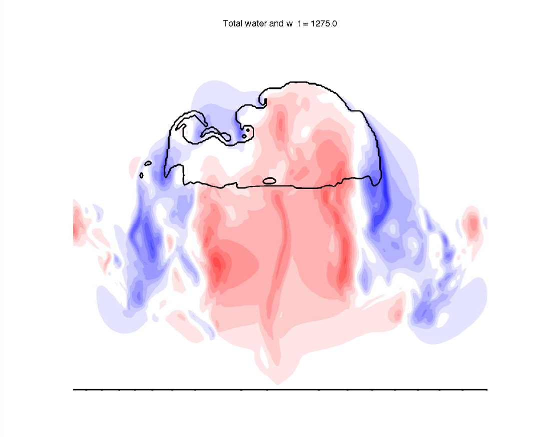
A ‘multi-fluid’ approach for the representation of convection



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ParaCon

Outline

- Some limitations of current convection schemes.
- Conditional filtering: the idea; the equations.
- Are these sensible equations? Conservation, normal modes, and instability.
- Relation to current approaches.
- Application to a single-column model of the dry convective boundary layer.

A typical mass flux convection scheme

Operates over a number of model levels in a single grid column, comprising:

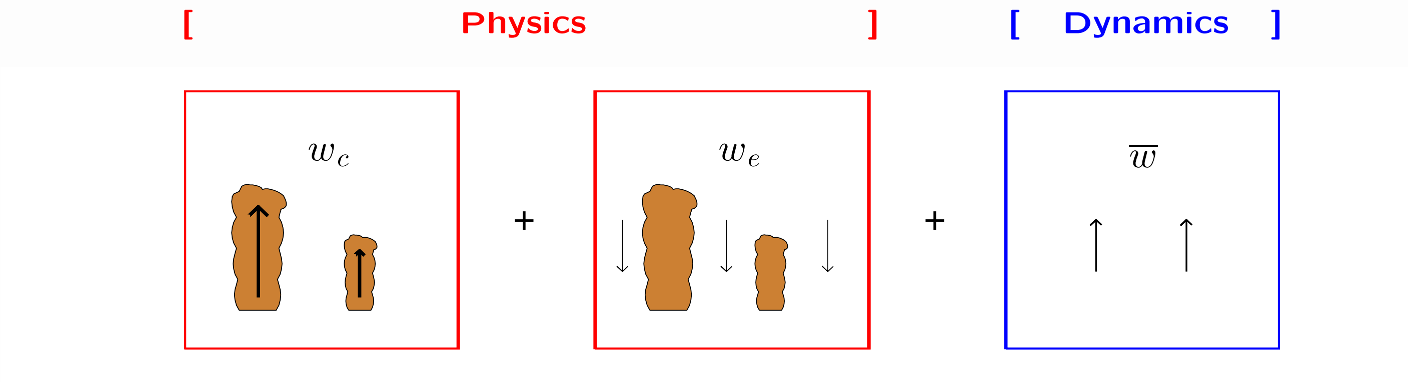
- Trigger and ‘closure’ assumptions to determine M at cloud base

- A steady entraining plume model $\partial M/\partial z = E - D$ etc. for the properties of updrafts M, w, η, q, ...

- Source terms for the grid-scale fields

Some limitations of typical mass flux convection schemes

1. Compensating subsidence is parameterized in the same grid column



2. There is no direct dynamical memory of the state of convection

3. In order to propagate horizontally convection must switch off and reform

4. The plume equations are generally not consistent with the equations used in the dynamical core

5. The grey zone problem: how do we switch off convection as $\Delta x \sim L_{\mathrm{conv}}$?

6. The closure problem: what controls the strength of convection?

Can we formulate governing equations that put convection and large scale dynamics on a more equal footing to obtain better physics-dynamics coupling?

Can we use the dynamical core to handle more of the convective dynamics in order to address some of the limitations mentioned?

Filtering

We should properly think of numerical models as solving filtered equations:

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u}\phi) = S$$

becomes

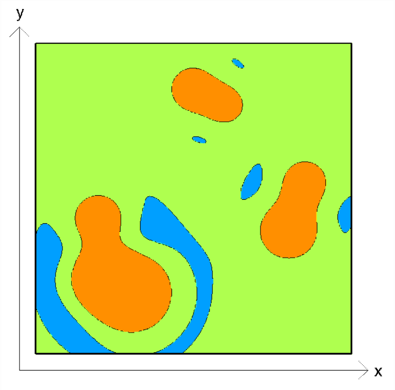
$$\frac{\partial \overline{\phi}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}\phi}) = \overline{S}$$

and then

$$\frac{\partial \overline{\phi}}{\partial t} + \nabla \cdot (\overline{\mathbf{u}}\overline{\phi}) = \overline{S} + \nabla \cdot (\overline{\mathbf{u}}\overline{\phi} - \overline{\mathbf{u}\phi}) = \overline{S} - \nabla \cdot \mathbf{F}^{\phi}_{\mathrm{SF}}$$

Conditional filtering

Extend this idea by first labelling the fluid with two (or more) Lagrangian indicator functions I_1, I_2, to pick out coherent structures...,
then applying the filter



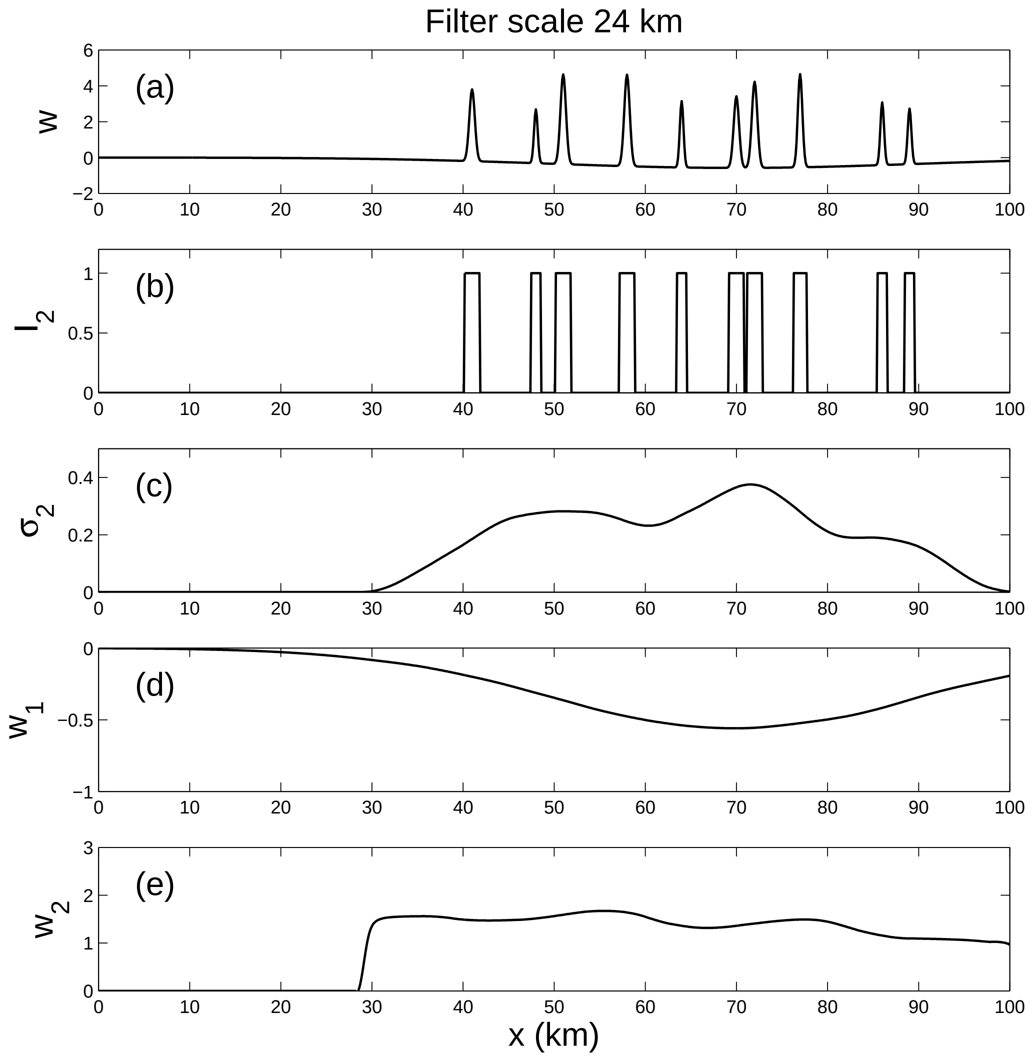
$$\frac{\partial}{\partial t}(\sigma_i \rho_i) + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) = 0$$

$$\frac{D_i \eta_i}{Dt} = -\frac{1}{\sigma_i \rho_i} \nabla . \mathbf{F}_{\mathrm{SF}}^{\eta_i}$$

$$\frac{D_i \mathbf{u}_i}{Dt} + \frac{1}{\rho_i} \nabla \overline{p} + \nabla \Phi = -\frac{1}{\sigma_i \rho_i} \left\{ \nabla \cdot \mathsf{F}_{\mathrm{SF}}^{\mathbf{u}_i} + \mathbf{b}_i + \sum_j \mathbf{d}_{i\,j} \right\}$$

Note there is only one pressure $\overline{p}$.

One-dimensional example:



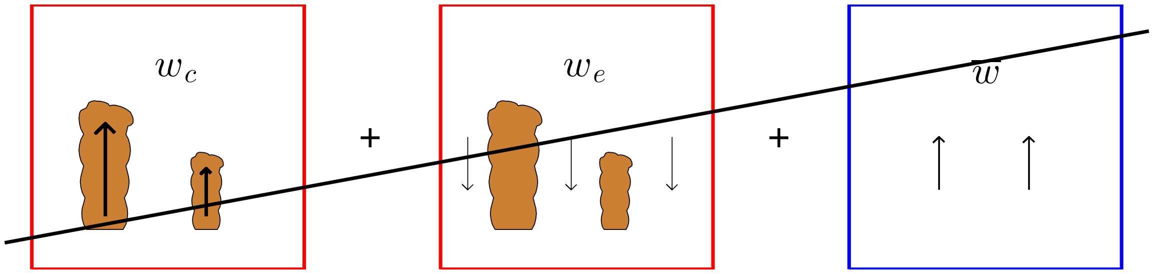
We must allow fluid to be relabelled, corresponding to entrainment, detrainment, and cloud base mass flux.

$$
\begin{aligned}
\frac{\partial}{\partial t}(\sigma_i \rho_i) + \nabla \cdot (\sigma_i \rho_i \mathbf{u}_i) &= \sum_{j \neq i} (\mathcal{M}_{ij} - \mathcal{M}_{ji}) . \\
\frac{D_i \eta_i}{Dt} &= \frac{1}{\sigma_i \rho_i} \left[\sum_{j \neq i} \{ \mathcal{M}_{ij} (\hat{\eta}_{ij} - \eta_i) - \mathcal{M}_{ji} (\hat{\eta}_{ji} - \eta_i) \} - \nabla . \mathbf{F}_{\mathrm{SF}}^{\eta_i} \right] . \\
\frac{D_i \mathbf{u}_i}{Dt} + \frac{1}{\rho_i} \nabla \overline{p} + \nabla \Phi &= \frac{1}{\sigma_i \rho_i} \left[\sum_{j \neq i} \{ \mathcal{M}_{ij} (\hat{\mathbf{u}}_{ij} - \mathbf{u}_i) - \mathcal{M}_{ji} (\hat{\mathbf{u}}_{ji} - \mathbf{u}_i) \} \right. \\
&\qquad \left. - \nabla \cdot \mathsf{F}_{\mathrm{SF}}^{\mathbf{u}_i} - \mathbf{b}_i - \sum_j \mathbf{d}_{i\,j} \right] .
\end{aligned}
$$

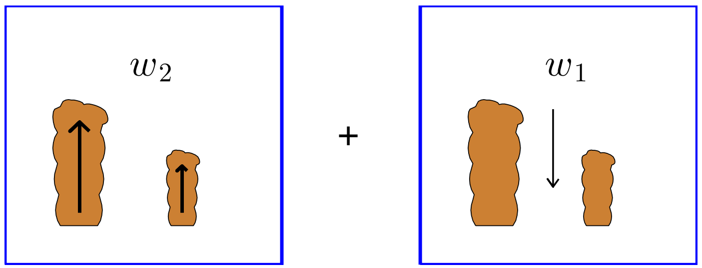
LHS resolved, RHS parameterized.

Better coupling of convection to large-scale dynamics?

[Physics] [Dynamics]



[Dynamics]



Are these sensible equations?

- Numbers of equations and unknowns
- Conservation properties
- Normal modes
- "Ill-posedness" / instability

Conservation properties

- The equations conserve mass, momentum, and $\int \rho\eta \, dV$.

- In the absence of the RHS, the filter-scale energy and the potential vorticity are conserved.

- In the presence of the RHS, the sink of filter-scale energy can be diagnosed and used as a source of subfilter-scale TKE.

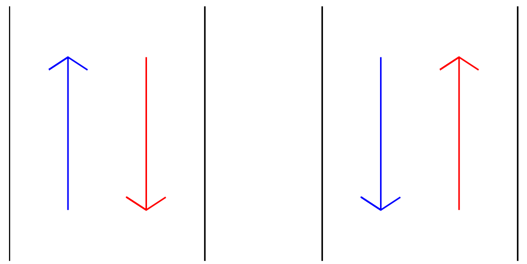
- In the absence of the RHS, we can derive the equations from a Lagrangian using Hamilton's principle.

Normal modes

Omit RHS, and linearize about a horizontally uniform isothermal state of rest.

We find the usual ‘single-fluid’ acoustic, gravity, and Rossby modes.

We also find



- multi-fluid gravity modes: $\omega^2 = N^2$, $p' = 0$, $\mathbf{c}_g = 0$, motion is purely vertical with $\sigma_1 \rho_1 w_1 = -\sigma_2 \rho_2 w_2$.

- multi-fluid inertial modes: $\omega^2 \approx f^2$, $p' = 0$, $\mathbf{c}_g \approx 0$, motion is purely horizontal with $\sigma_1 \rho_1 \mathbf{u}_1 = -\sigma_2 \rho_2 \mathbf{u}_2$.

“Ill-posedness”

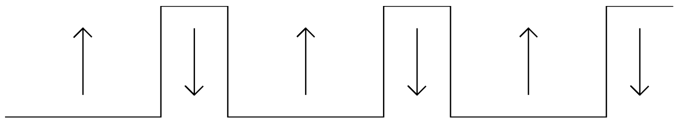
Writing the 1D equations (without RHS) as

$$\mathbf{\Psi}_t + \mathbf{A}\mathbf{\Psi}_x = 0$$

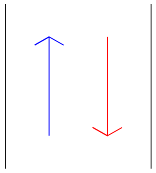
we find that some eigenvalues of $\mathbf{A}$ can be complex:
the characteristic speeds are complex.

Instability

2D basic state unstable to KH instability



1D two-fluid representation



$$\frac{\omega}{m} = \begin{cases} \left(\sigma_1^0 W_1 + \sigma_2^0 W_2\right) \pm \mathrm{i}\sqrt{\sigma_1^0 \sigma_2^0}\,(W_2 - W_1) \\ \left(\sigma_2^0 W_1 + \sigma_1^0 W_2\right) \pm \mathrm{i}\sqrt{\sigma_1^0 \sigma_2^0}\,(W_2 - W_1). \end{cases}$$

$$\frac{\omega}{m} = \left(\sigma_2^0 W_1 + \sigma_1^0 W_2\right) \pm \mathrm{i}\sqrt{\sigma_1^0 \sigma_2^0}\,(W_2 - W_1).$$

Growth rate $\propto m\sqrt{\sigma_1^0 \sigma_2^0}\,|W_2 - W_1|$

A realistic eddy diffusion of w (damping rate $\propto m^2$) can suppress the instability

Relation to current approaches - boundary layer

$\nabla . \mathbf{F}_{\mathrm{SF}}^{\eta_i}$ and $\nabla \cdot \mathsf{F}_{\mathrm{SF}}^{\mathbf{u}_i}$ represent *local* turbulent transports and can be modelled, for example, by an eddy diffusivity as in most boundary layer schemes.

Relation to current approaches - mass flux scheme

Assume steady, neglect horizontal fluxes, assume $\sigma_2 << 1$, neglect $\mathbf{F}_{\mathrm{SF}}^{\eta_i}$ and $\mathbf{F}_{\mathrm{SF}}^{\mathbf{u}_i}$ in updrafts.

This leaves typical mass flux equations for updraft properties

$$\frac{\partial M}{\partial z} = \mathcal{M}_{21} - \mathcal{M}_{12} = E - D, \qquad\qquad M = \sigma_2 \rho_2 w_2$$

$$M \frac{\partial \eta_2}{\partial z} = E(\eta_1 - \eta_2),$$

and the effective source for the filter-scale mean is $-\partial F_{\mathrm{CF}}^{\eta} / \partial z$ where

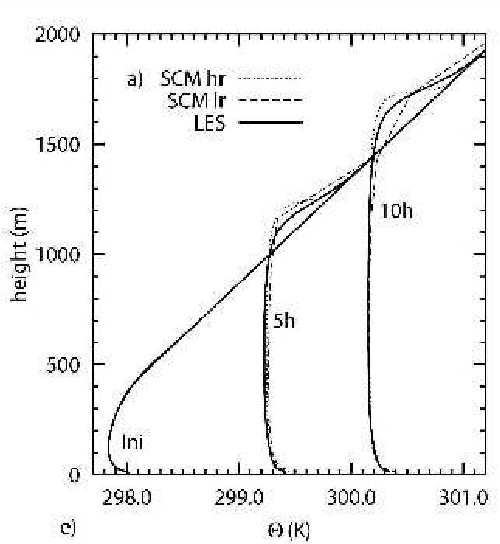
$$F_{\mathrm{CF}}^{\eta} = \sigma_2 \rho_2 w_2 (\eta_2 - \overline{\eta}^*) = M(\eta_2 - \overline{\eta}^*)$$

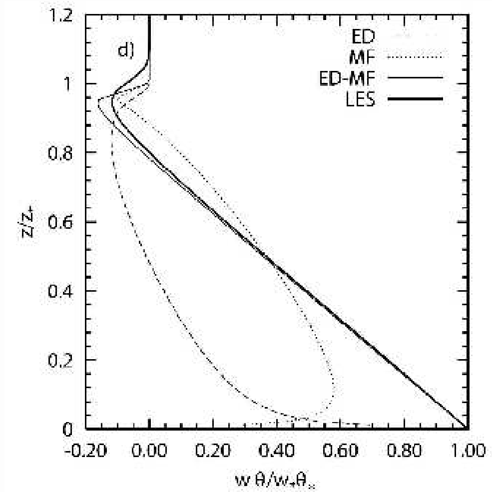
Single column dry CBL

Inspired by EDMF (Siebesma et al. 2007)

$\overline{w'\phi'} = -K\frac{\partial\overline{\phi}}{\partial z} + M(\phi_2 - \overline{\phi})$

Use steady entraining plume model for w_2 to get z_* and for θ_2 to get $M(\theta_2 - \overline{\theta})$.





Two-fluid column equations

For $i = 1, 2$ ...

$$\frac{\partial}{\partial t}(\sigma_i \rho_i) + \frac{\partial}{\partial z}(\sigma_i \rho_i w_i) = \mathcal{M}_{ij} - \mathcal{M}_{ji}$$

$\mathcal{M}_{ij}$ is rate of relabelling fluid j as fluid i.

$$\sigma_i \rho_i \frac{D_i \theta_i}{Dt} = \frac{\partial}{\partial z}\left(\sigma_i \rho_i K_i \frac{\partial \theta_i}{\partial z}\right) + \mathcal{M}_{ij}(\hat{\theta}_{ij} - \theta_i) - \mathcal{M}_{ji}(\hat{\theta}_{ji} - \theta_i)$$

K_i from Siebesma et al.,

$$\frac{D_i w_i}{Dt} + C_p \theta_i \frac{\partial \Pi}{\partial z} + \frac{\partial \Phi}{\partial z} = \frac{1}{\sigma_i \rho_i}\left\{-\mathcal{P}_i + \mathcal{M}_{ij}(\hat{w}_{ij} - w_i) - \mathcal{M}_{ji}(\hat{w}_{ji} - w_i)\right\}$$

Plume base and Diffusion profile

At the bottom model level mass and θ are conservatively 'moved' between fluid 1 and fluid 2 to leave

$$\sigma_2(z_1) = 0.12 \qquad\qquad \theta_2(z_1) = \theta_1(z_1) + 1.5\frac{Q_*}{w_{\rm sd}(z_1)}$$

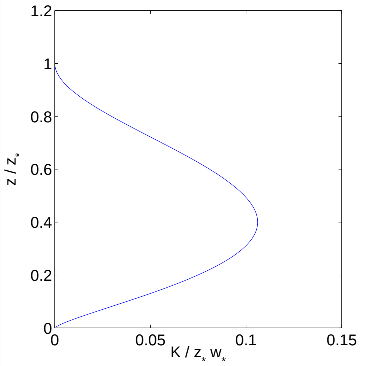
where

$$w_{\rm sd}(z) = 1.3w_*\left[\left(\frac{u_*}{w_*}\right)^3 + 0.6\frac{z}{z_*}\right]^{1/3}\left(1-\frac{z}{z_*}\right)^{1/2}$$

with $u_* = 0$.

The diffusion coefficient for θ and w is given by

$$K = z_* w_* k\left[\left(\frac{u_*}{w_*}\right)^3 + 39k\frac{z}{z_*}\right]^{1/3}\frac{z}{z_*}\left(1-\frac{z}{z_*}\right)^2$$



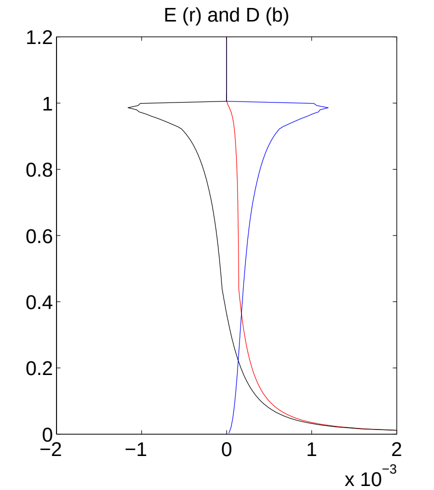
Entrainment

Write

$$\mathcal{M}_{21} = \frac{\sigma_1 \sigma_2 \rho_{21}}{\tau_{21}}$$

where

$$\frac{1}{\tau_{21}} = \max\left(\frac{w_*}{z_*}, \frac{c_e w_2}{z}\right)$$



Detrainment

$$\mathcal{M}_{12} = \frac{\sigma_2 \sigma_1 \rho_{12}}{\tau_{12}}$$

where

$$\frac{1}{\tau_{12}} = \frac{c_d w_2}{z_* - z} + \max\left(0, -\frac{2b}{|w_2|}\right)$$

The second contribution helps avoid a spike in σ_2.

Solution method: ENDGame-like SISL scheme

- SLICE advection scheme for conservative mass transport

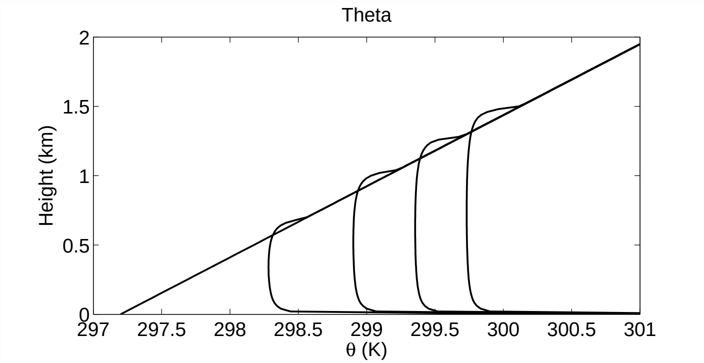
- Simple conservation fixer for transport of θ

- Extended solver to handle implicit treatment of diffusion terms $\Rightarrow$ hepta-diagonal linear solve at each solver iteration

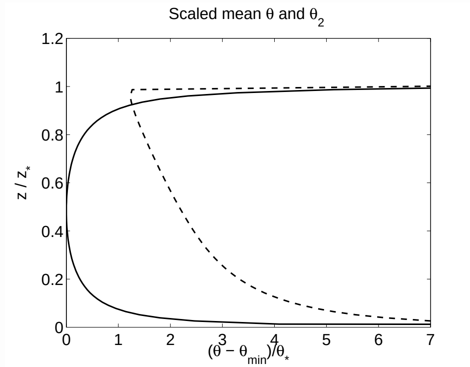
- Some fields are singular or near singular at the ground and at the boundary layer top - solution is sensitive to numerics

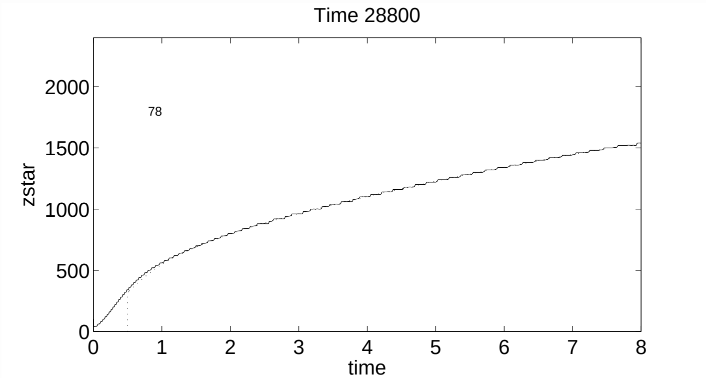
Some example results

$\Delta z = 20\,\mathrm{m}$, $\Delta t = 6\,\mathrm{s}$



$\overline{\theta}^*$ profiles at 1.5, 3.5, 5.5 and 7.5 hrs.

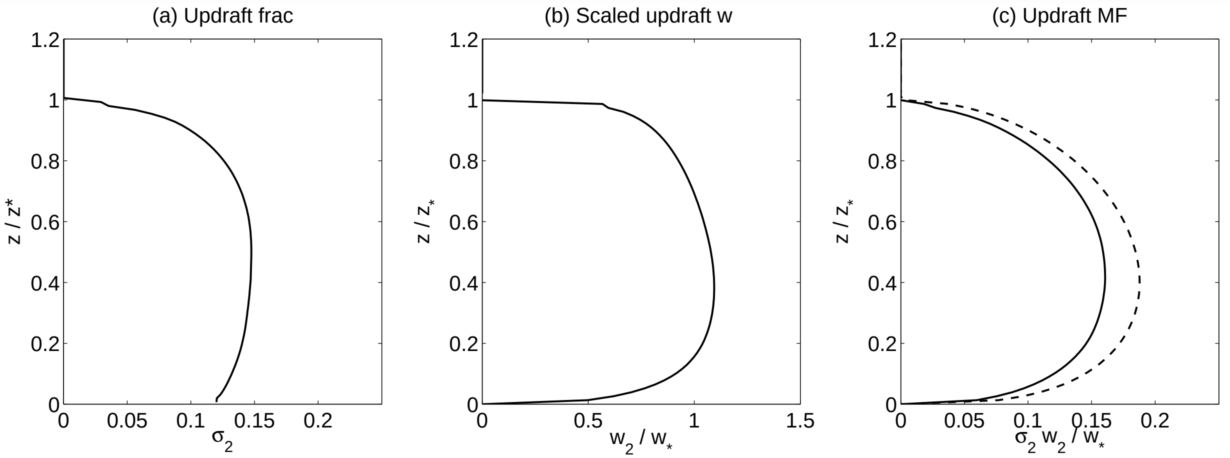


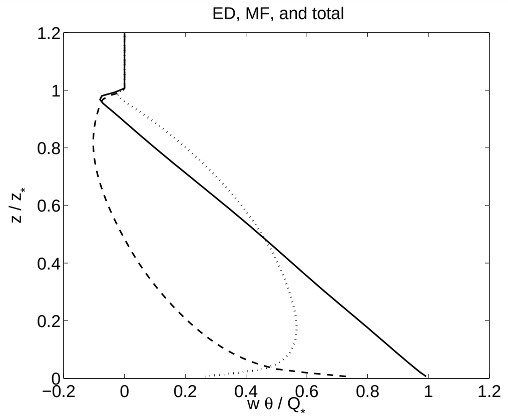


Time series of z_*

Scaled profiles of $\overline{\theta}^*$ and θ_2

Updraft fraction   vertical velocity   mass flux

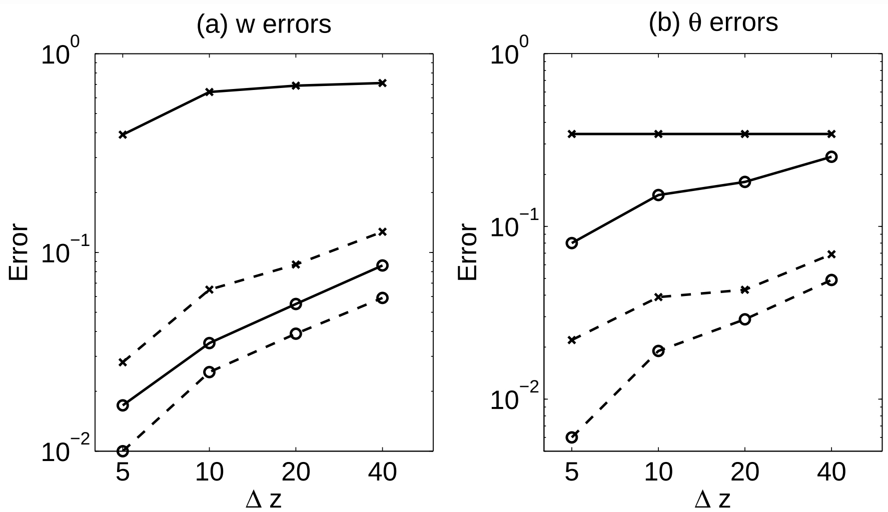




Contributions to θ flux profile

Convergence test

Δz (m)	Δt (s)	z_* (m)	Minimum θ_2 (K)	Maximum w_2 (ms^{-1})	Maximum $\sigma_2 w_2$ (ms^{-1})	$z(\min \overline{\theta}^*)$ (m)	t(ent flx) (hr)
2.5	0.75	1515	299.89	1.62	0.301	412.5	0.92
5	1.5	1513	299.89	1.61	0.291	470	0.75
10	3	1501	299.88	1.59	0.276	540	≈0.6
20	6	1495	299.87	1.58	0.257	620	≈0.4
40	12	1471	299.85	1.53	0.232	720	≈0.4



o - fluid 1

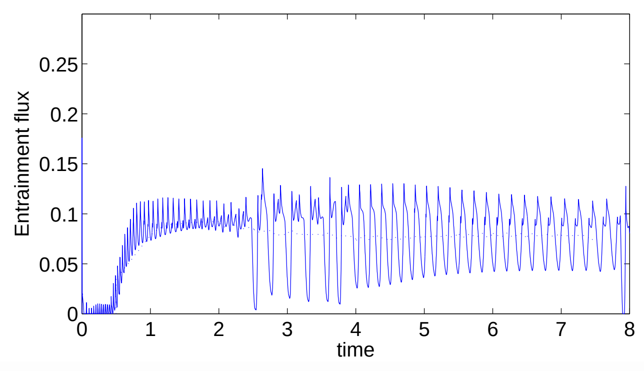
x - fluid 2

solid - L_∞

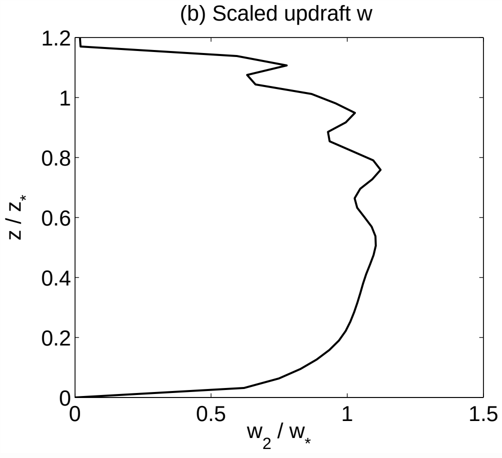
dashed - L_2

Some outstanding issues

• Singularities in the solution are problematic for the numerics; we hope to try a TKE-based approach for entrainment, detrainment, and diffusion.



• With large time steps we see an instability like a $2\Delta t$ numerically distorted acoustic mode, exacerbated by $\partial w_2/\partial z < 0$ in the upper boundary layer.



Next steps

- Include moisture
- Implement and test in 3D

Summary

Conditionally filtered equations provide a framework that encompasses traditional parameterizations for the boundary layer and convection while permitting generalization

Allowing the dynamical core to handle the dynamics of coherent structures has the potential to overcome some limitations of current approaches

The conditionally filtered equations have physically reasonable conservation and normal mode properties, but support a KH-like instability

A single-column dry CBL model shows that, with suitable parameterizations, the equations can be solved numerically and produce plausible solutions

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Any questions?