New covariance statistics of model error for use in weak-constraint 4DVar

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April 25, 2016

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Outline

1. Motivation
2. Calculating the Model Error Covariance Matrix
3. Experimentation and Results

Weak-Constraint 4DVar

- Data assimilation for NWP has reached a level of accuracy where model errors can no longer be neglected.
- Taking model error into account in 4D-Var requires that we specify a covariance matrix for model error.

$$
\begin{aligned}
J(\mathbf{x}) = \frac{1}{2}(x_0 - x_b)^{\mathrm{T}}\mathbf{B}^{-1}(x_0 - x_b) \\
+ \frac{1}{2}\sum_{k=0}^{N}(\mathcal{H}_k(x_k) - y_k)^{\mathrm{T}}\mathbf{R}_k^{-1}(\mathcal{H}_k(x_k) - y_k) \\
+ \frac{1}{2}\sum_{k=0}^{N}(\mathcal{M}(x_{k-1}) - x_k)^{\mathrm{T}}\mathbf{Q}_k^{-1}(\mathcal{M}(x_{k-1}) - x_k)
\end{aligned}
$$

Weak-Constraint 4DVar with model error forcing

$$J(\mathbf{x}_0, \eta) = \frac{1}{2}(x_0 - x_b)^{\mathrm{T}}\mathbf{B}^{-1}(x_0 - x_b) + \frac{1}{2}\sum_{k=0}^{N}(\mathcal{H}_k(x_k) - y_k)^{\mathrm{T}}\mathbf{R}_k^{-1}(\mathcal{H}_k(x_k) - y_k) + \frac{1}{2}\eta^{\mathrm{T}}\mathbf{Q}^{-1}\eta$$

- with $x_k = \mathcal{M}(x_{k-1}) + \eta_k$
- η_k is propagated by the model
- η_k represents the instantaneous model error

Weak-Constraint 4DVar with cycling term

- Model error is both random and systematic.
- For the systematic part the cost function is:

$$J(\mathbf{x}_0, \eta) = \frac{1}{2}(x_0 - x_b)^{\mathrm{T}}\mathbf{B}^{-1}(x_0 - x_b) + \frac{1}{2}\sum_{k=0}^{N}(\mathcal{H}_k(x_k) - y_k)^{\mathrm{T}}\mathbf{R}_k^{-1}(\mathcal{H}_k(x_k) - y_k) + \frac{1}{2}(\eta - \eta_b)^{\mathrm{T}}\mathbf{Q}^{-1}(\eta - \eta_b)$$

- η_b is like a background to the model error
- In the following experiments a constant forcing over the assimilation window is used.

Weak-Constraint 4DVar

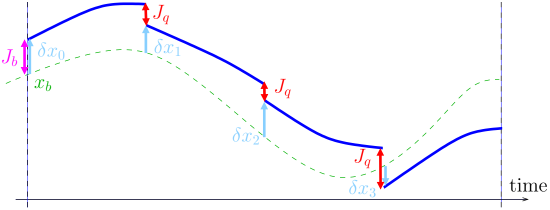


Figure: longwindow.

- Model integrations within each time-step (or sub-window) are independent:
  - Information is not propagated across sub-windows by TL/AD models,
  - Natural parallel implementation
- Tangent linear and adjoint models:
  - Can be used without modification,
  - Propagate information between observations and control variable within each sub-window.

EPS Experiment

- 50 member ensemble + control
- T_L 399 resolution
- 12 hour forecast
- Cycle 40R3
- 20 days of forecasts: 2013083100 - 2013091900
- Identical initial conditions (ensemble members are not perturbed)
- Stochastic parametrisation SPPT and SKEB

Methodology

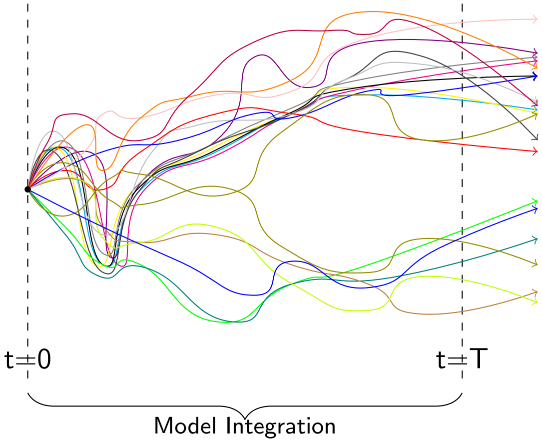
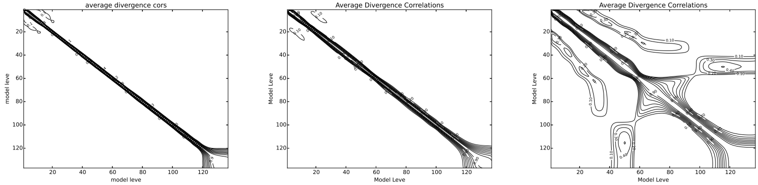


Figure: Cartoon of EPS members with identical initial conditions but different realisations of model error. In this experiment T is chosen to be 12 hrs because this is the length of the 4DVAR assimilation window.

Average Divergence Correlation



(a) Background (b) SPPT & SKEB **Q** (c) 4D-Var Model Error Estimation

Figure: Comparison between background, EPS model error at 12 hrs and 4D-Var Model Error Estimation. Maximum off diagonal correlation contour 0.5.

Choice of αQ

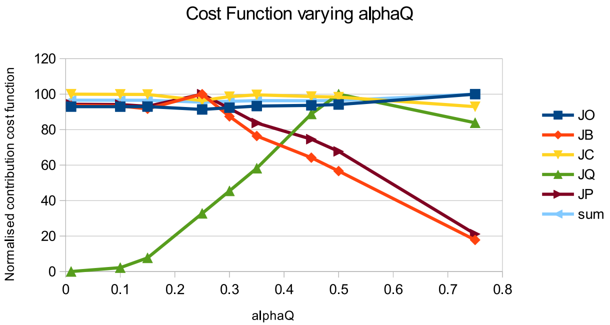


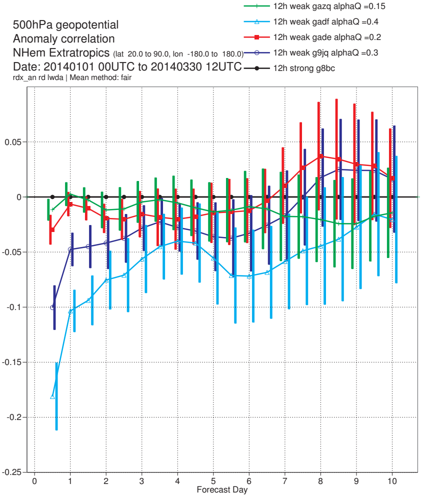
Figure: Normalised cost function contributions

- choose to experiment with $\alpha = 0.15, 0.2, 0.3$ and 0.4
- We want the Q term to be significant but not to dominate over other terms

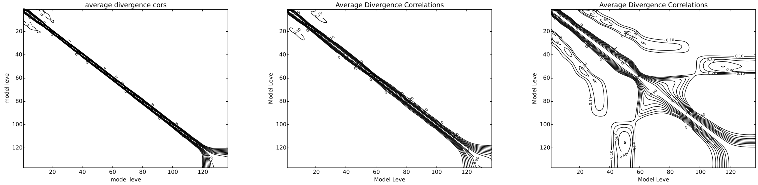
Weak-constraint 4DVar experimentation:

- 10 day forecast
- 12 hour weak constraint 4DVar with model error forcing
- $\alpha = 0.15, 0.2, 0.3$ and 0.4
- 3 months JFM 2014
- Using new stochastic $\mathbf{Q}$ matrix
- CY41R1

Scores - against operational analysis



Average Divergence Correlation (Repeat slide)



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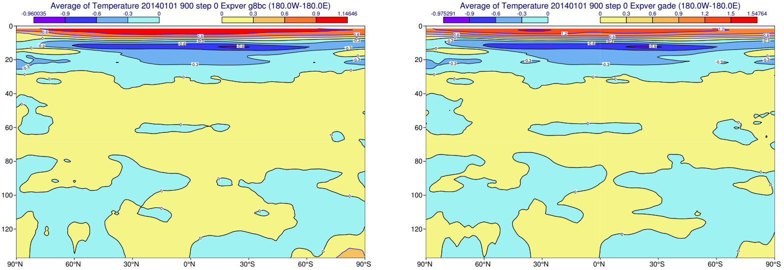
Zonal Means of Analysis Increment and Estimated Model Error Forcing



(a) Analysis Increment Zonal Mean (Strong)

(b) Analysis Increment Zonal Mean (Weak)

Figure: Fig (a) Analysis increment for strong constraint min value -0.96 K, max value 1.15 K, fig (b) Analysis increment for weak constraint min value -0.98 K, max value 1.55 K.

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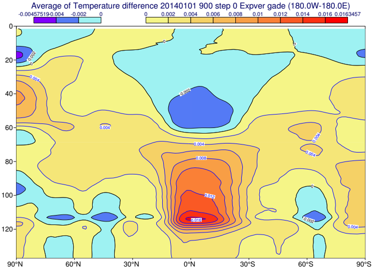
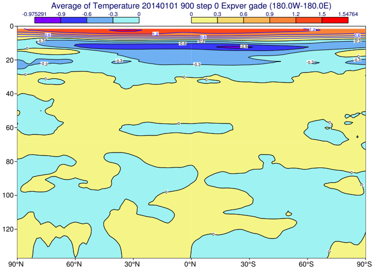
Zonal Means of Analysis Increment and Estimated Model Error Forcing



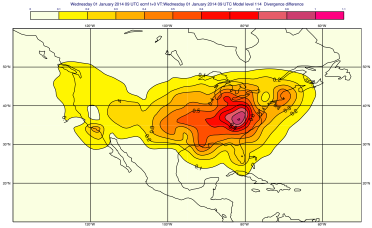
(a) Model Error Zonal Mean (Weak)



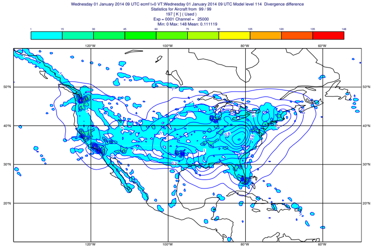
(b) Analysis Increment Zonal Mean (Weak)

Figure: Fig (a) Model error zonal mean for weak constraint min value -0.0046 K/hr, max value 0.0163 K/hr, fig (b) Analysis increment for weak constraint min value -0.98 K, max value 1.55 K.

Misinterpretation of AIREP Data



(a) Divergence Covariances



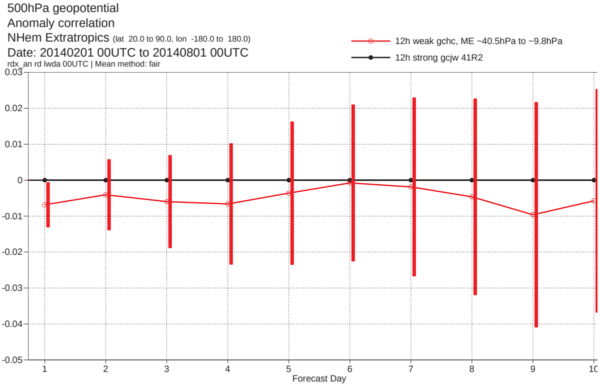
(b) AIREP 250hPa - 500hPa

Figure: (a) Divergence covariances between level 52 and 114 over the USA (multiplied by 10E14) (b) AIREP temperature data (averaged over January 2014) overlaid with (a).

Misinterpretation of AIREP Data

- Observation errors misinterpreted as model error
- To avoid erroneous aliasing of errors restrict model error forcing to above $\sim$ 40.5 hPa

Scores - against own analysis



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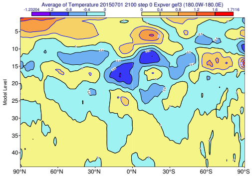
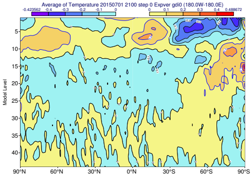
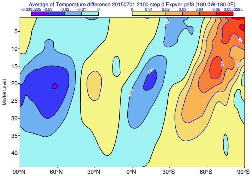
Operational resolution CY41R2



(b) Weak-constraint analysis inc



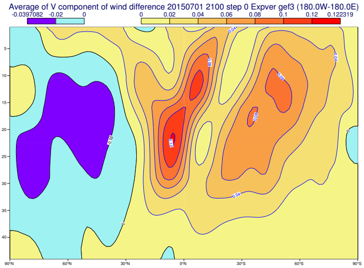
(c) strong-weak analysis inc



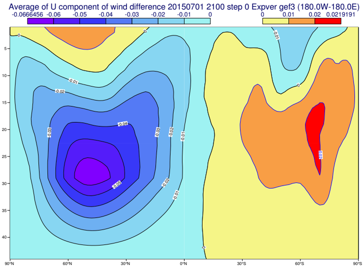
(d) Model error forcing

Figure: Temperature analysis increments and model error forcing comparison - July 2015

Operational resolution CY41R2



(a) V wind



(b) U wind

Figure: Model error forcing $\mathrm{ms}^{-1}\mathrm{hr}^{-1}$

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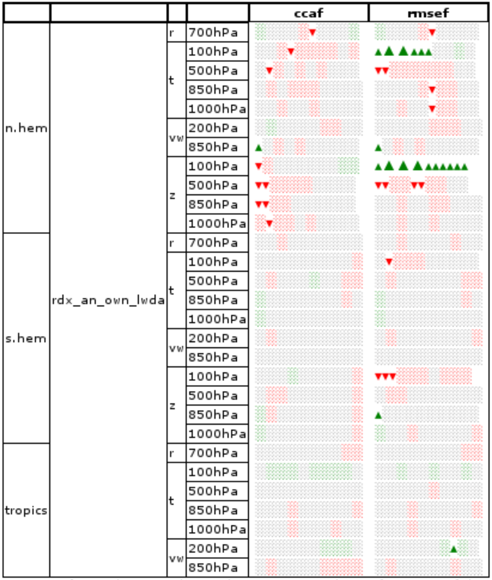
Operational resolution CY41R2 -MJJA own analysis



Conclusions

- Overall results are fairly neutral with some positives above 100hPa
- RMS forecast error is being shifted above 100hPa
- Model error forcing is trying to fix large scale circulation errors in the stratosphere
- Very difficult to verify results but gpsro verification may be an option

. . . Any Questions?