Upscale and Downscale Error Growth

Dale Durran

University of Washington, Atmospheric Sciences

Jonathan Weyn, Mark Gingrich

Edward Lorenz

Flavors of predictability

- Chaos
  - Sensitive dependence to initial conditions, bounded phase space
- Butterfly effect
  - Finite limit on predictability due to upscale cascade of errors

Which Lorenz equation set has a "butterfly effect"?

Nonlinear

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(r - z) - y$$
$$\dot{z} = xy - bz$$

Chaos (Lorenz, 1963)

Linear (with nonlinear saturation)

$$\ddot{Z}_k = \sum_{l=1}^{21} C_{k,l} Z_l$$

Butterflies (Lorenz, 1969)

``The predictability of a flow which possesses many scales of motion"

$$\ddot{Z}_k = \sum_{l=1}^{n} C_{k,l} Z_l$$

- Z_k ensemble mean error KE at wavenumber k
- $C_{k,l}$ strength of interaction of error KE at l with the background flow to force errors at k
- $C_{k,l}$ and speed of upscale error propagation depend on the slope of the background KE spectrum

Upscale error propagation: scaling argument

- *E(k)*: background KE per unit wavenumber ($m^3\ s^{-2}$)
- Dimensional analysis: time scale is $\tau(k) = k^{-3/2} E^{-1/2}$
  - Assumed to be the time required for errors propagate from *k* to *k/2*
- T_{up} : time for error propagation upscale from $2^N k_L$ to k_L is sum of time scales $\tau(k)$ over geometrically increasing wavelengths

$$\tau(2^N k_L) + \tau(2^{N-1} k_L) + \ldots + \tau(2k_L)$$

Palmer et al., 2014: The real butterfly effect

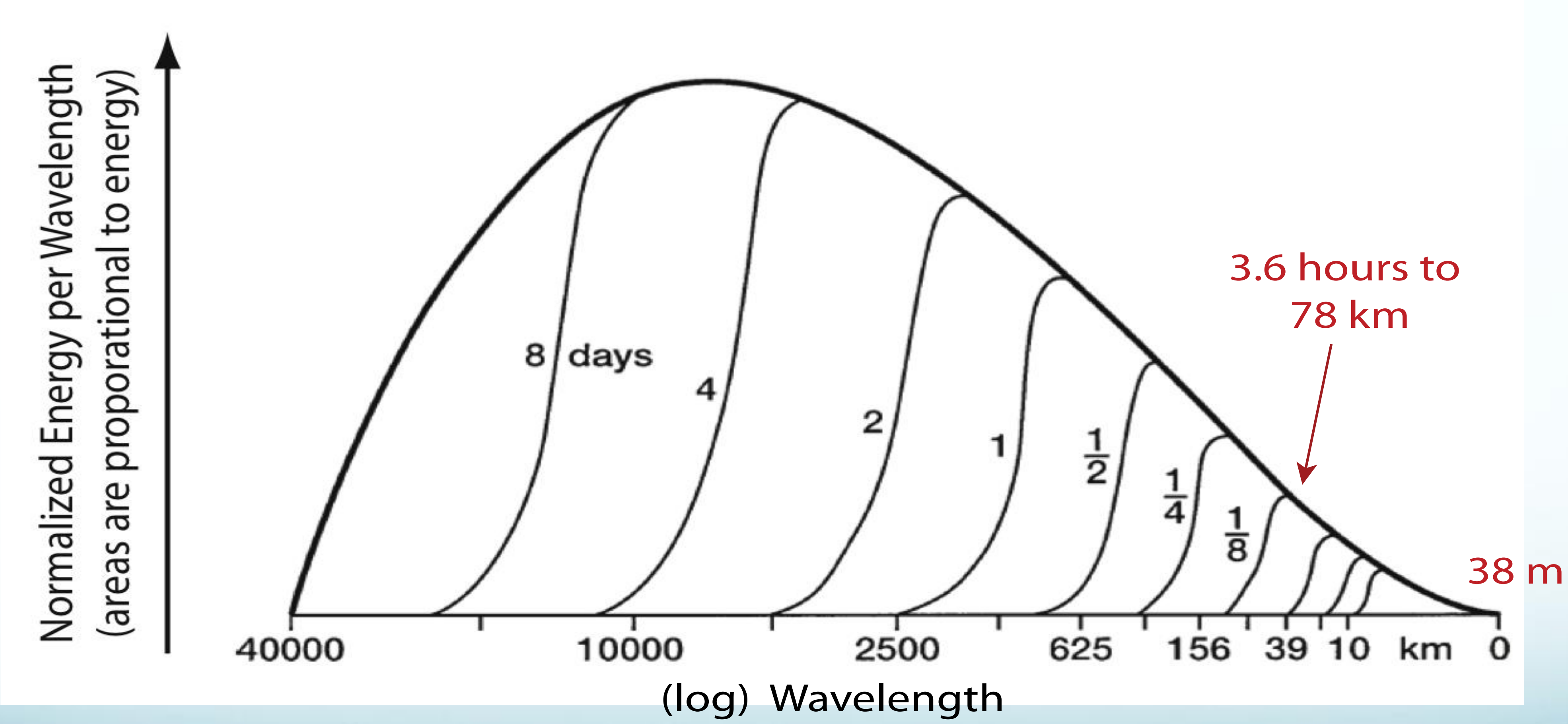
Upscale error propagation: scaling argument

- For background KE spectrum proportional to k^{-p} ,

$$\tau(2^n k_L) \propto [2^{(p-3)/2}]^n$$

- As $N \to \infty$, T_{up} converges to a **finite value** if $p < 3$.
- Finite limit on predictability when $p < 3$.

Lorenz: 1969. Errors propagate upscale in turbulent flows with a $k^{-5/3}$ KE spectrum.



Lorenz, 1969: The predictability of a flow which possesses many scales of motion. Tellus, 21, 289-307.

Butterflies before Lorenz

W.S. Franklin in 1898 review of book by Pierre Duhem

"An infinitesimal cause produces a finite effect. Long range detailed weather prediction is therefore impossible… the flight of a grasshopper in Montana may turn a storm aside from Philadelphia to New York!"

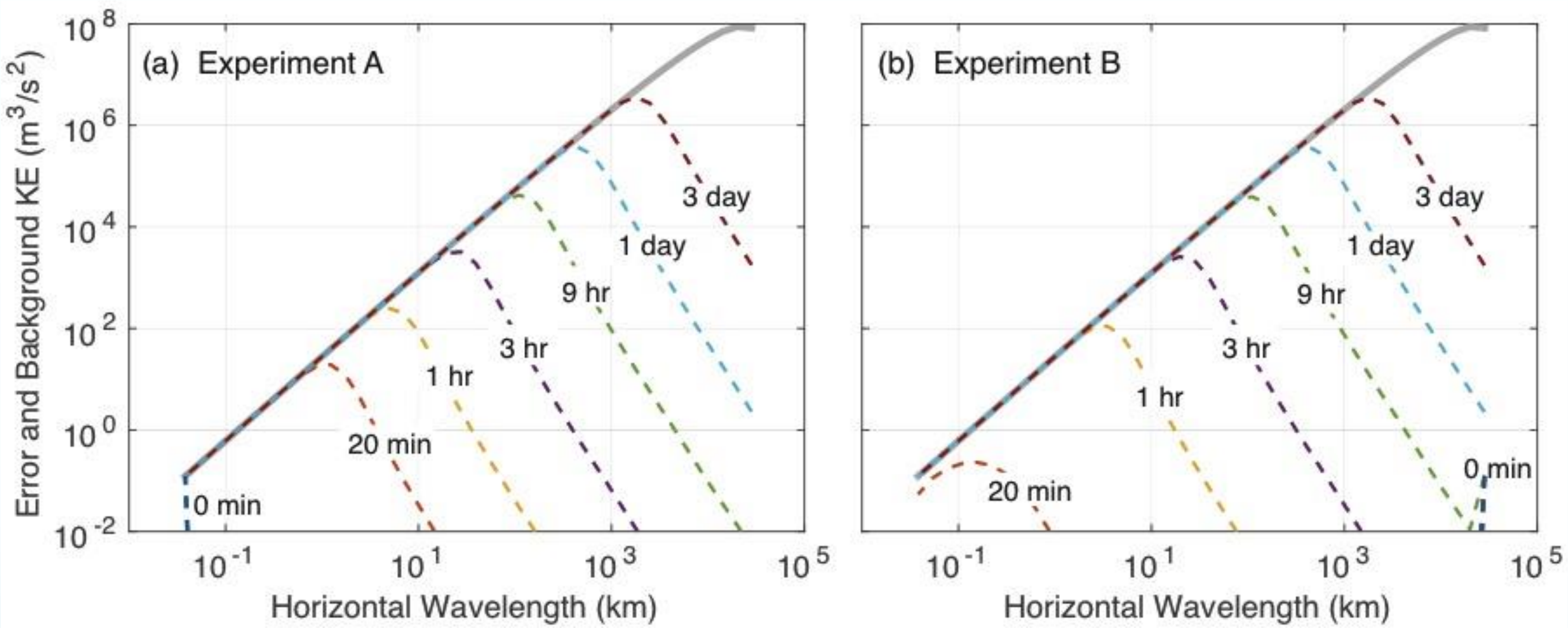
G. Vallis, 2006, p. 372

Should we associate the butterfly with small-amplitude or small-scale perturbations?

Consider two *different* questions

- Is upscale error growth important?
  - (even if it is not exactly a “spectral cascade”)
- Given initial errors of *fixed absolute magnitude*, does their *horizontal scale* influence predictability?

Lorenz, 1969: Experiments A & B



"Evidently when the initial error is small enough, its spectrum has little effect upon the range of predictability."

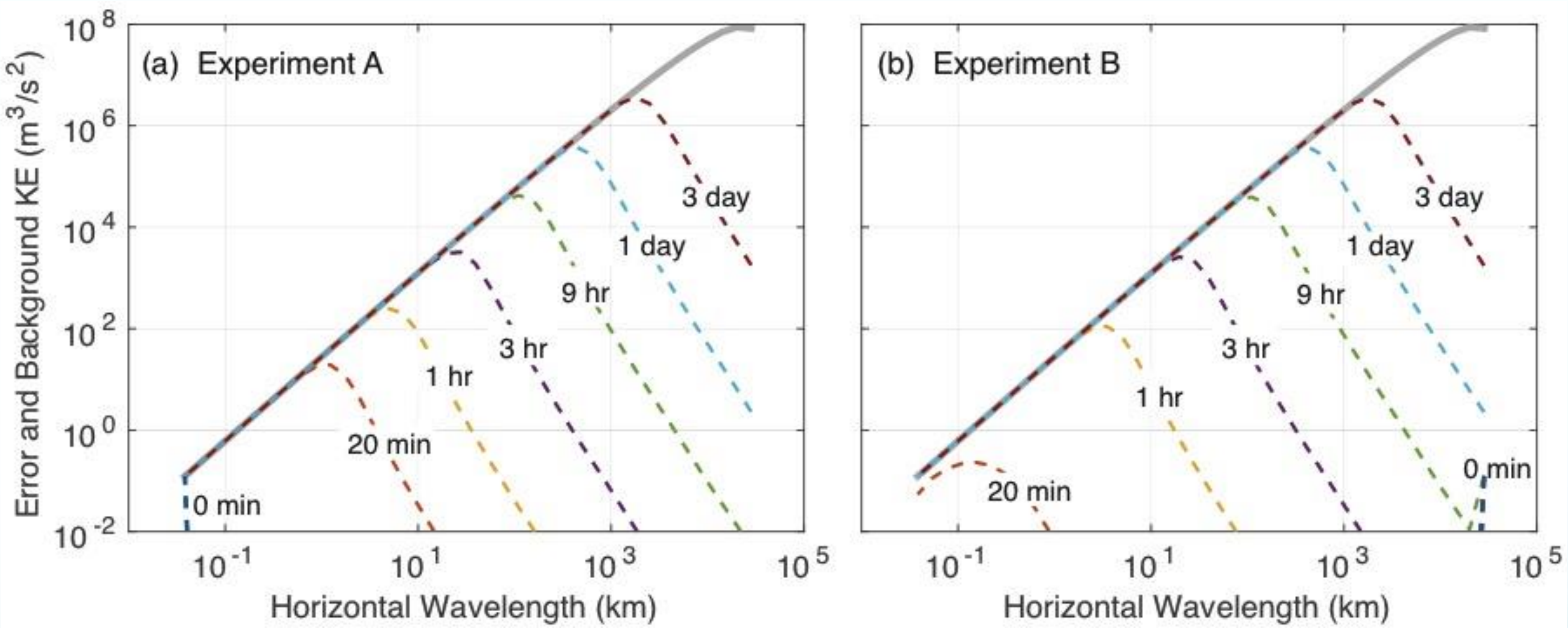
Implications of Experiment B were largely overlooked.

Anthes, 1985: The large scale makes mesoscale predictability possible.

Downscale error quenching

- Estimates of mesoscale predictability from classical turbulence theory are too pessimistic.
  - Physical forcing at the earth's surface, such as mountains, may contribute to extended predictability.
  - Mesoscale phenomena, such as fronts, can evolve from purely large-scale initial conditions.
- Fine-resolution forecast models rely on large-scale and surface forcing to create small-scale features that cannot be initialized based directly on observations.
- *How accurate must the large-scale forecast be?*

Small *relative errors* in the large-scales can destroy predictability.



Relative error in the velocity
100% at λ=38 m.

Relative error in the velocity
1% at λ=28,000 km.

The large scale must be known with extreme accuracy to correctly

- Forecast the *mesoscale* distribution of orographic precipitation (Nuss and Miller, 2001)
- Forecast downslope winds (Reinecke and Durran, 2009)
- Differentiate between lowland rain and snow in Pacific Northwest winter storms (Durran et al., 2013)

The large-scale does exert a strong control, but small errors in the large scale often interfere with mesoscale predictability.

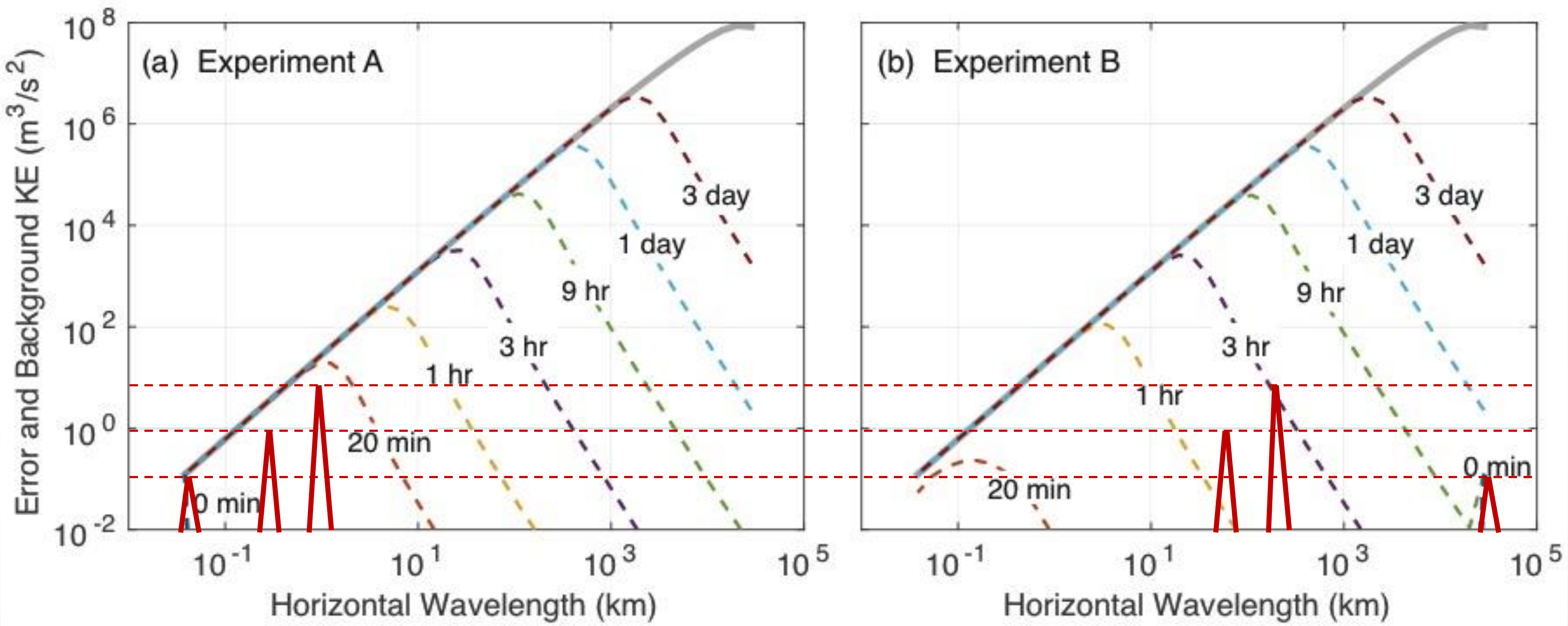
On the scale of initial errors

- Lorenz (1969): “We have proposed that certain formally deterministic fluid systems possessing many scales of motion may be observationally indistinguishable from indeterministic systems, in that they possess an *intrinsic range of predictability* which cannot be lengthened by reducing the error of observation to any value greater than zero.”
- Limits from intrinsic predictability become apparent as saturated errors appear at progressively larger scales.
- Scale of the initial error is largely irrelevant (for wavelengths < 400 km)

More implied by Lorenz 1969

- Small-amplitude large-scale errors rapidly propagate downscale (for $k^{-5/3}$ background KE spectrum) and then propagate back up scale.
- *Net effect appears as if the error originated in the smallest scales.*
- *Rate of upscale error growth does determine the theoretical limit of intrinsic predictability.*

Initial amplitude, not initial scale

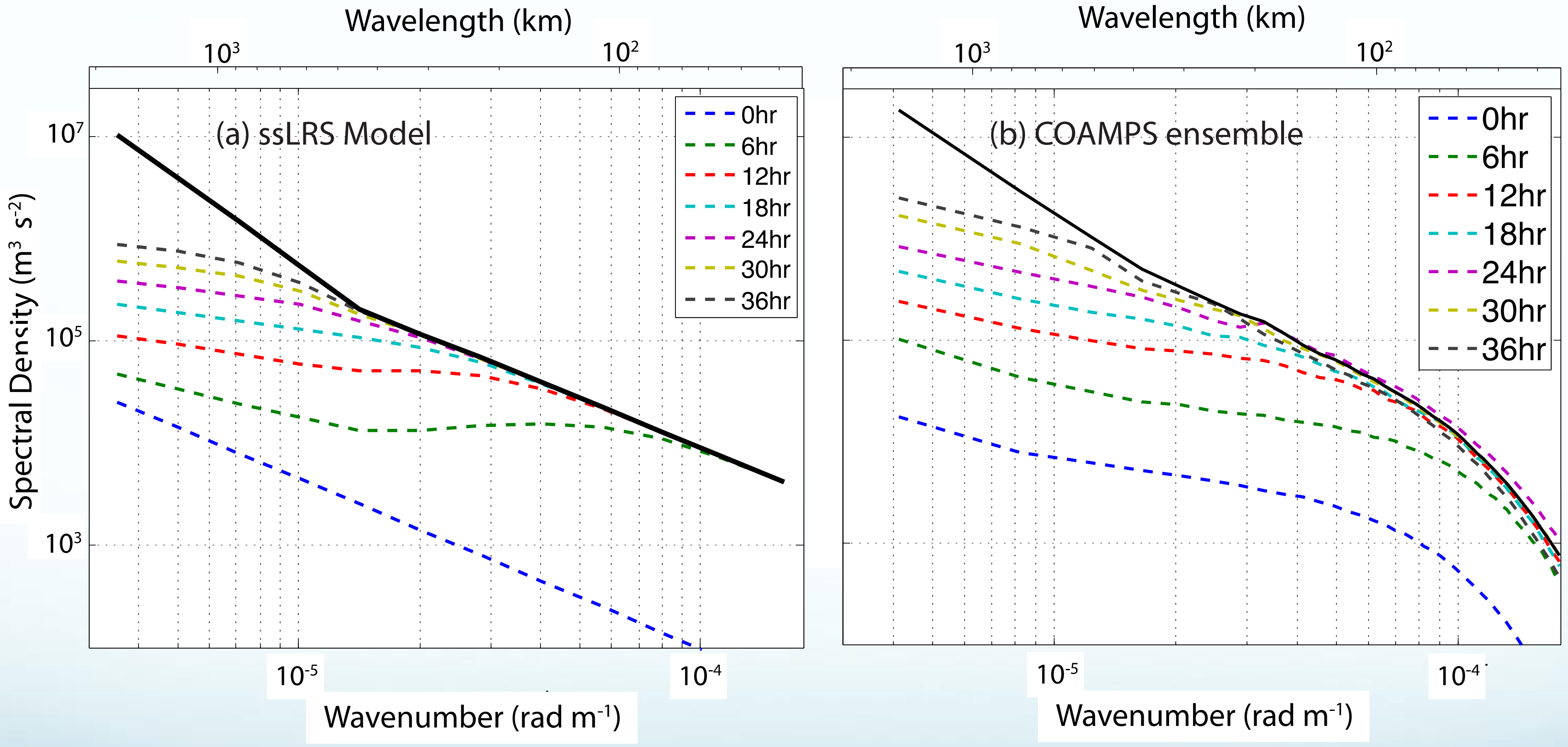


- After downscale propagation, up-scale error growth begins at a smaller scale if the error is smaller amplitude.
  - Faster eddy turnover time on smaller scales.

How relevant is the Lorenz model?

- It does not include
  - Baroclinic instability
  - Deep convection
  - Inhomogeneity and nonstationarity
- Nonlinear effects are incorporated only crudely.
- Incorrectly assumed $k^{-5/3}$ slope for the background KE spectrum at large-scales.
- *Nevertheless*, when given appropriate initial errors (~1% relative errors at all scales), it predicts error growth similar to that in 100-member COAMPS forecasts of east-coast snowstorms.

Lorenz model compared with 5-km-resolution COAMPS ensemble simulations of east-coast snowstorm



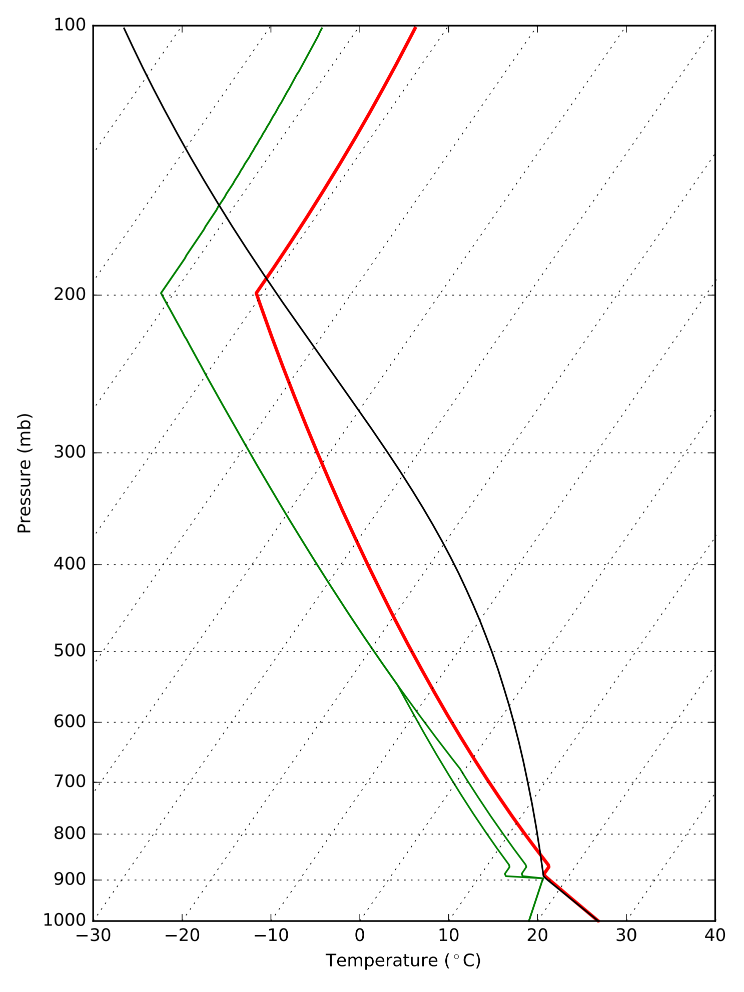
Error growth is more up-amplitude than up-scale

Does the scale of initial mesoscale errors matter?

- Idealized convective systems
- Actual convective cases
- Idealized moist baroclinic instability

20-member ensemble simulations of deep convection

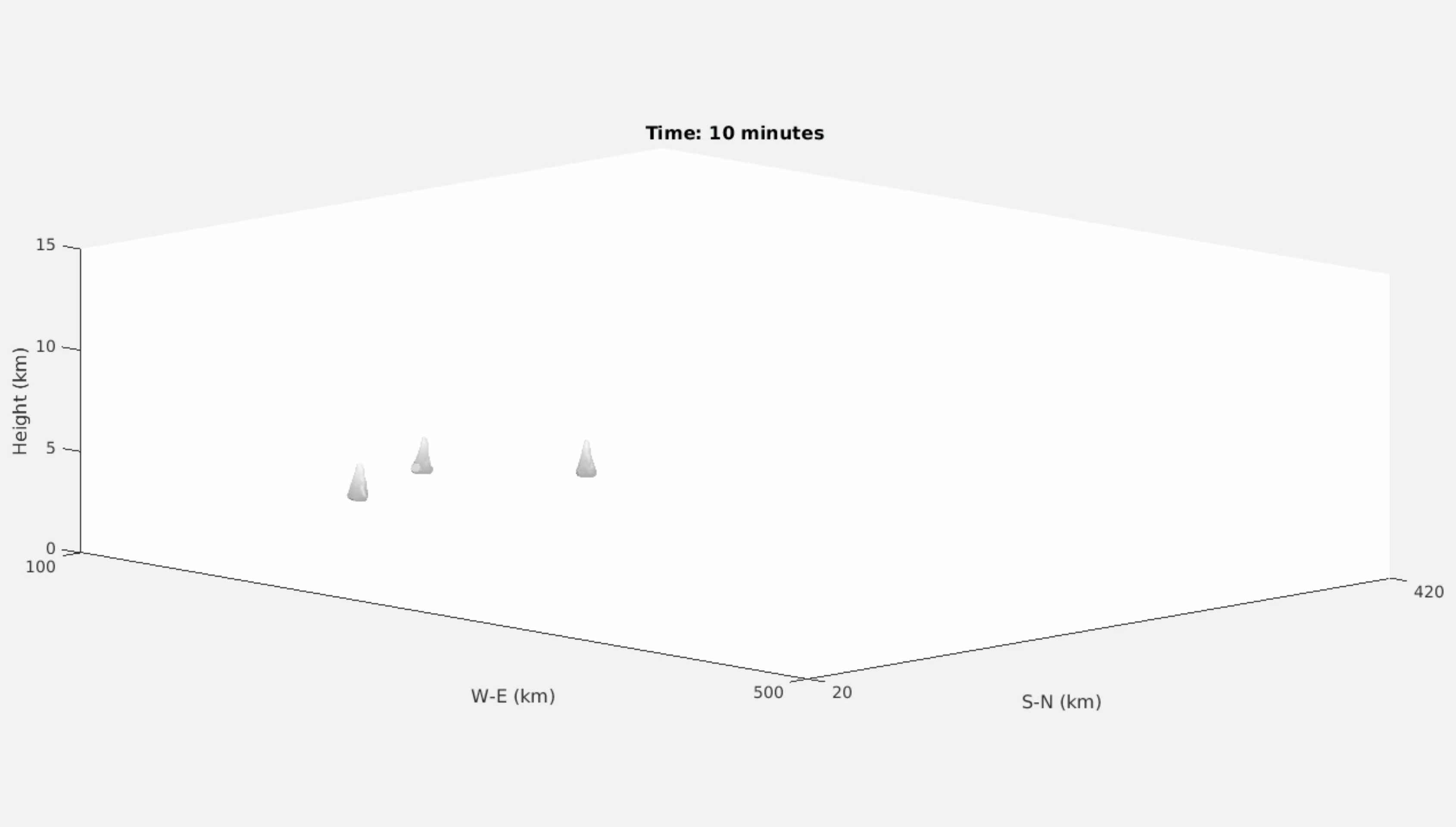
- Modified Weismann and Klemp 1983 idealized sounding
  - Unidirectional shear from 0 to 10, 20 or 30 m s^{-1} over 5 km
  - Shear favors organization of the convection into a squall line
- 512 km x 512 km doubly periodic horizontal domain
  - Facilitates spectral analysis
  - 1 km horizontal, 40 to 500 m vertical grid spacing
  - Surface friction, but no surface heat fluxes
- Coriolis force neglected



Ensemble strategy

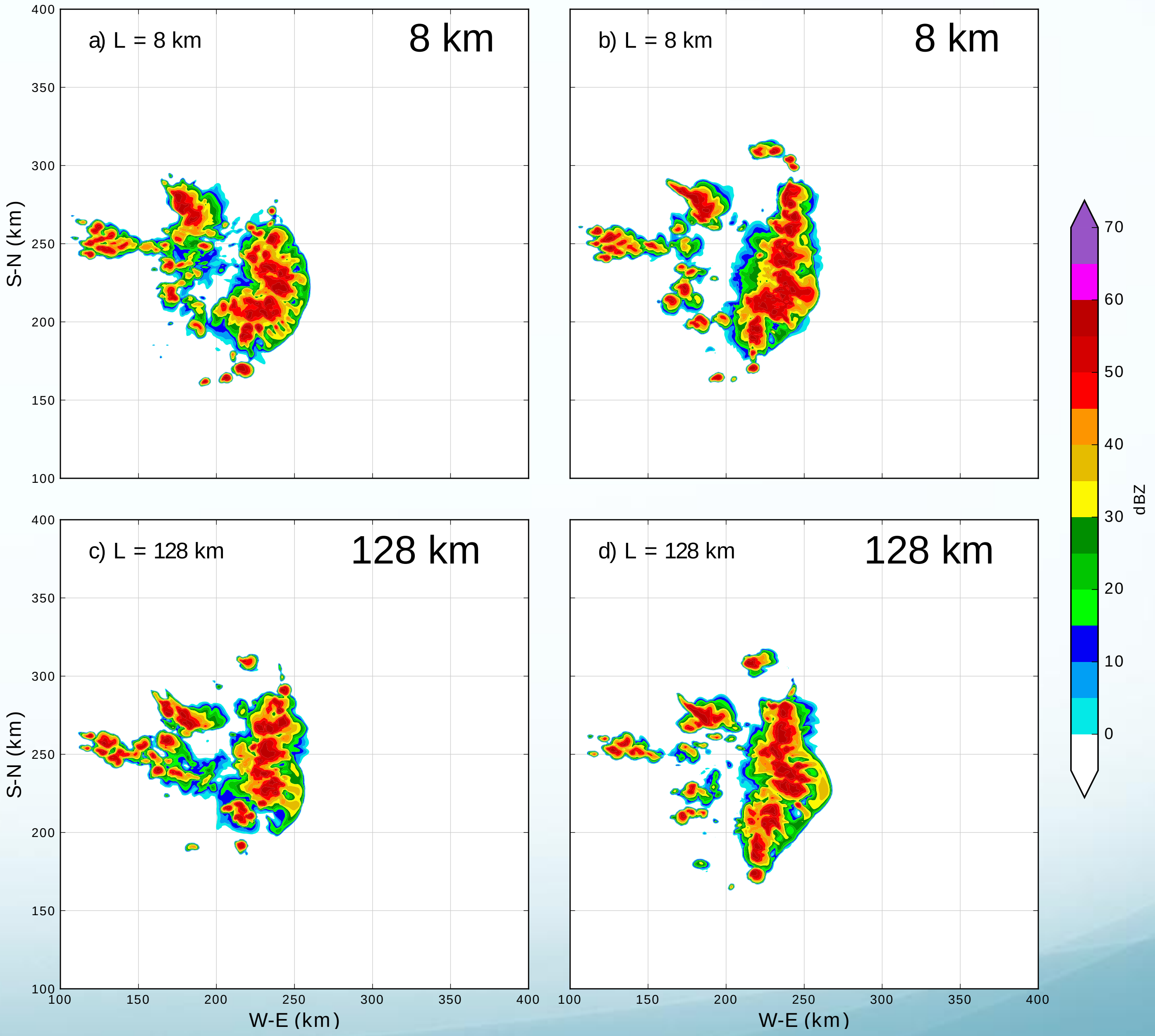
- All members initialized with 3 identical 2 K warm bubbles in the *same* location
- *Different* background perturbations among members in the near-surface *moisture* field
  - Monochromatic square wave in horizontal, random phase
    - **Small-scale ensemble**: *wavelength* 8 km
    - **Large-scale ensemble**: *wavelength* 128 km
  - Perturbation amplitude of 0.1 g kg^{-1}
  - 1-km *e*-folding decay scale away from the surface
- Simulate for 6 hours

Evolution of one member: *w* and cloud



Variability among ensemble members

- Synthetic reflectivity (colors) and anvil-level cloud water (gray)
- T=5 hours
- 10 m s^{-1} shear



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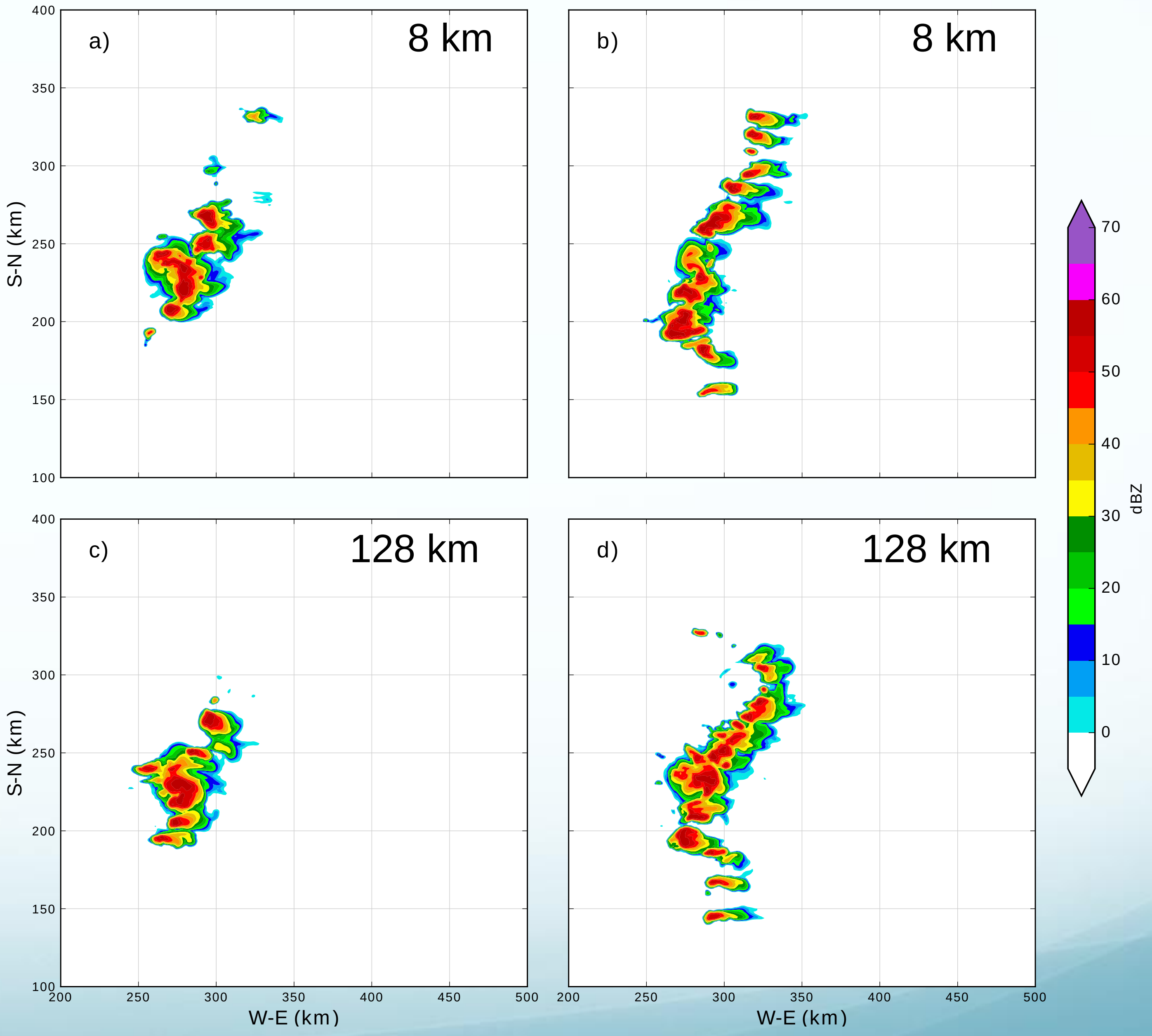
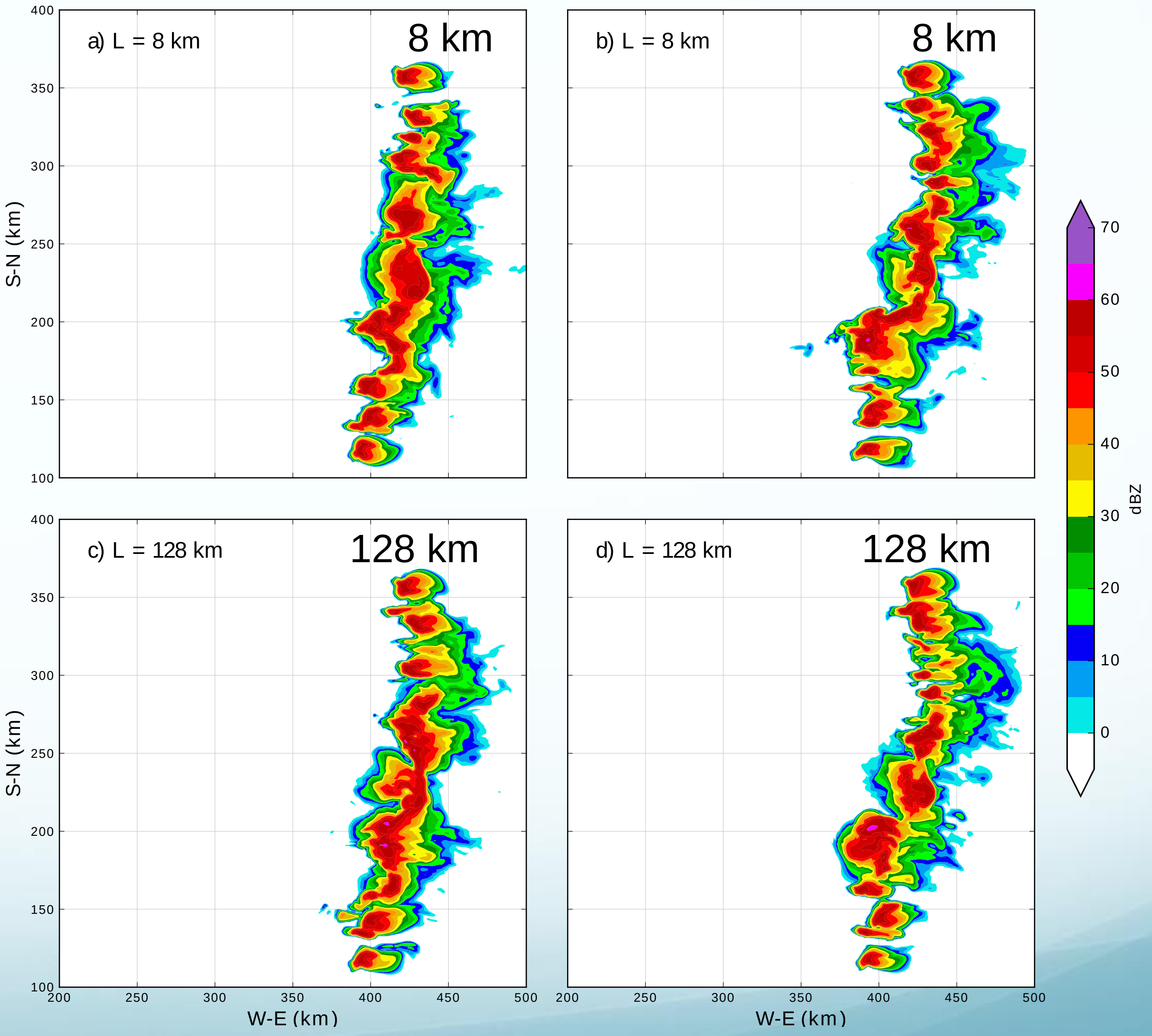
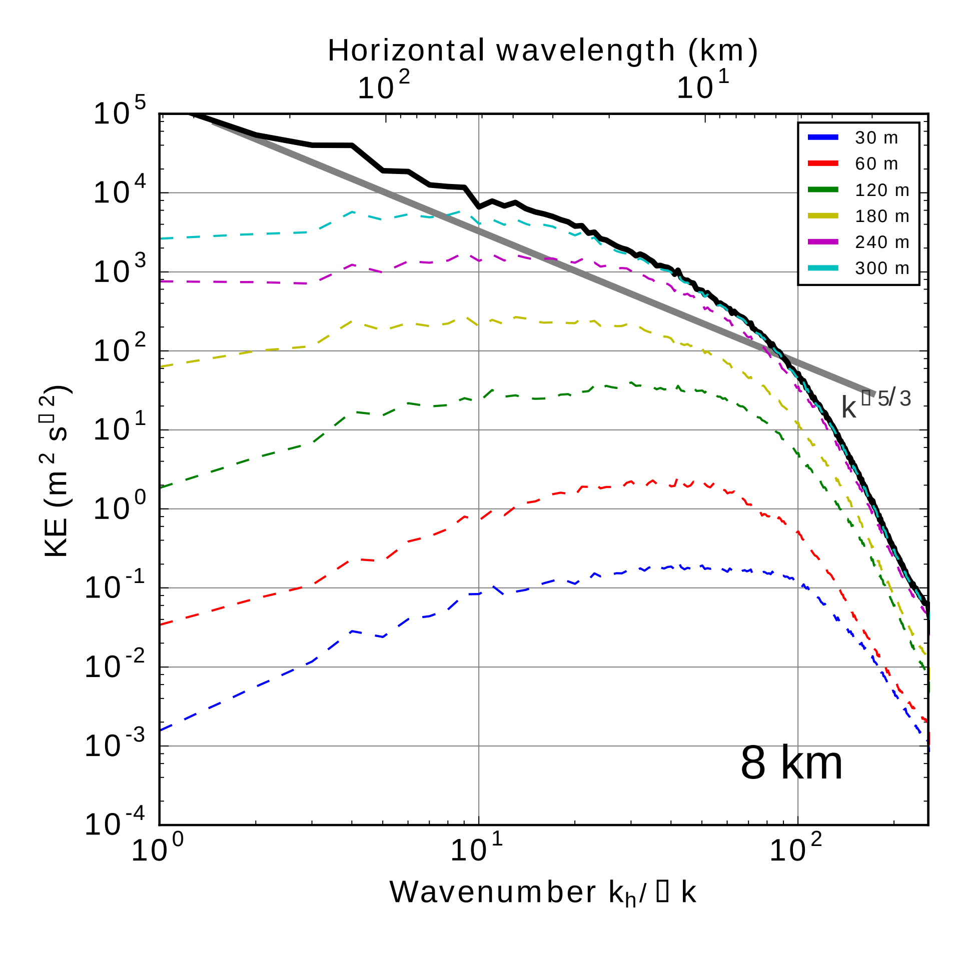


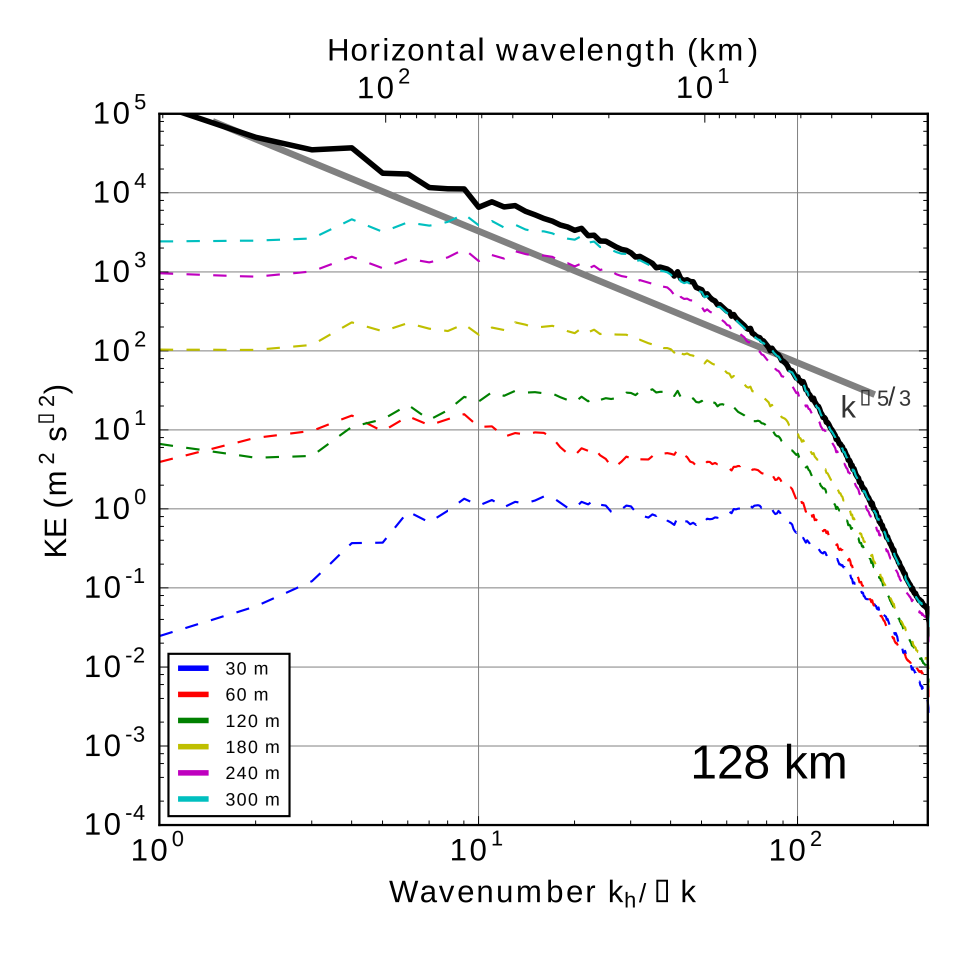
Variability among ensemble members

- Synthetic reflectivity and anvil-level cloud water
- T=5 hours
- 30 m s^{-1} shear



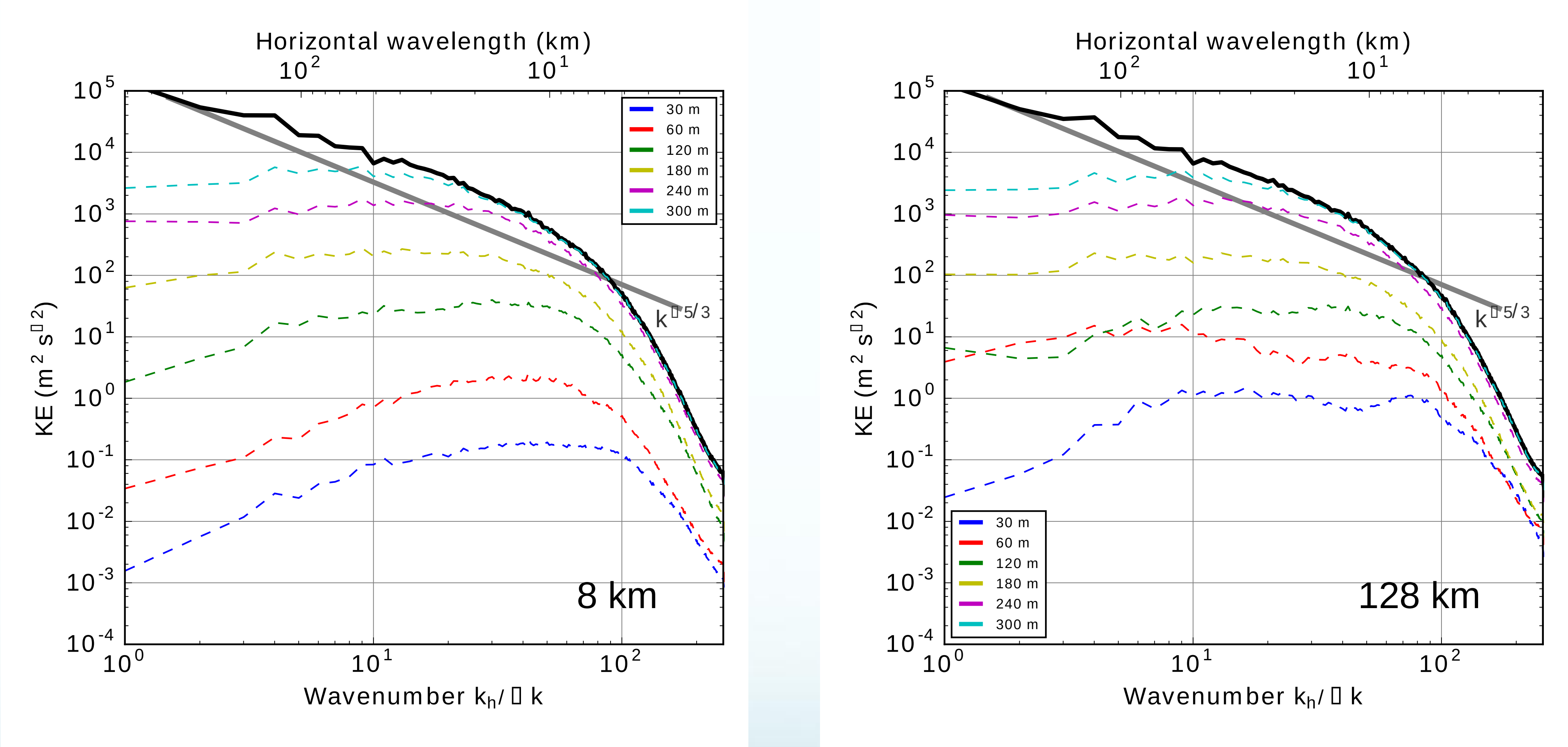
Ensemble mean KE' (error) and KE for 20 m s^{-1} shear





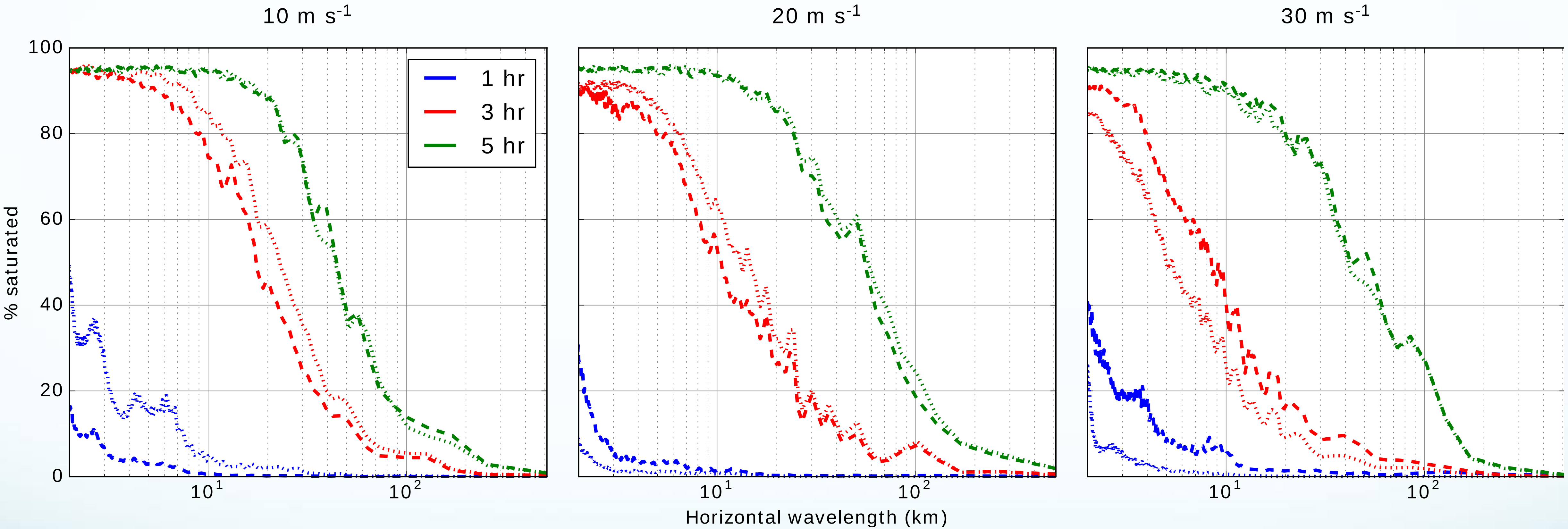
KE' dashed; black line is KE at 5 hr, gray line is observed background KE

Ensemble mean KE’ (error) and KE for 20 m s^{-1} shear



Error growth is up-amplitude, not an up-scale cascade!

Error saturation (KE'/KE) in layer 10 < *z* < 12 km



- 8-km ensemble dotted; 128 is dashed
- *By 5 hours the degree of saturation is independent of initial scale*
    - 8-km ensemble starts saturating faster for weak shear
    - 128-km ensemble starts faster for strong shear

Intrinsic Predictability

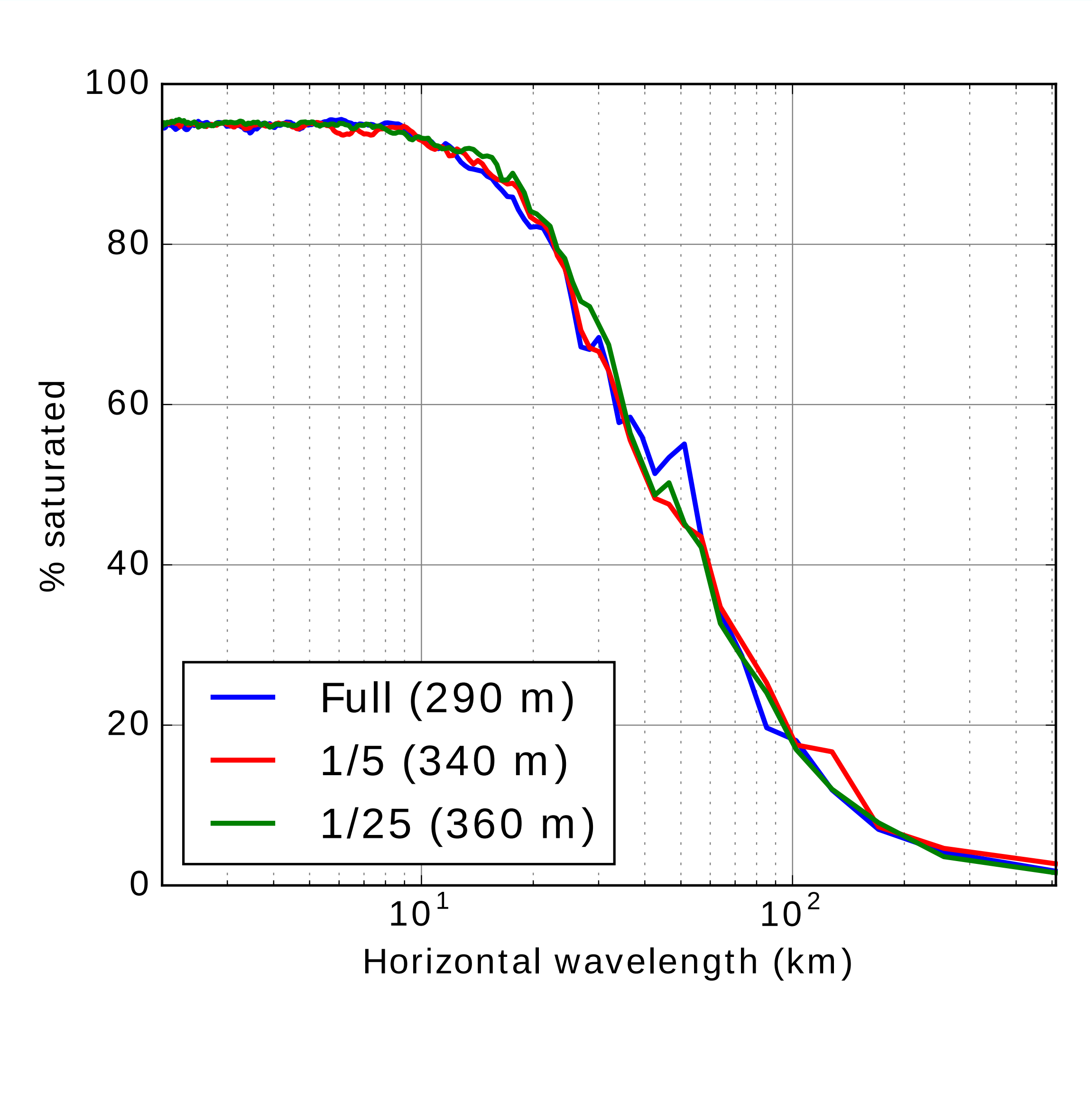
What happens if the initial errors are lower amplitude?

- Multiply the magnitude of the 0.1 g kg^{-1} moisture perturbations by factors of 1/5 and 1/25
- Repeat the 8 and 128-km ensemble simulations for the 20 m s^{-1} shear case

Similar predictability lead times from different initial amplitudes

Combined results for 8- and 128-km ensembles (40 members)

Gain 50 min, then just 20 min with the second reduction.

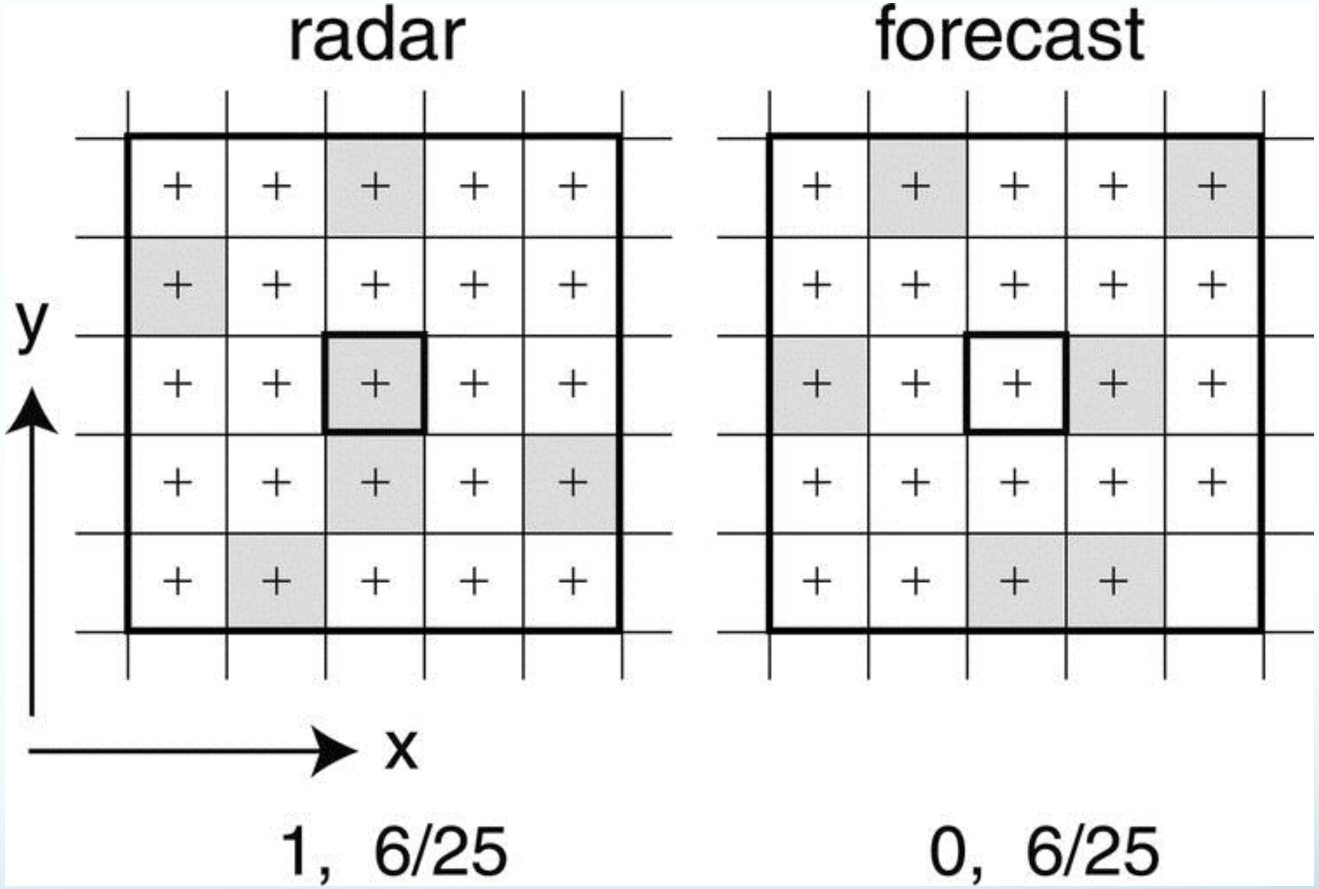


Would a forecaster view the three shear cases as having similar predictability?

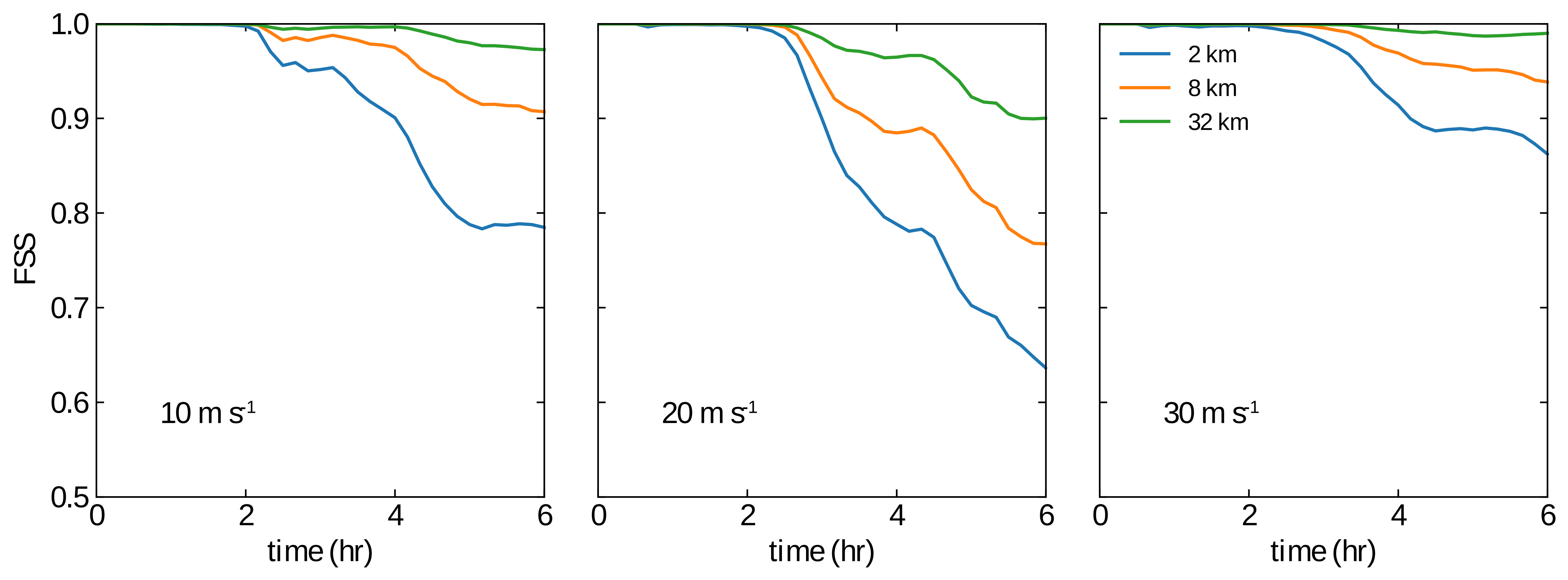
Another measure of predictability

Fractions skill score

(Roberts and Lean, *MWR,* 2008)



FSS as a function of environmental shear

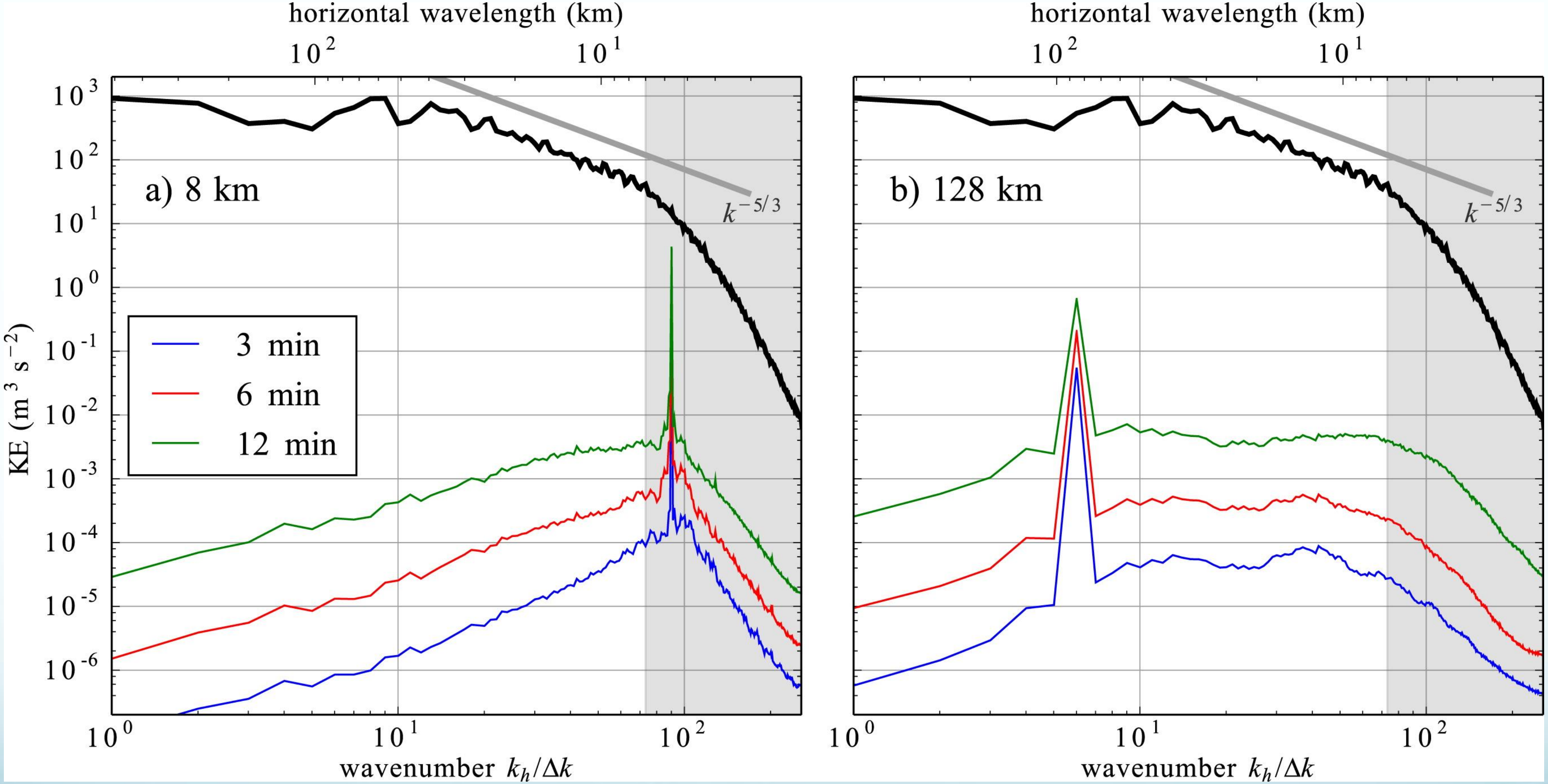


FSS score captures degree of variability suggested by visual inspection of the ensemble

After large-scale errors spread downscale, is there a butterfly effect connected with subsequent upscale propagation?

- Add perturbations after 1 hour when the background KE spectrum has largely developed.
- 20 m s^{-1} shear case
- Pertubation wavelengths
  - 8-km square wave has 2D wavelength 5.7 km
  - 128-km square wave has 2D wavelength 90 km

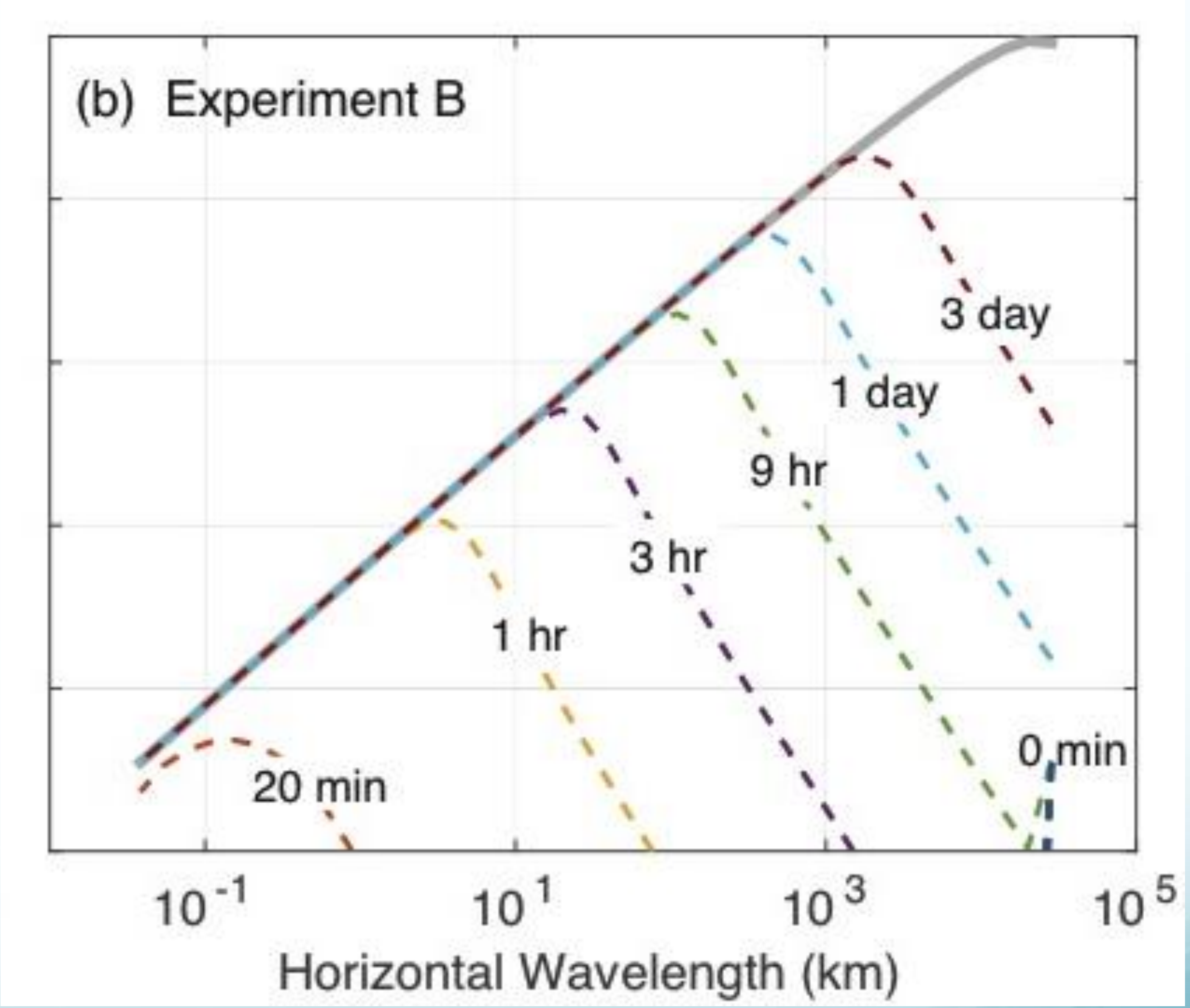
Error growth at early times



Growth is primarily up amplitude.

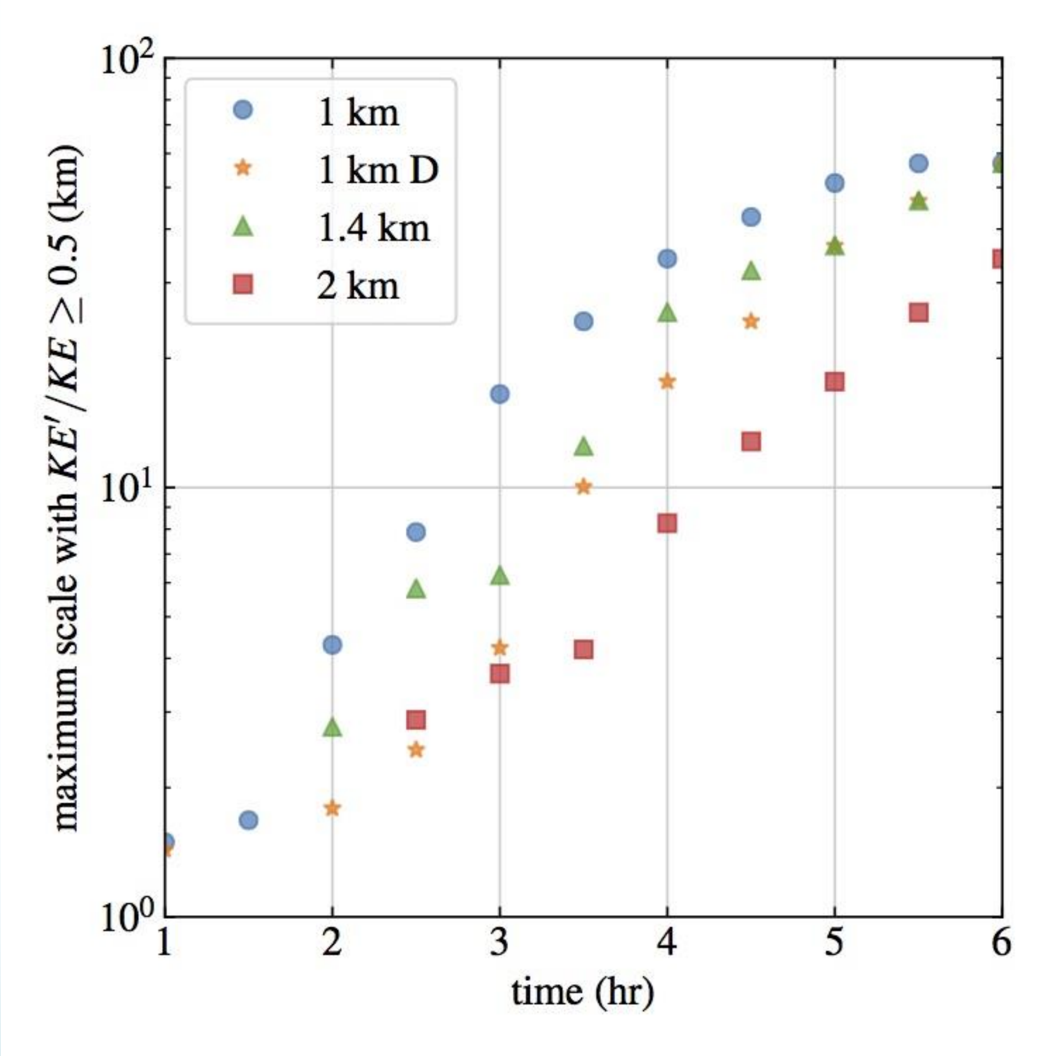
Do the small scales still play an important role?

Upscale error growth after error propagates downscale in Lorenz’s experiment B



Upscale evolution of relative errors

- 25 m s^{-1} environmental shear
- 1, 1.4 and 2 km grid spacing
- 1-km case with extra diffusion
- Initial errors are large scale (128 km square waves)
- Maximum wavelength for which KE'/KE > 0.5 plotted as a function of time
- Errors propagate more rapidly when small-scale motions are captured.



Implications for data assimilation: I

Parseval's relation

$$\int_S u^2(x)\,dx = \int_{-\infty}^{\infty} \hat{u}(k)\hat{u}^*(k)\,dk$$

KE in wavenumber band *(k_1,k_2)*

$$E(k_1, k_2) = \int_{k_1}^{k_2} \hat{u}(k)\hat{u}^*(k) + \hat{v}(k)\hat{v}^*(k)\,dk$$

Implications for data assimilation: II

- $k^{-5/3}$ KE spectrum

$$\frac{E(k_1,k_2)}{E(k_3,k_4)} = \frac{\lambda_1^{2/3} - \lambda_2^{2/3}}{\lambda_3^{2/3} - \lambda_4^{2/3}}$$

- Ratio of velocities in 200-400-km band to those in 2-4-km band is 0.21
- Which is the easier goal?
  - Reduce errors at 200-400 km below 10%
  - Reduce errors at 2-4 km below 50%

Conclusions

- The large scales exert significant control on small-scale weather (Anthes), but that control also includes the introduction of the many serious initial errors.
- Small relative errors in the largest scales at which the background KE spectrum follows a $k^{-5/3}$ slope (100--400 km) rapidly propagate down to the smallest resolved scale.
- Those small-scale errors subsequently propagate back upscale as if they had simply originated in the small scales.
    - Upscale growth is responsible for the finite limit to intrinsic predictability
- No easy way to diagnose the scale of the “original errors”.

References

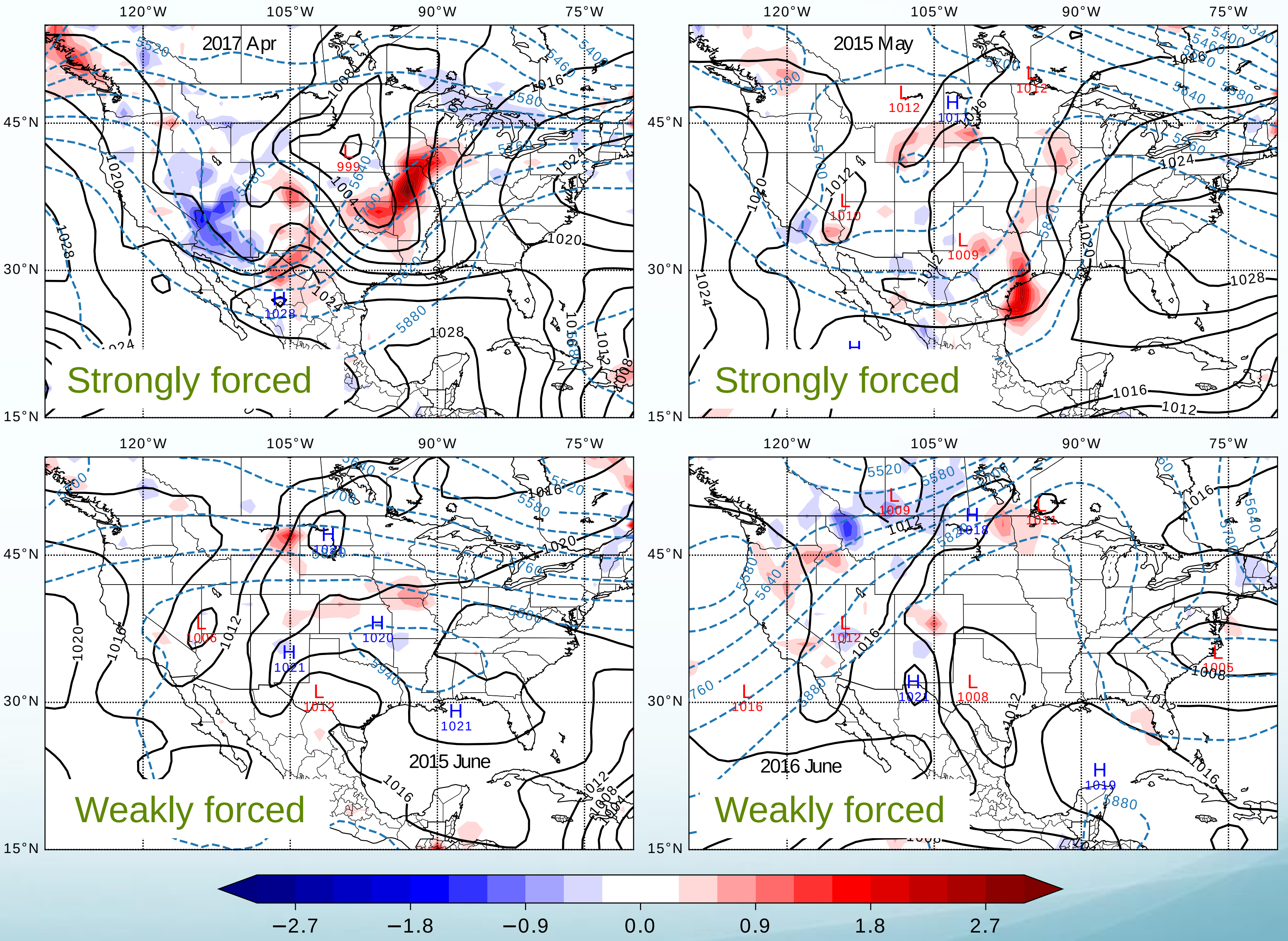
Weyn, J.A.. and D.R. Durran, 2017: The dependence of the predictability of mesoscale convective systems on the horizontal scale and amplitude of initial errors in idealized simulations. *J. Atmos. Sci.***,** **74**, 2191-2210.

Durran, D.R. and J.A. Weyn, 2016: Thunderstorms don't get butterflies. *Bull. Amer. Meteor. Soc.*, **97,** 237-243.

Durran, D.R. and M.A. Gingrich, 2014: Atmospheric Predictability: Why Butterflies Are Not of Practical Importance. *J. Atmos. Sci.*, **71,** 2476-2488.

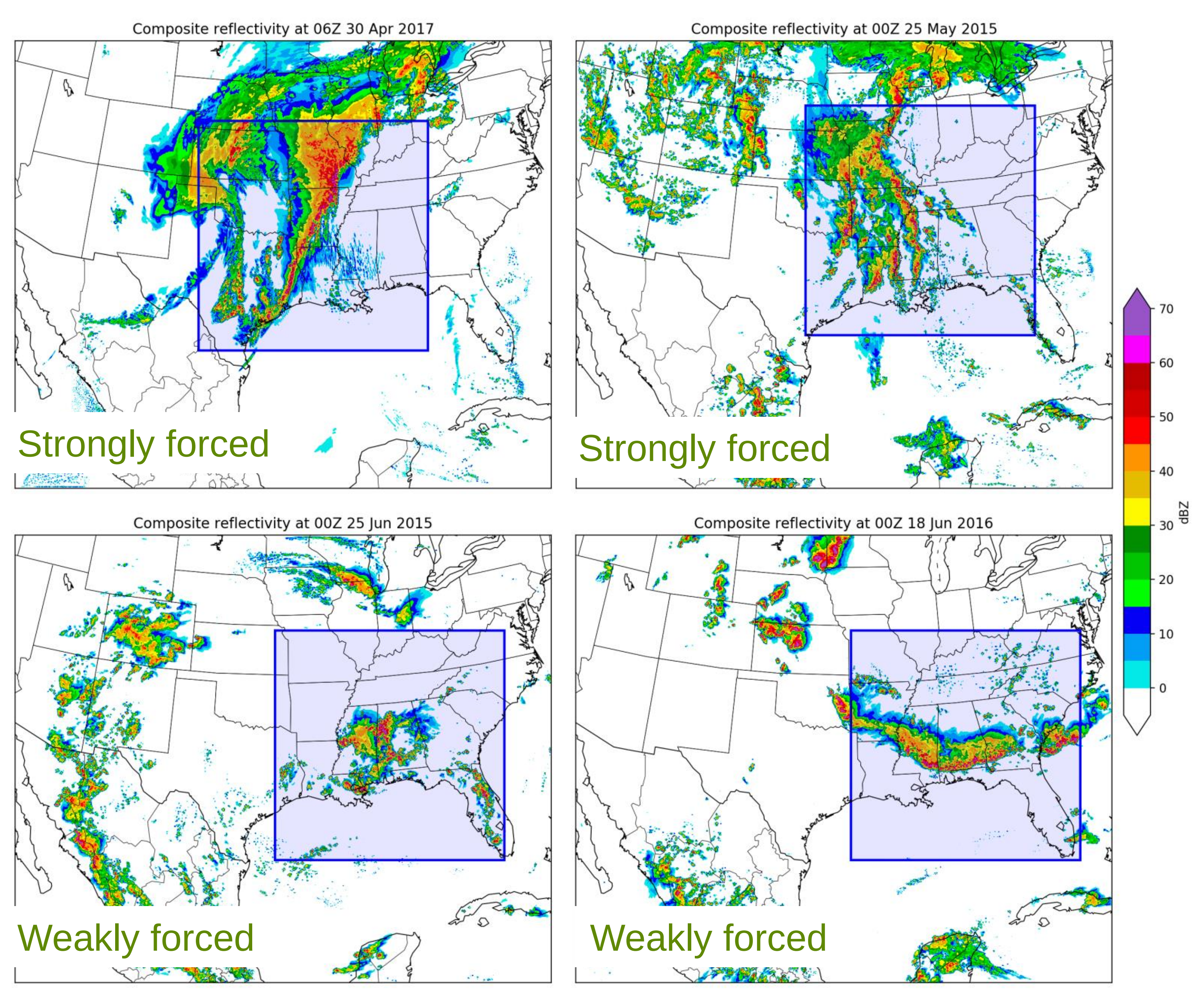
Case studies

- Sea-level pressure
- 500 hPa heights
- 500 hPa vertical velocity (contours)



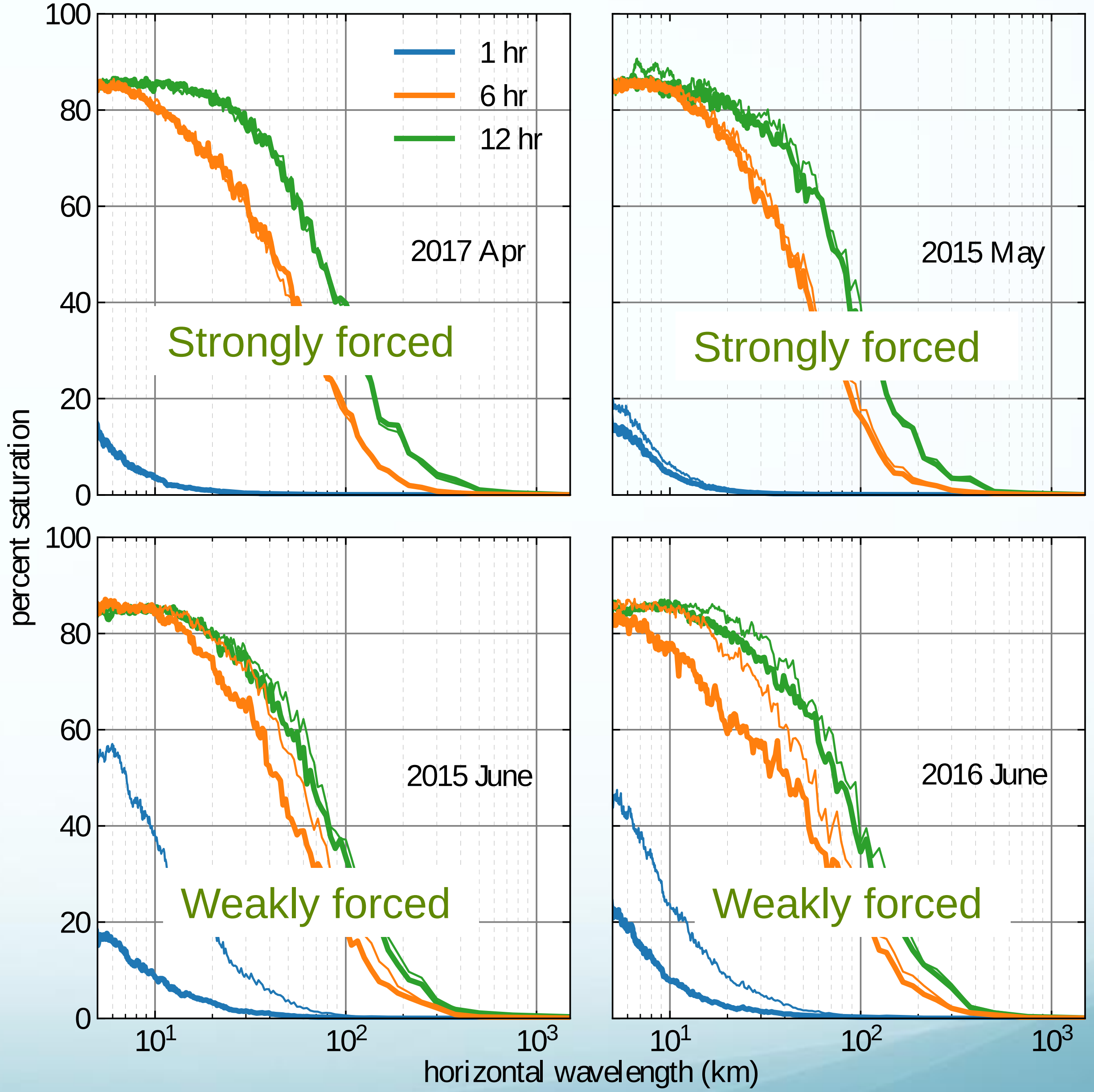
Control Simulations

- Simulated composite reflectivity
- 6 other ensemble members
- 12 hours after initialization from GFS
- 20 or 200-km square wave moisture perturbations are introduced at hour 6
- 2.5 km horiz. resolution



Error saturation (KE'/KE) in layer 10 < *z* < 12 km

- Similar errors at 12 hr in all cases
- Small-scale errors produce more saturation at 6 hr in the weakly forced cases
  - More variation in CI



FSS: Case Studies

As in the KE’/KE saturation plots, in weakly forced cases:

- 20-km perturbations lead to larger early-time errors than 200-km perturbations.