Historical Review: Foundational Studies in Atmospheric Predictability

Joe Tribbia

NCAR

ECMWF Annual Seminar

Ensemble Prediction: Past, Present and Future

11 September 2017

Program

- Preamble- “All history is lies agreed upon” Attributed to Napolean
- Act I - Forming the Question (1949-1960)
- Act II -The Question becomes important (1960-1969)
- Act III-Resolving the Question (1970-1982)
- Epilogue: What’s next?

1949 -1960

- 1949 First successful Numerical Weather ‘Prediction’
- Also, these lines were published in QJRMS:

V. The Ultimate Limitations of Weather Forecasting

for such a system? Suppose we attempt to formulate the problem as one of determining a final (forecast) state from a given initial one. The initial state is in practice “given” only within a certain margin of error. For concreteness let us consider pressure at a given point, known within a margin δp. Let ϑ_1 be the maximum growth-rate of unstable disturbances of the system. Then in the final (forecast) state we can guarantee pressure correct only within a margin $\delta p \cdot e^{\vartheta_1 t}$ since disturbances below the margin of error initially (and therefore completely unknown) will have attained this size. It is clear that “guaranteed”

Who was this Prophet?

This is the title and Author

Long Waves and Cyclone Waves

By E. T. EADY, Imperial College of Science, London

(Manuscript received 28 Febr. 1949)

The Abstract concludes with…

system. The characteristic disturbances (forms of breakdown) of certain types of initial system (approximating to those observed in practice) are identified as the ideal forms of the observed cyclone waves and long waves of middle and high latitudes. The implications regarding the ultimate limitations of weather forecasting are discussed.

By 1956 operational NWP was being done by the JNWP in the US

Uncertainty of Initial State as a Factor in the Predictability of Large Scale Atmospheric Flow Patterns

By PHILIP DUNCAN THOMPSON, Lt. Colonel, U. S. Air Force, Joint Numerical Weather Prediction Unit, Washington, D.C.

(Manuscript received February 5, 1957)

Abstract

This article deals with the predictability of the atmosphere or, more exactly, with the gradual growth of "inherent" errors of prediction, due to errors in an initial state that is reconstructed from measurements at a finite number of points. By investigating the initial time-derivatives of the error arising from *random* analysis error, it is found that the increase of the RMS (root-mean-square) wind error in predictions over periods of a few days depends on:

1) the period of the forecast
2) the *initial* RMS vector wind error
3) the difference between the characteristic scale of the initial error field and the scale of fluctuations in the true initial flow pattern
4) the area average of the vertical wind shear between 250 and 750 mb
5) the RMS vector deviation of the wind at about 500 mb from its area average and
6) the average static stability of the atmosphere.

PDT noticed poorer forecasts if Obs from Ship 'P' not received

$$\frac{\partial E}{\partial t} = \frac{2}{A}\int_A \nabla R \cdot \nabla \frac{\partial R}{\partial t}\, dA$$

$$= \frac{2}{A}\int_A \left(\nabla \cdot R\nabla \frac{\partial R}{\partial t} - R\nabla^2 \frac{\partial R}{\partial t}\right) dA$$

$$\frac{\partial E}{\partial t} = \frac{2}{A}\int_A RJ(\psi,\ \nabla^2 R)\, dA$$

$$\equiv \frac{2}{A}\int_A \mathbf{k} \cdot \nabla\psi \times R\nabla(\nabla^2 R)\, dA \qquad (8)$$

E= error KE. Thompson uses characteristic Length scales for the true and error fields .

He then uses a Fjørtoft-like constraint to analyze error growth in 2D and QG model

Fjørtoft constraint in QG turbulence

$$q_t + J(\psi, q) = 0,$$

$$q = \nabla^2\psi + (f^2 / N^2)\psi_{zz} \equiv L\psi$$

Fjørtoft Constraints

$$\text{Total Energy } E = -\int(\psi L\psi)dV$$

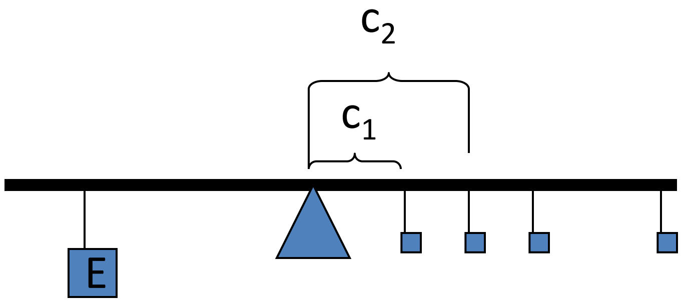
$$\text{Pot. Enstrophy } EN = \int(L\psi)^2 dV$$

$$L\varphi_n = -c_n^{\,2}\varphi_n$$

$$\psi = \sum a_n\varphi_n,\ \ E = \sum |a_n|^2 c_n^{\,2}$$

$$\text{Let } e_n \equiv |a_n|^2 c_n^{\,2}$$

$$E = \sum e_n \ \text{ and } EN = \sum c_n^{\,2} e_n$$



Cannot maintain balance if
Energy moves toward large n
Energy cannot be cascaded
to small scale

1960's

- Lorenz studies Deterministic Nonperiodic Flow as a paradigm of NWP

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

$$X^{\cdot} = -\sigma X + \sigma Y, \tag{25}$$

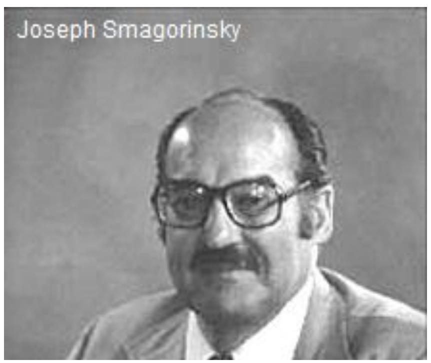
$$Y^{\cdot} = -XZ + rX - Y, \tag{26}$$

$$Z^{\cdot} = XY - bZ. \tag{27}$$

Lorenz '63 a landmark study for several reasons

- Demonstrated 'chaos' in a small finite dimensional system
- Complete rigorous dynamical systems analysis (Birkhoff)
- Convincing demonstration of Eady's conjecture of persistent instability

GARP and two week predictability



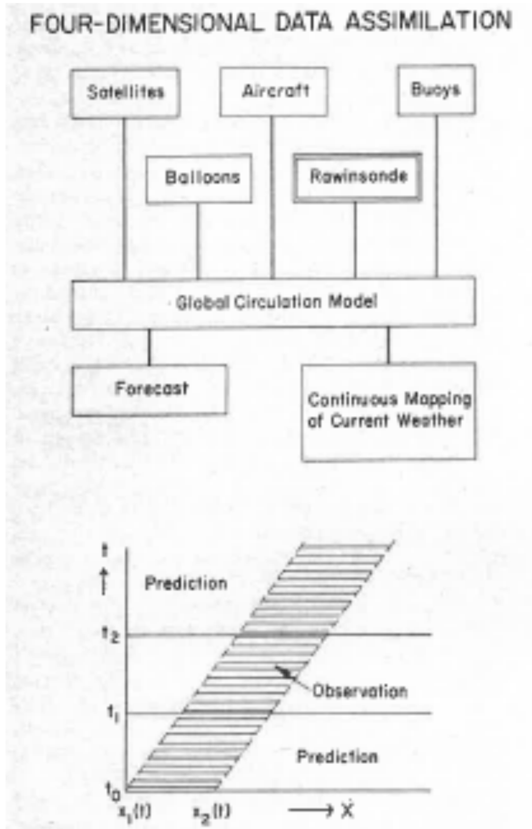


FIG. 2. Four-dimensional data assimilation and numerical weather prediction. The prediction model serves as an integrator of various observed data. Bottom part illustrates how observed data taken from a moving platform will be mixed with prediction.

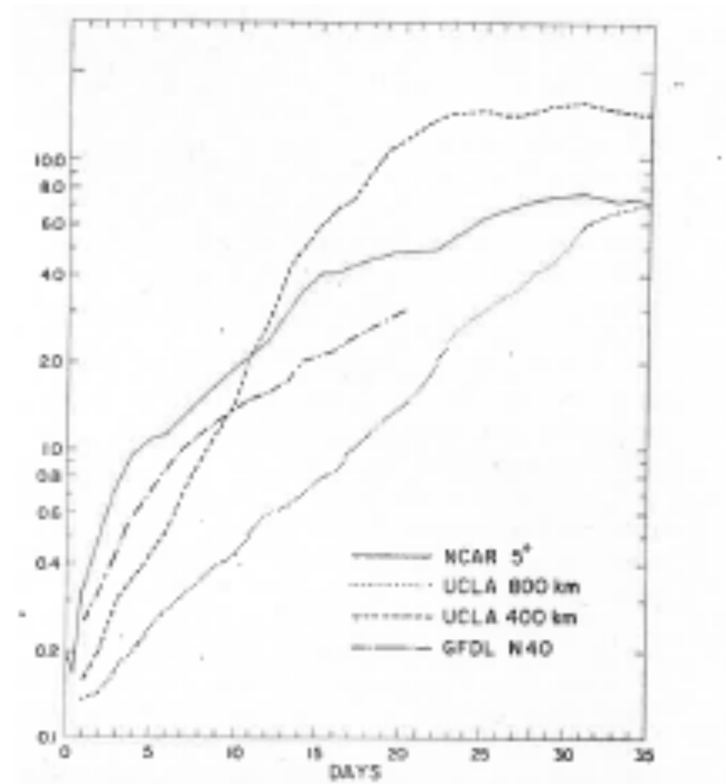


FIG. 3. Predictability experiments with various general circulation models. The ordinate shows a measure for the difference between the control and perturbated cases which is different for different experiments. For details see

A controversy brews

- Lorenz in ‘63 published results from a 28 variable model:
- Starts wondering about ‘turbulence’ predictability

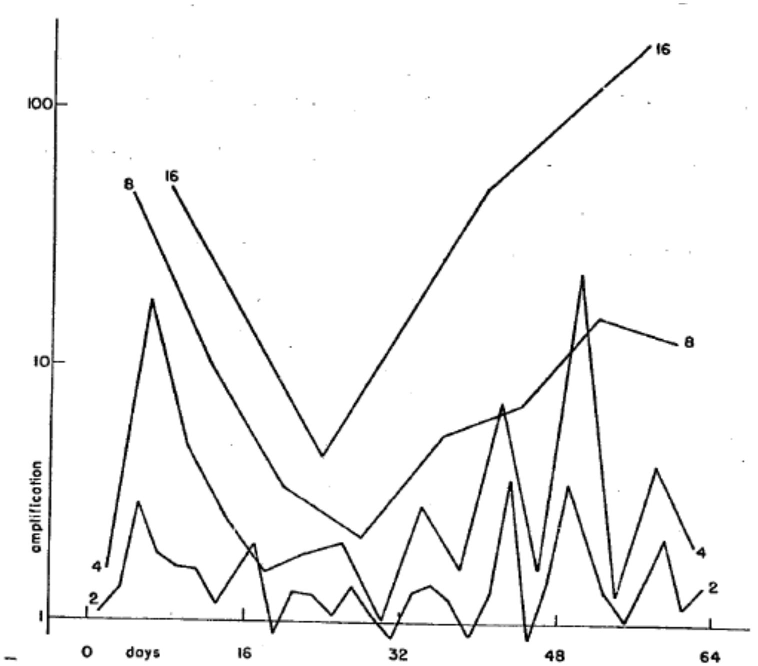


FIGURE 6. Mean amplification factors for small random errors during a particular 64-day interval. Curves labeled 2,4,8, and 16 indicate amplifications during successive 2-,4-,8-, and 16-day periods.

Reprinted from Transactions of The New York Academy of Sciences
Ser. II, Volume 25, No. 4, Pages 409-432
February 1963

THE PREDICTABILITY OF HYDRODYNAMIC FLOW*†

Edward N. Lorenz
Massachusetts Institute of Technology, Cambridge, Mass.

the results of this numerical study are at all applicable to the atmosphere, they suggest a wide discrepancy between practical predictability and attainable predictability at ranges up to one week. Good forecasts several days in advance do not seem to be prevented simply by current errors in measurement. If, however, we are genuinely interested in forecasting a few weeks in advance, we should give serious consideration to enlarging our network of observing stations, particularly over the oceans.

Perhaps these conclusions are too optimistic. The real atmosphere possesses significant fluctuations of shorter period than any which occur in the numerical model. Maybe what we have called one week in the model is more like two or three days in the real atmosphere. If this is so, we have already reached the maximum range which present errors in measurement will allow.

The predictability of a flow which possesses many scales of motion

By EDWARD N. LORENZ, *Massachusetts Institute of Technology*[1]

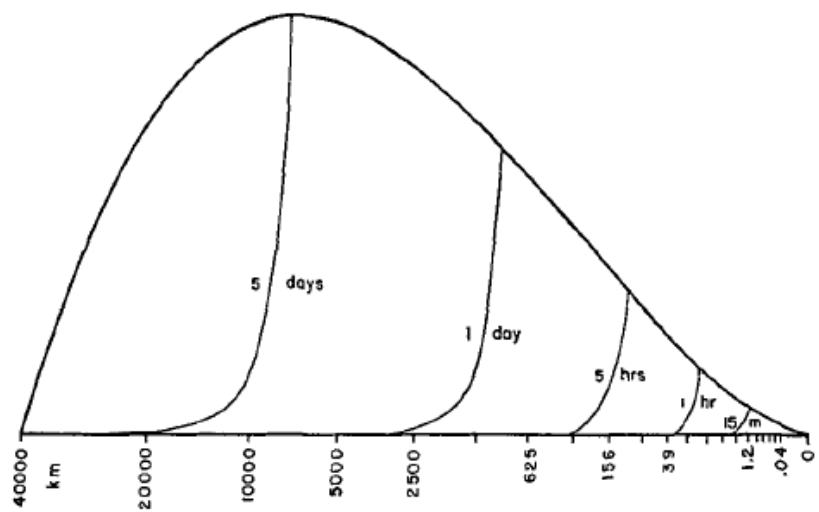
- Lorenz develops a closure for 2-dimensional homogeneous turbulence and uses it for the energy of the error

$$\partial(\nabla^2\varepsilon')/\partial t = -J(\psi, \nabla^2\varepsilon') - J(\varepsilon', \nabla^2\psi),$$

$$Z_k = \int_{a_{k-1}}^{a_k} Z(K)\,d(\log K),$$

$$d^2Z_k/dt^2 = \sum_{l=1}^{n} C_{kl} Z_l.$$

$$Z'_k = Z'_k(t_0)\cosh\lambda_k(t-t_0)$$



Error growth ends when Z_k reaches the Initial spectrum

Some current projects for global meteorological observation and experiment

By G. D. ROBINSON, Ph.D., F.Inst.P.

$$\left| u \frac{\partial u}{\partial x} \right| \simeq K \frac{\partial^2 u}{\partial y^2}$$

and find

$$K \simeq LU/8.$$

The rate of dissipation of the energy of the large-scale motion is

$$\epsilon = \bar{u} K \frac{\partial^2 u}{\partial y^2} = \tfrac{16}{3} K \frac{U^2}{L^2}$$

so that

$$\epsilon = \tfrac{2}{3} \frac{U^3}{L} \text{ and } K = (\tfrac{3}{2})^{1/3} \tfrac{1}{8} \epsilon^{1/3} L^{4/3}.$$

In the atmosphere we may be fairly confident that the frictional dissipation of kinetic energy lies between 1 and 10 cm^2 sec^{-3} (we can established this from the observed radiation field without appeal to details of atmospheric motion). On our assumptions these are limiting values of ϵ, so that

$$0{\cdot}15\, L^{4/3} < K < 0{\cdot}3\, L^{4/3}$$

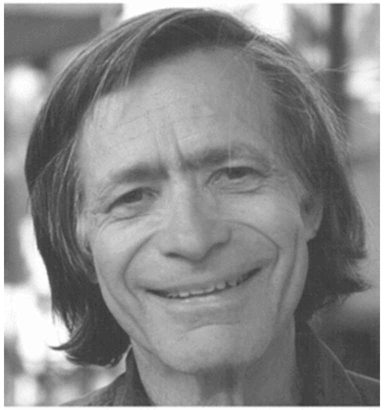
which we may compare with L. F. Richardson's (1926) finding of

$$K \simeq 0{\cdot}2\, L^{4/3}$$

TABLE 1. VELOCITY, DIFFUSIVITY COEFFICIENT, AND PREDICTABILITY TIME APPROPRIATE TO VARIOUS SCALES OF MOTION AND DISSIPATION RATES

Scale length L cm	Dissipation rate ϵ cm^2 sec^{-3}	Diffusivity K cm^2 sec^{-1}	Velocity U cm sec^{-1}	Predictability time T sec
5×10^8	10	$1{\cdot}2 \times 10^{11}$	$1{\cdot}9 \times 10^3$	$2{\cdot}6 \times 10^5$
	1	$5{\cdot}8 \times 10^{10}$	$9{\cdot}4 \times 10^2$	$5{\cdot}4 \times 10^5$
5×10^7	10	$5{\cdot}8 \times 10^9$	$9{\cdot}4 \times 10^2$	$5{\cdot}4 \times 10^4$
	1	$2{\cdot}7 \times 10^9$	$4{\cdot}2 \times 10^2$	$1{\cdot}2 \times 10^5$
5×10^6	10	$2{\cdot}7 \times 10^8$	$4{\cdot}2 \times 10^2$	$1{\cdot}2 \times 10^4$
	1	$1{\cdot}2 \times 10^8$	$2{\cdot}0 \times 10^2$	$2{\cdot}5 \times 10^4$
5×10^5	10	$1{\cdot}2 \times 10^7$	$2{\cdot}0 \times 10^2$	$2{\cdot}5 \times 10^3$
	1	$5{\cdot}8 \times 10^6$	$9{\cdot}2 \times 10^1$	$5{\cdot}4 \times 10^3$

Given Kraichnan’s 2D Turbulence
Robinson and Lorenz reconsider



Inertial Ranges in Two-Dimensional Turbulence

ROBERT H. KRAICHNAN

Two-dimensional turbulence has both kinetic energy and mean-square vorticity as inviscid constants of motion. Consequently it admits two formal inertial ranges, $E(k) \sim \epsilon^{2/3}k^{-5/3}$ and $E(k) \sim \eta^{2/3}k^{-3}$, where ϵ is the rate of cascade of kinetic energy per unit mass, η is the rate of cascade of mean-square vorticity, and the kinetic energy per unit mass is $\int_0^\infty E(k)\ dk$. The $-\frac{5}{3}$ range is found to entail backward energy cascade, from higher to lower wavenumbers k, together with zero-vorticity flow. The -3 range gives an upward vorticity flow and zero-energy flow. The paradox in these results is

The predictability of a dissipative flow

By G. D. ROBINSON
The Center for the Environment and Man, Inc., Hartford, Connecticut

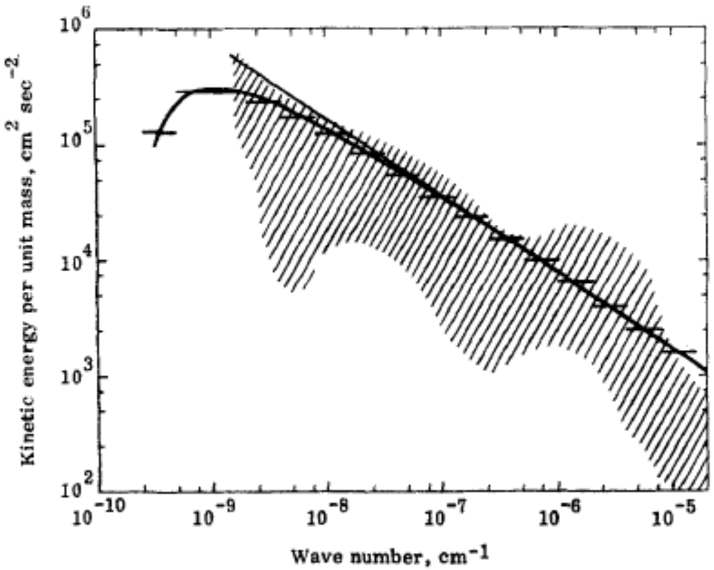
constraining spectrum which is not an intrinsic property of the basic flow studied. Lorenz, for example, concludes that a flow constrained by a spectrum of type

$$KE = \text{constant } l^{-n}, \quad |n| \geqslant 3$$

is inherently predictable in that indefinite reduction of the initial error indefinitely increases the predictability time. He also finds that in unpredictable flows, the predictability time varies with the total energy of the flow.

What is the KE spectrum of the atmosphere?

Pinus et al used by Lorenz



Atmospheric energy spectrum. The hatched area covers the observations of Pinus, Sl
ko. The horizontal lines are Lorenz's spectrum (octave averages). The continuous lines
interpolated Lorenz spectrum and its '-2/3' extrapolation.

Julian et al 1969

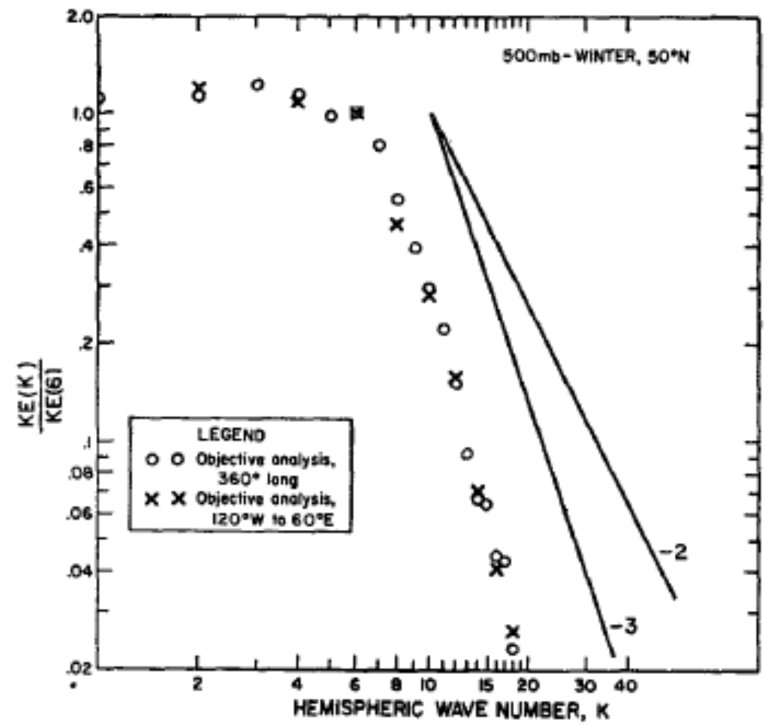
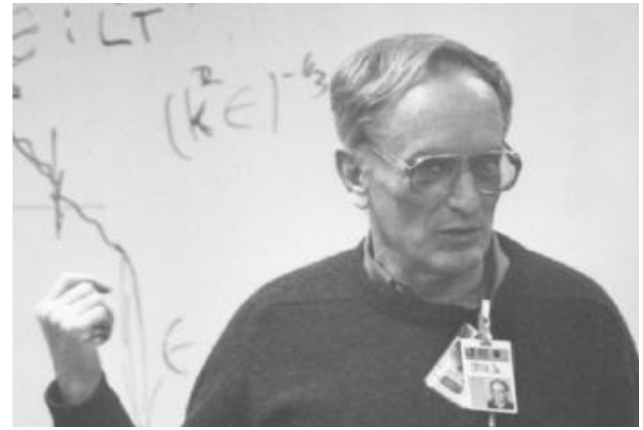


FIG. 1. Kinetic energy spectra, 500 mb, 50N, for the winter season (data sets Iw and IIw, Table 1) plotted on a full logarithmic scale. Shown are the spectral estimates using data from all longitudes and for a 180° segment from 120W to 60E. For purposes of comparison all estimates have been standardized by division by the estimate for wavenumber 6.

1970's
Leith and Kraichnan



Atmospheric Predictability and Two-Dimensional Turbulence

C. E. LEITH

National Center for Atmospheric Research,[1] Boulder, Colo.

- Chuck Leith uses Kraichnan DIA-like EDQNM closure

Planetary Waves Predictable For 2 weeks

The second-moment equation reduces to

$$[d/dt+2\nu k^2-2\alpha(k)]U(k,t)=S(k,t)$$
$$=2\int d\mathbf{p}D(k,p,q,t)B(k,p,q)$$
$$\times[U(q,t)U(p,t)-U(q,t)U(k,t)];$$

$$[d/dt+2\nu k^2-2\alpha(k)]\Delta(k,t)=S'(k,t)$$
$$=2\int d\mathbf{p}D(k,p,q,t)B(k,p,q)[\Delta(p,t)U(q,t)+\Delta(q,t)U(p,t)$$
$$-\Delta(p,t)\Delta(q,t)-U(q,t)\Delta(k,t)]. \quad (12)$$

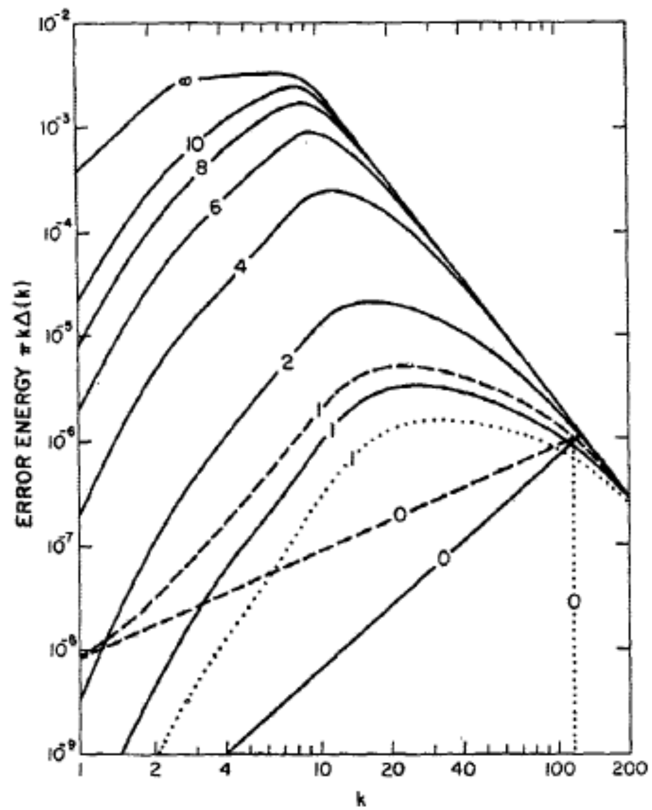
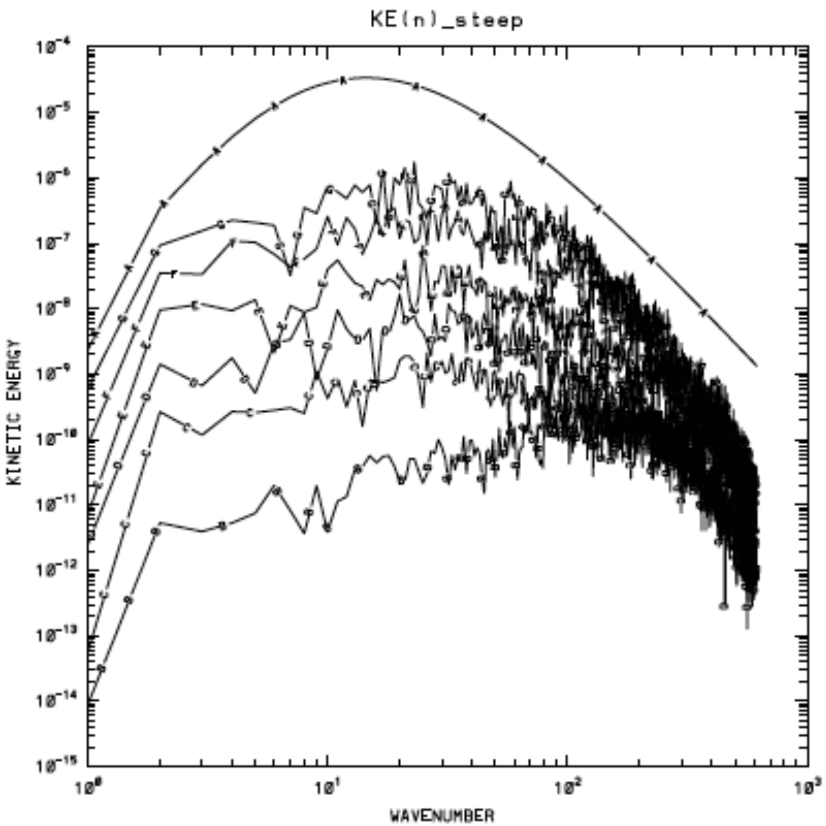


FIG. 6. Error energy spectra (rad² day⁻²) for $k_0=128$ and $\gamma=0.8$ labelled by days after observation with random error (dashed), alias error (solid), and no error (dotted) for $k<k_0$.

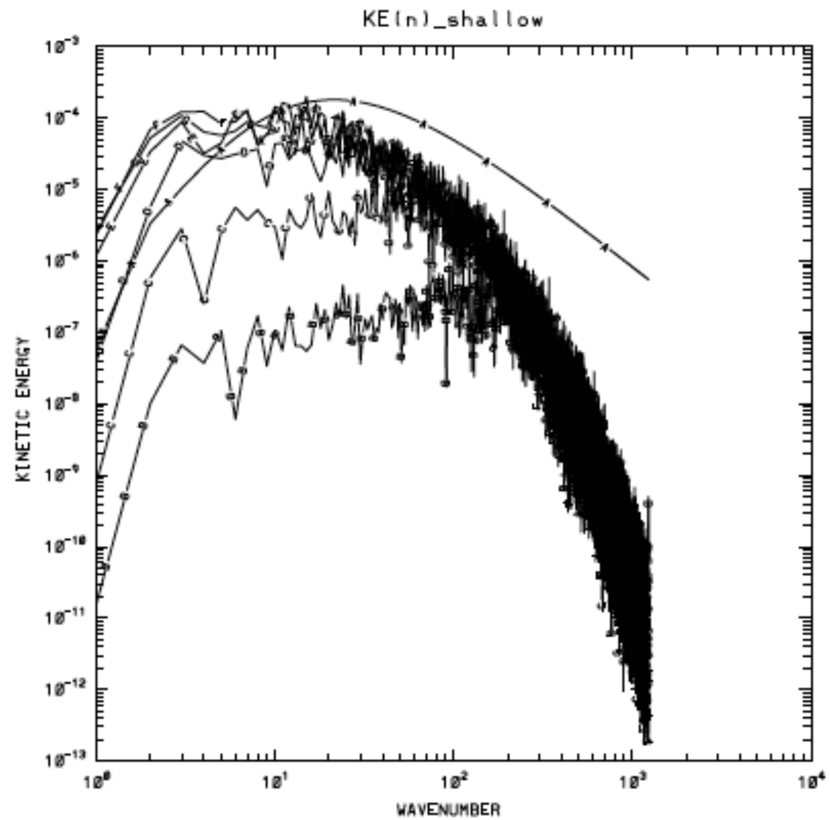
Does spectal slope really matter?

Bartello-Warn 1D turbulence model

Steep (K^{-3}) Spectral Slope



Shallow ($K^{-5/3}$) Spectral Slope



Non-local Cascade

Stochastic Dynamic Prediction: A formalism for probabilistic NWP

- Ed Epstein

Fundamentally,
Classical statistical
Moment prediction
Need to close the
system

Stochastic dynamic prediction[1]

By EDWARD S. EPSTEIN, *University of Michigan, Dept. of Meteorolog[y] and Oceanography, Ann Arbor, Mich.*[2,3]

$$\dot{x}_i = \sum_{j,k} a_{ijk} x_j x_k - \sum_j b_{ij} x_j + c_i$$

where

$$\sum_{i,j,k} a_{ijk} x_i x_j x_k = 0 \quad \text{and} \quad \sum_{i,j} b_{ij} x_i x_j \geqslant 0$$

$$\begin{aligned} \dot{\mu}_i &= \sum_{j,k} a_{ijk} \varrho_{jk} - \sum_j b_{ij} \mu_j + c_i \\ &= \sum_{j,k} a_{ijk} (\mu_j \mu_k + \sigma_{jk}) - \sum_j b_{ij} \mu_j + c_i \end{aligned}$$

and

$$\begin{aligned} \dot{\varrho}_{ij} = E[\sum_{k,l} (a_{jkl} x_i x_k x_l + a_{ikl} x_j x_k x_l)] \\ - \sum_k (b_{ik} \varrho_{jk} + b_{jk} \varrho_{ik}) + c_i \mu_j + c_j \mu_i \end{aligned}$$

Stochastic dynamic predictions have significantly smaller mean square errors than deterministic procedures, and also give specific information on the nature and extent of the uncertainty of the forecast. Also the range of time over which useful forecasts can be obtained is extended. However, they also require considerably more extensive calculations.

Closure not the biggest problem
2nd moment prediction is order N^2

Closure assumption

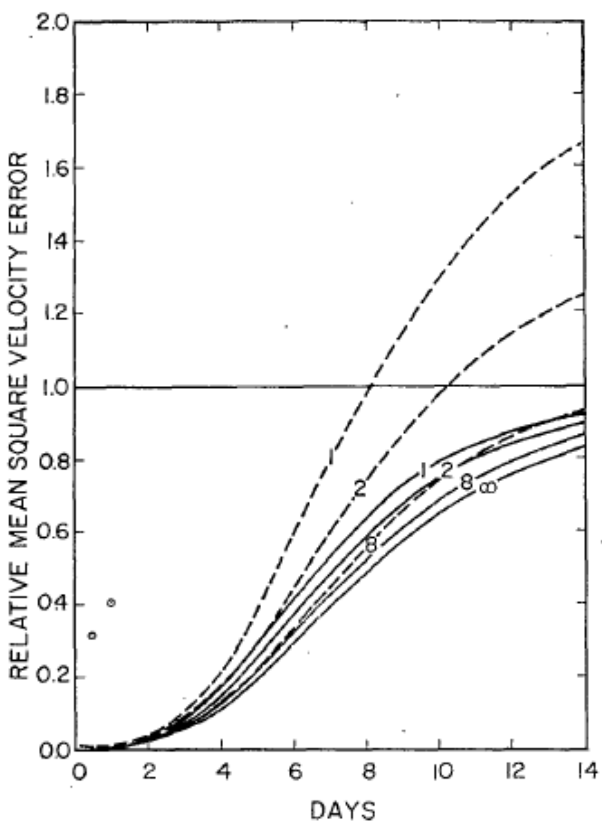
$$\sum_{k,l} (a_{jkl} \tau_{ikl} + a_{ikl} \tau_{jkl}) = 0$$

Leith suggests a practical method for Stochastic Dynamic Prediction

Theoretical Skill of Monte Carlo Forecasts

C. E. LEITH

National Center for Atmospheric Research,[1] *Boulder, Colo. 80302*



We conclude in general that a Monte Carlo forecasting procedure represents a practical, computable approximation to the stochastic dynamic forecasts proposed by Epstein (1969). Adequate accuracy should be obtained for the best mean estimate of the forecast field with sample sizes as small as 8. Improvement in skill is appreciable for Monte Carlo forecasts as compared to conventional single forecasts although much of this improvement comes from the filtered nature of the forecasts and is obtainable with a linear regression step applied to a single forecast.

The question of what sample size is adequate for the detailed determination of forecast error needed for optimal data assimilation has not been decided by the present theoretical study and will require experiments with real data applications of the Monte Carlo procedure. Such experiments are planned using a spectral barotropic model applied to forecasts of the 500-mb height field.

Surprising: # A Note on Predictability

STANLEY J. JACOBS

Dept. of Meteorology and Oceanography, and Dept. of Engineering Mechanics, The University of Michigan, Ann Arbor 48104

1. Introduction

The problem of predictability involves forecasting the future state of a physical system given imperfect knowledge of the initial state. For meteorological purposes this consists of finding a vector $z_i(t)$, $i=1, 2, \ldots N$, satisfying the system

$$\dot{z}_i=\sum_j a_{ij}z_j+\sum_{j,k} b_{ijk}z_jz_k+q_i(t), \tag{1}$$

and the initial condition

$$z_i(0)=c_i+h_i, \tag{2}$$

where c_i is a best estimate of the initial value and h_i is an error term, in general unknown. The forecast $z_i^0(t)$ solves (1) with initial condition $z_i^0(0)=c_i$. Assuming that the reduction of the hydrodynamic equations to the form (1) and the numerical integration procedure for computing z_i^0 do not involve appreciable errors, the forecast is accurate if

$$x_i=z_i-z_i^0 \tag{3}$$

is small in some sense.

2. Error estimates

Let $\mathbf{x}(t)$ and $\mathbf{h}$ be the column vectors whose components are $x_i(t)$ and h_i. Substitution of (3) into (1) leads to the vector equation

$$\dot{\mathbf{x}}=\mathbf{A}(t)x+\mathbf{f}(x), \quad \mathbf{x}(0)=\mathbf{h}, \tag{4}$$

where the matrix $\mathbf{A}$ and vector $\mathbf{f}$ have components

$$A_{ij}=a_{ij}+\sum_k (b_{ijk}+b_{ikj})z_k^0(t), \tag{5a}$$

$$f_i=\sum_{j,k} b_{ijk}x_jx_k. \tag{5b}$$

Since $\dot{\mathbf{x}}$ is a polynomial in x, the solution $\mathbf{x}(t)$ is unique, and is a continuous function of $\mathbf{h}$. Therefore, $\mathbf{h}=0$ implies $\mathbf{x}=0$, and $|\mathbf{h}|\leqslant\epsilon$ implies $|\mathbf{x}(t)|\leqslant k$, for any constant $k>\epsilon$, at least in some time interval $0\leqslant t\leqslant T(k,\epsilon)$. We will seek an upper bound for $|\mathbf{x}(t)|$ and a lower bound for T, given the upper bound ϵ for $|\mathbf{h}|$.

The norm for a vector $\mathbf{y}$ is defined to be

$$|\mathbf{y}|=(\mathbf{y}^T\mathbf{F}\mathbf{y})^{\frac{1}{2}}, \tag{6}$$

where $\mathbf{F}$ is a constant positive definite symmetric matrix and $\mathbf{y}^T$ is the transpose of $\mathbf{y}$. This is a convenient norm, since it allows us to weight the components of $\mathbf{x}$ differently. Let $r=|\mathbf{x}(t)|$. Then

$$\dot{r}=r^{-1}\{\mathbf{x}^T[\tfrac{1}{2}(\mathbf{FA}+\mathbf{A}^T\mathbf{F})]\mathbf{x}+\mathbf{x}^T\mathbf{F}\mathbf{f}(\mathbf{x})\}, \quad r(0)\leqslant\epsilon. \tag{7}$$

Now, for any vector $\mathbf{y}$,

$$\mathbf{y}^T[\tfrac{1}{2}(\mathbf{FA}+\mathbf{A}^T\mathbf{F})]\mathbf{y}\leqslant\alpha(t)\mathbf{y}^T\mathbf{F}\mathbf{y}, \tag{8}$$

where $\alpha(t)$ is the largest eigenvalue of $\frac{1}{2}(\mathbf{A}+\mathbf{F}^{-1}\mathbf{A}^T\mathbf{F})$. Hence,

$$\dot{r}\leqslant\alpha(t)r+[\mathbf{x}^T\mathbf{F}\mathbf{f}(\mathbf{x})]/(\mathbf{x}^T\mathbf{F}\mathbf{x})^{\frac{1}{2}}, \tag{9}$$

Finally, a predictability study with at real forecast model

- Ed Lorenz, of course
- Made possible only because of ECMWF forecast archive policy
- Demonstrated how go the forecasts were and how good they potentially could be

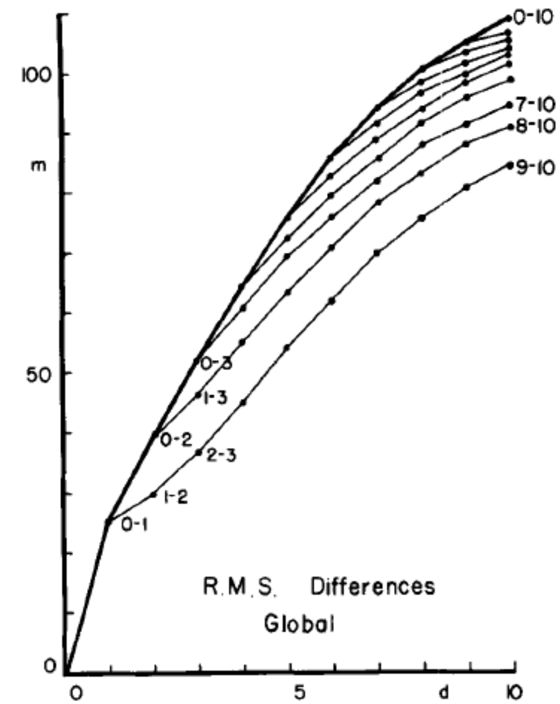


Fig. 1. Global root-mean-square 500-mb height differences E_{jk}, in meters, between j-day and k-day forecasts made by the ECMWF operational model for the same day, for $j < k$, plotted against k. Values of (j,k) are shown beside some of the points. Heavy curve connects values of E_{0k}. Thin curves connect values of E_{jk} for constant $k - j$.

Epilogue

These predictability studies opened the agenda for ECMWF

- First showing that it was, in principle, possible to predict the weather beyond a few days
- Then, opening the field of dynamical probabilistic forecasting
- Finally, pointing the way to practical solutions to the prediction of the mean and covariance leading to ensemble predictions and singular vector initial uncertainty sampling