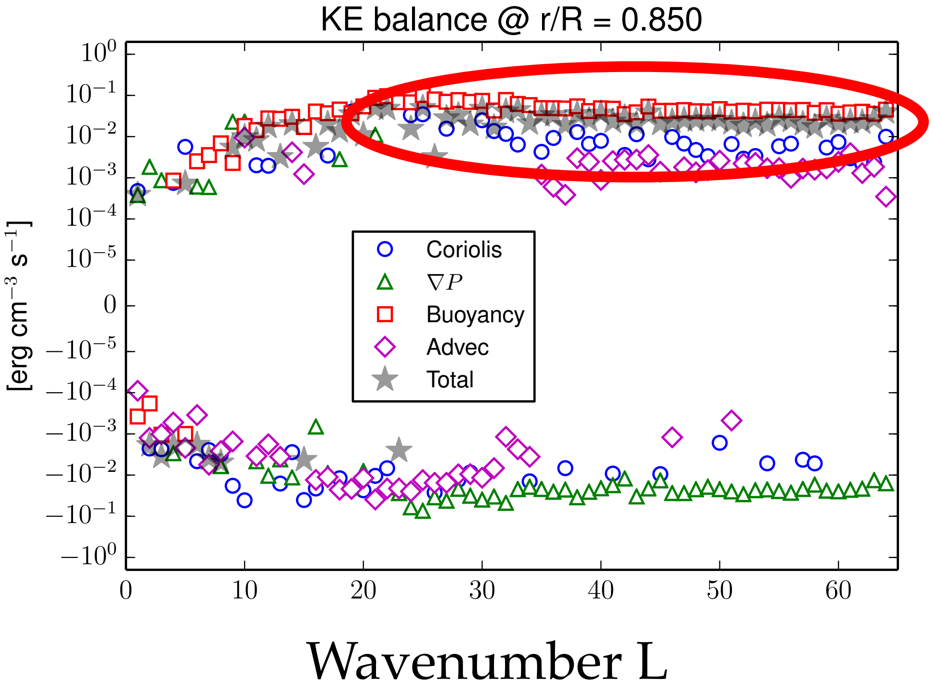
Spectral energy transfer in EULAG simulation

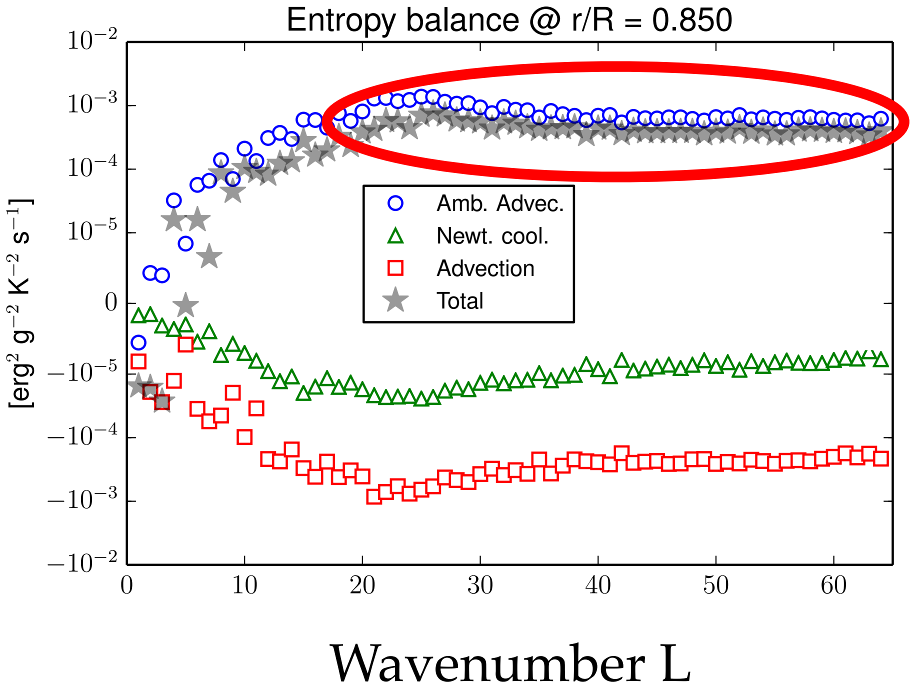


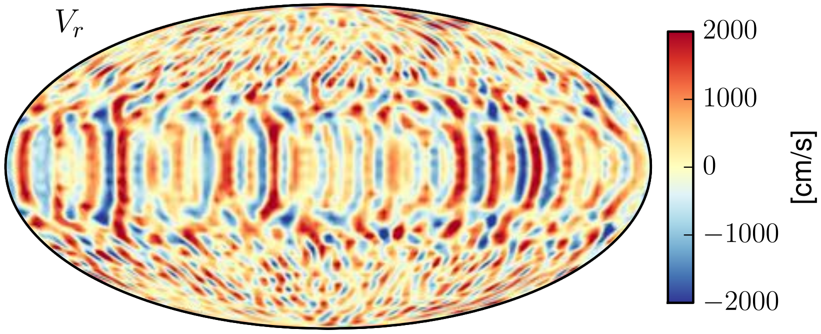


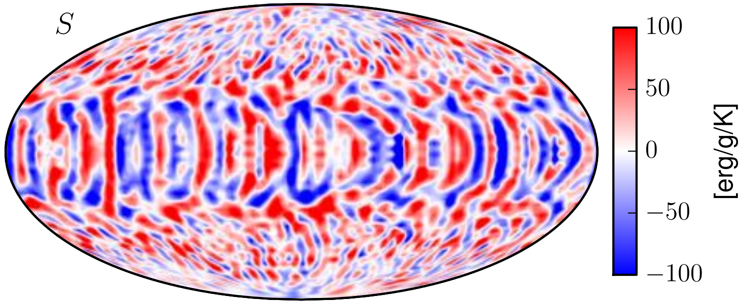


Spectral energy transfer in EULAG simulation





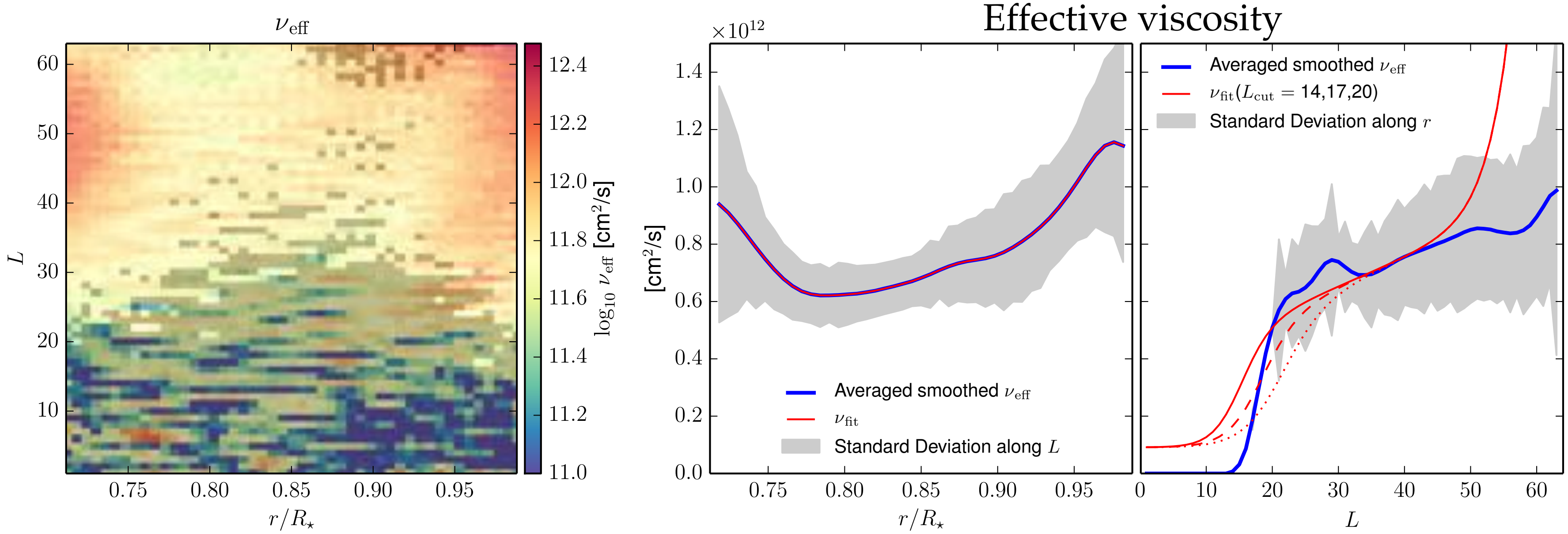




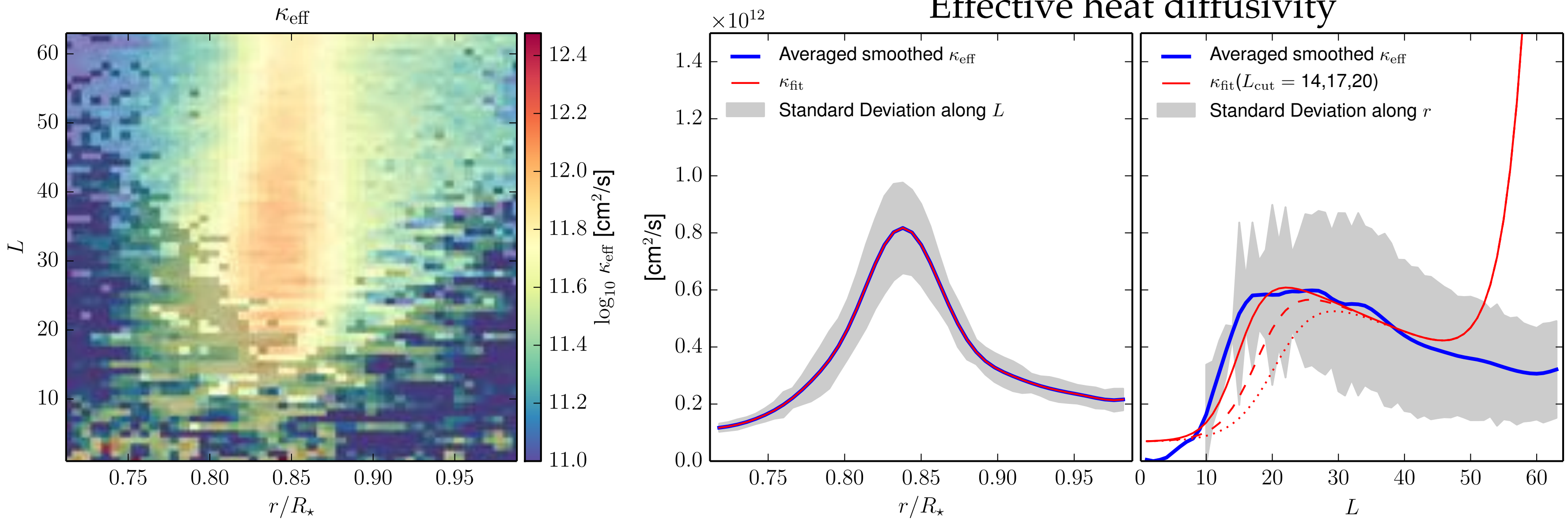
[Strugarek+ 2016]

Effective dissipation coefficients in EULAG

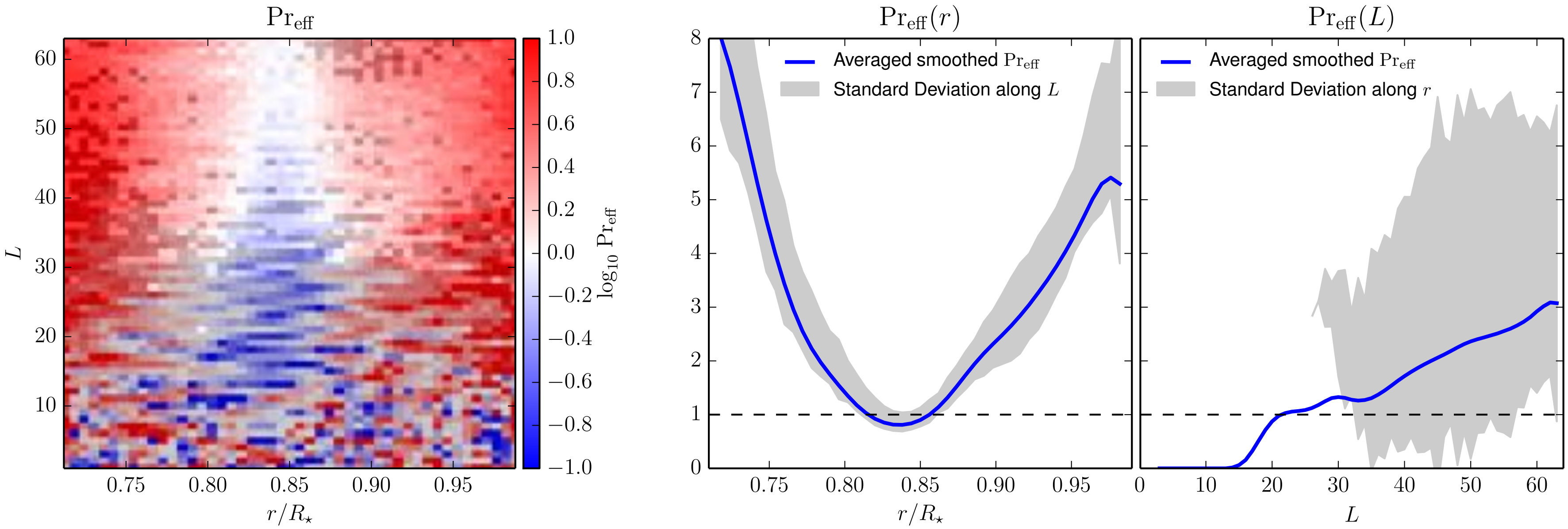
Effective viscosity



Effective heat diffusivity

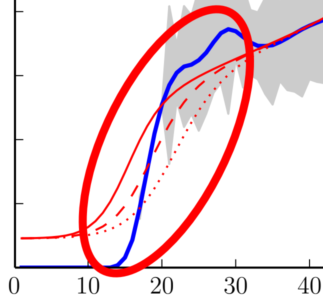


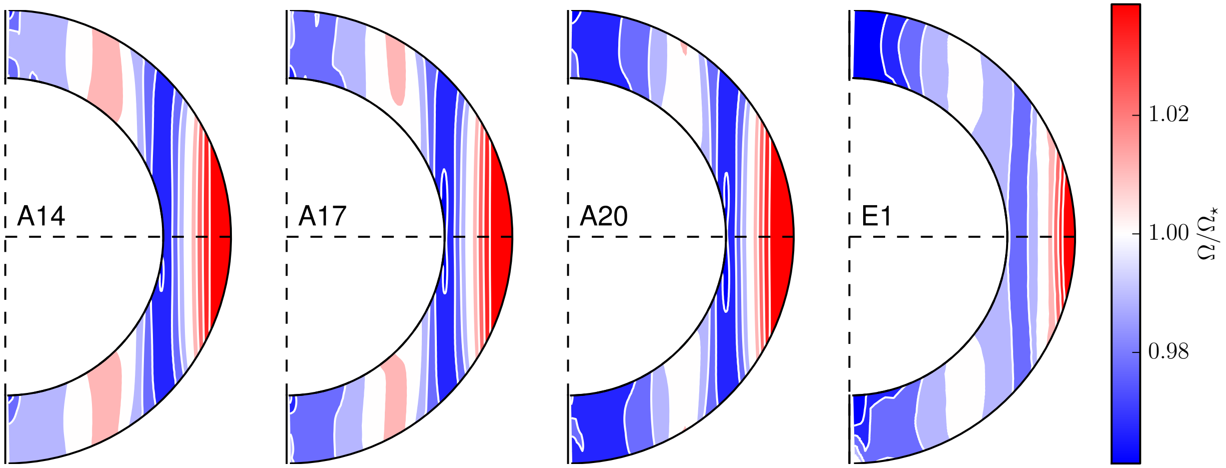
Effective dissipation coefficients in EULAG

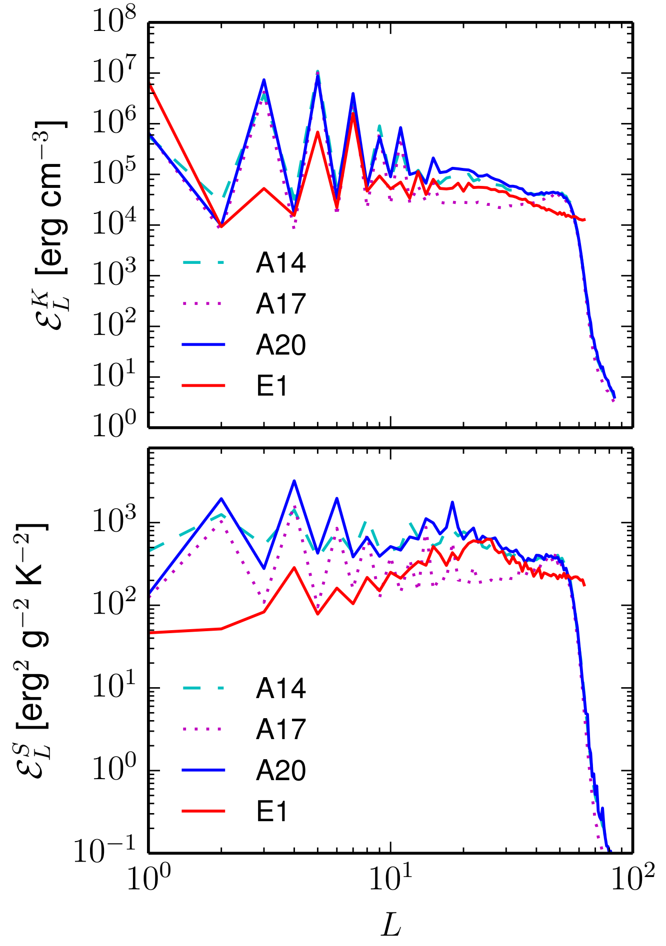


$$\mathrm{Pr}_{\mathrm{eff}} = \frac{\nu_{\mathrm{eff}}}{\kappa_{\mathrm{eff}}}$$

Comparison with an ‘equivalent’ ASH simulation







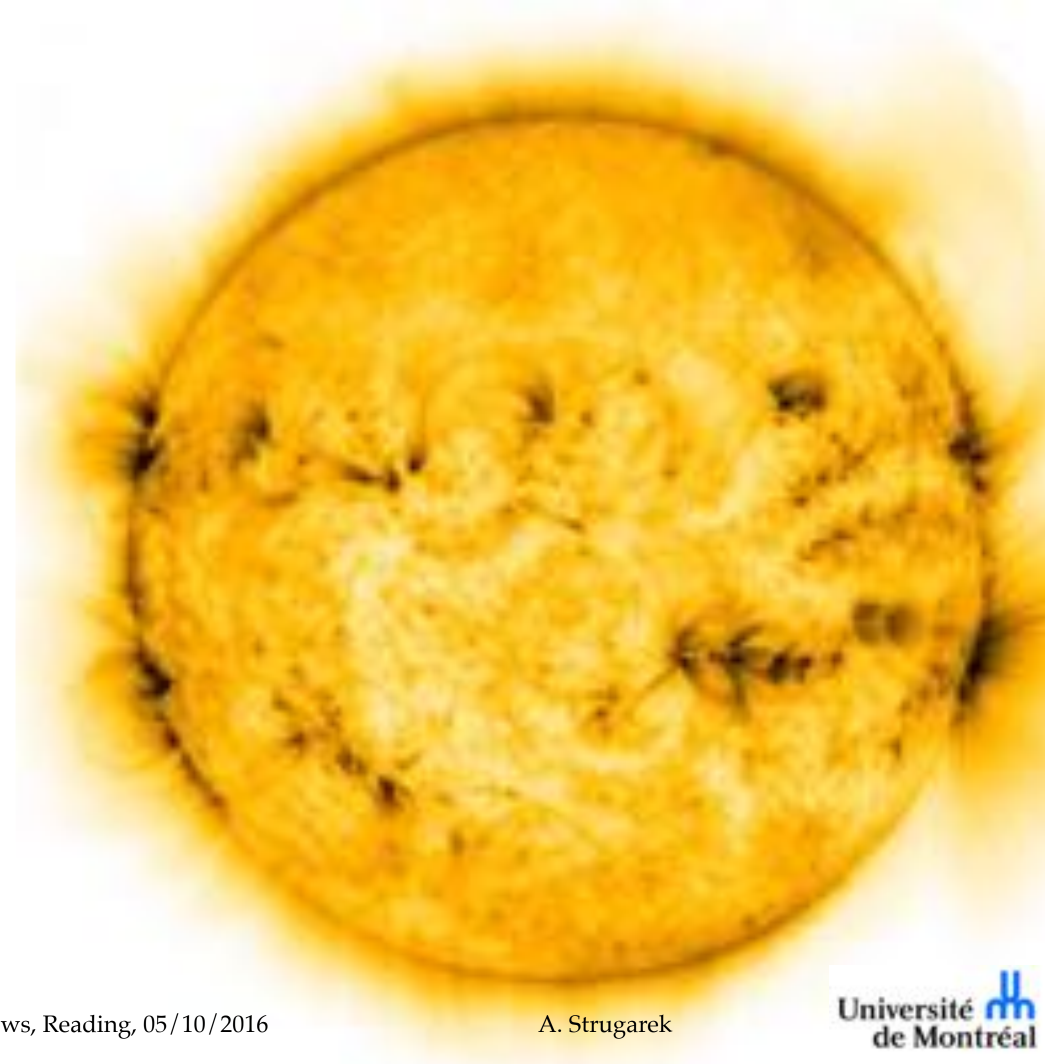
In the ASH simulation, we have degrees of liberty as to what amount of dissipation to put at **large** and **small scales**

Encouraging results: **qualitatively similar DR profile** obtained with an ASH simulation with fitted κ,ν

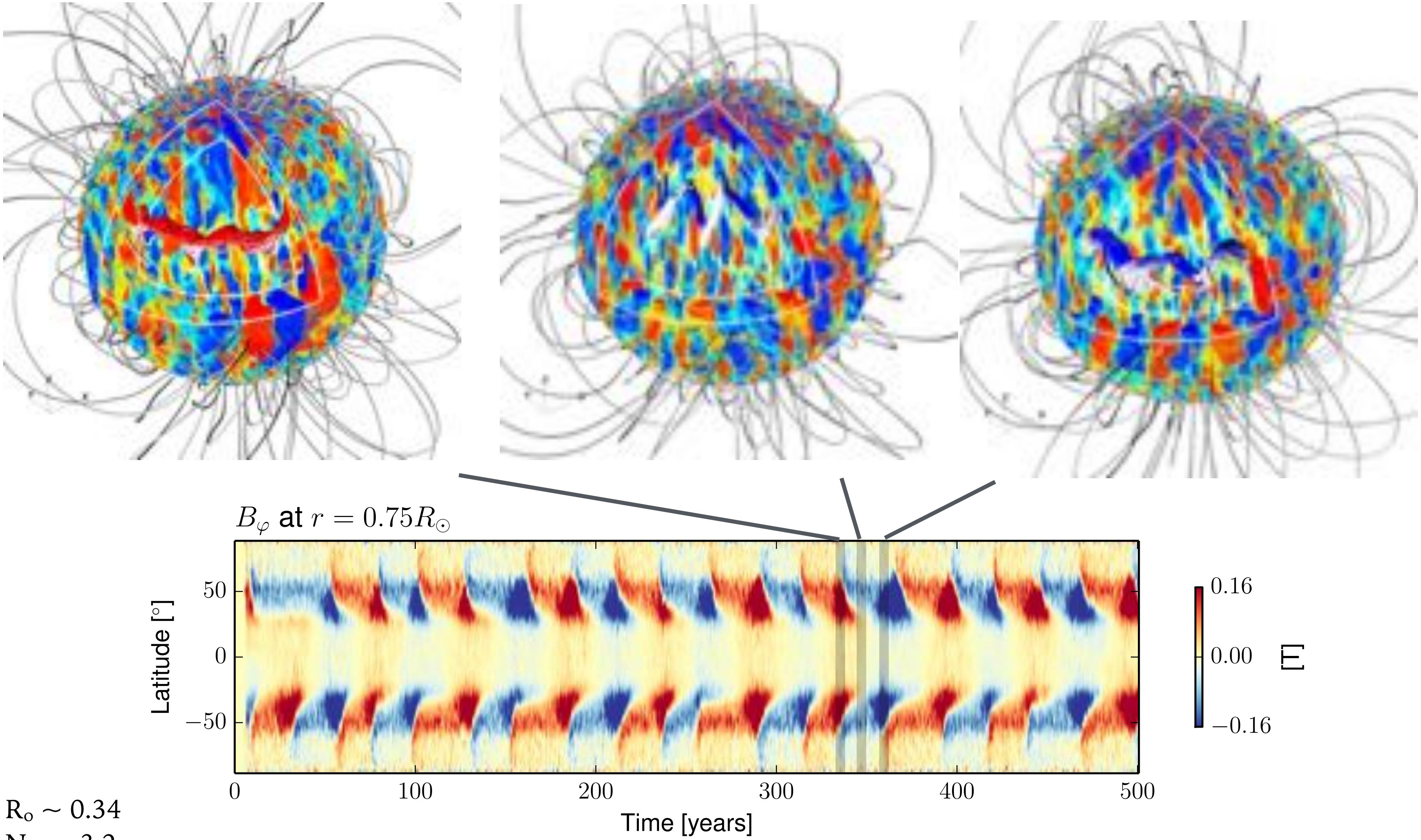
Next: comparing dynamo cases with the same formalism…

[Strugarek+ ASR 2016]

A new take on stellar magnetic cycles



Prototype cyclic dynamo in a convective enveloppe



$R_o \sim 0.34$
$N_\rho = 3.2$
$\Delta S = 10^4$ erg/K/g

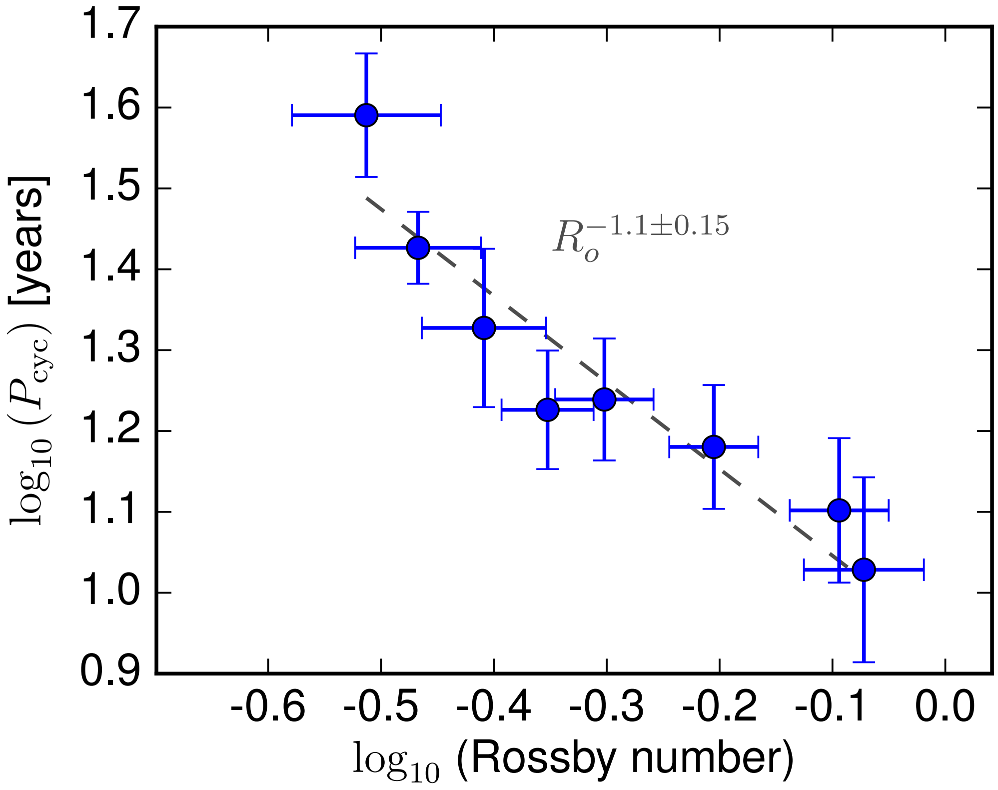
Cycle period is inversely proportional to the Rossby number

Basic ingredients of stellar dynamos

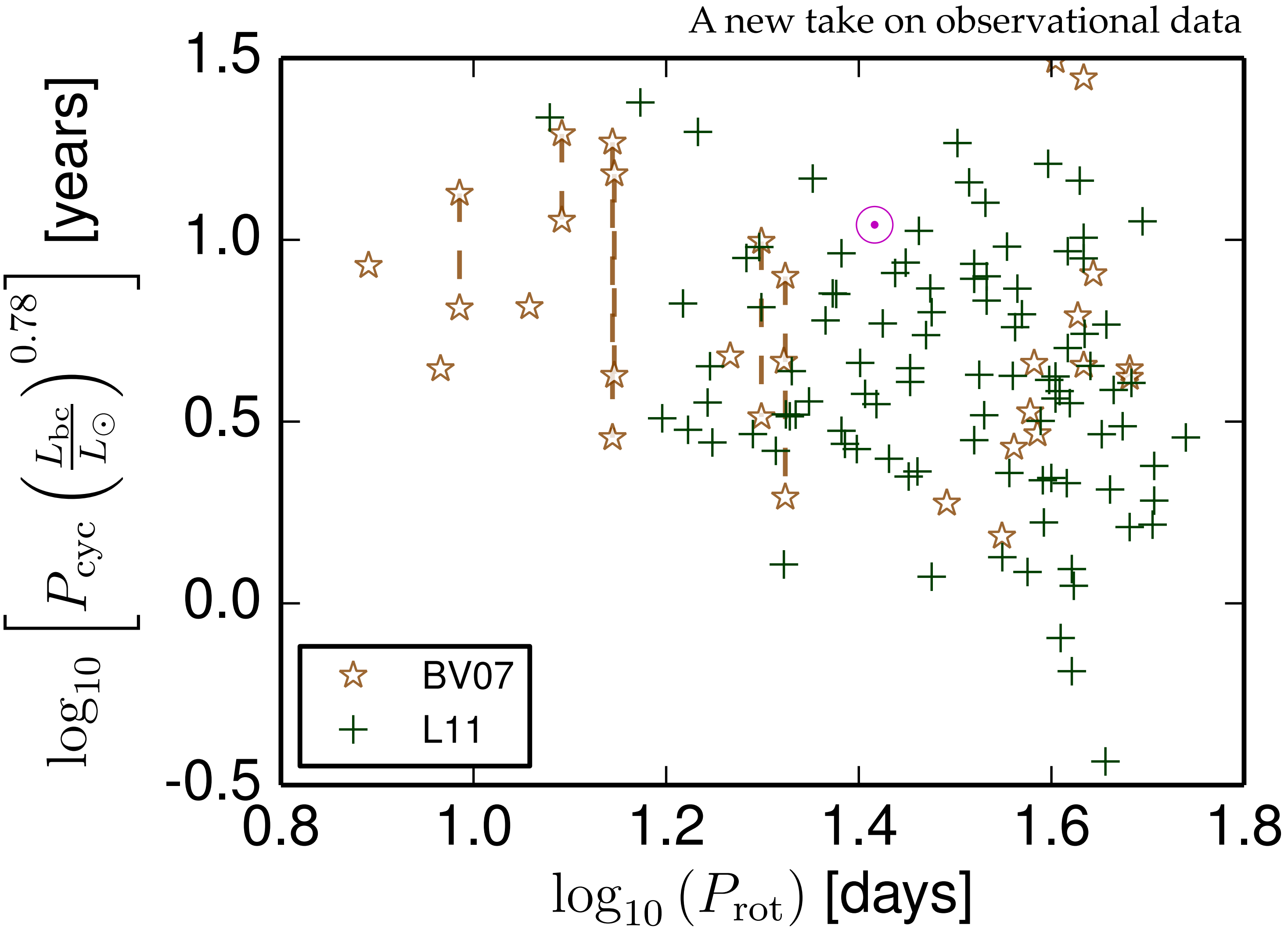
- Differential rotation
- Cyclonic turbulence

'Go to' parameter is the **Rossby number**

$$R_o = \frac{\text{Convection}}{\text{Coriolis}} \sim \frac{|\boldsymbol{\nabla} \times \mathbf{U}|}{2\Omega_\star}$$

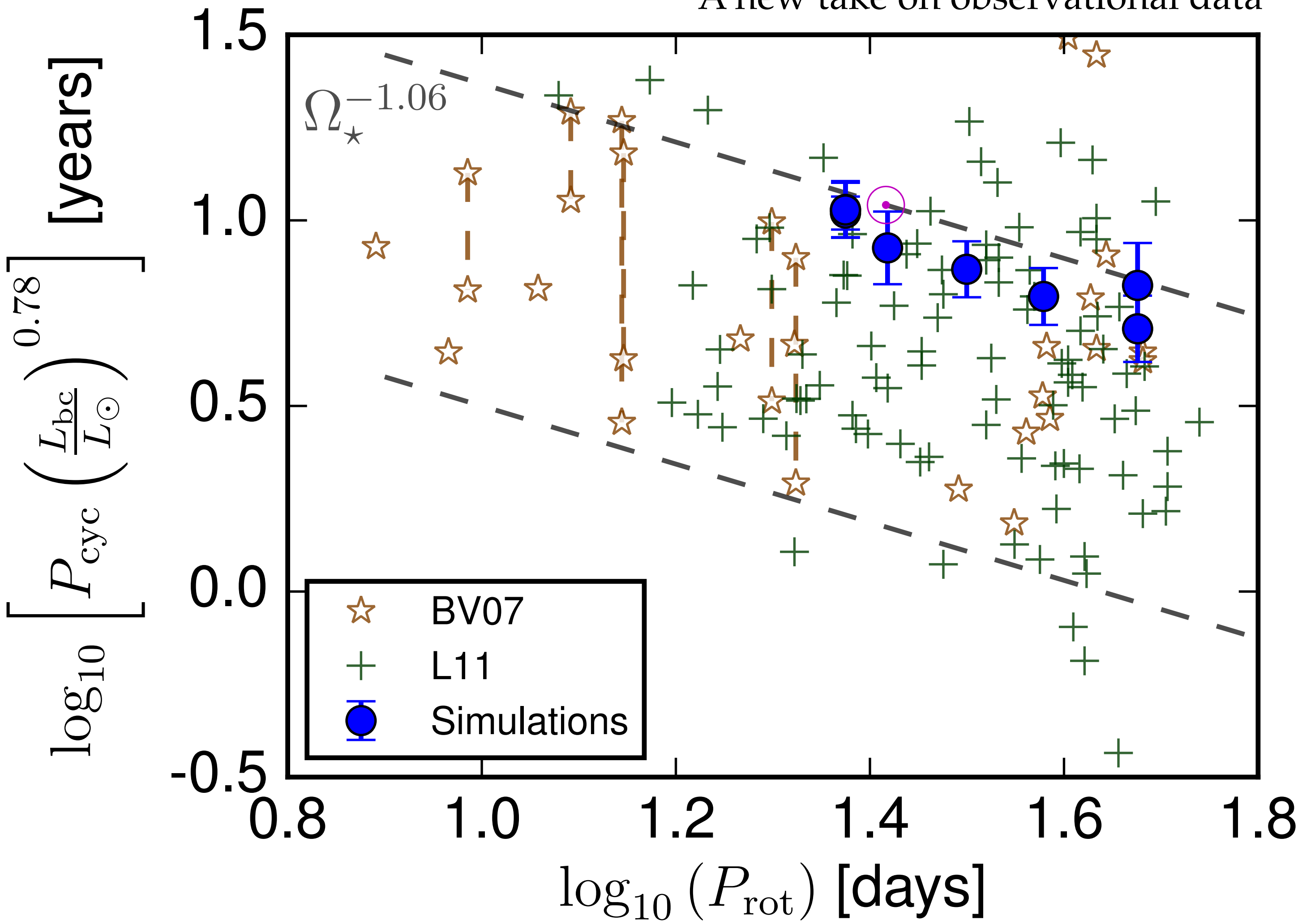


Magnetic cycles in a stellar context



Magnetic cycles in a stellar context

A new take on observational data



Conclusions & perspectives

A **convective dynamo benchmark** has been successfully developed to specifically study the impact of sub-grid scales modelling (dynamo comparison to be achieved soon)

A **powerful method** based on **spectral transfers analysis** is proposed to relate **implicit** and **explicit** sub-grid scales models

For the first time we are able to simulate **cycles in 3D turbulent convection zone** that **vary systematically** with the large scale parameters of the star (**rotation, luminosity**)

Very promising first comparisons with observational data

Future prospects: vary the **convection zone aspect ratio**, and explore **states with higher degree of turbulence** (closer to real stars)